very well be on a higher indifference curve (curve II) than the nondiscriminatory position at R.

The superiority from the point of view of the profit-takers of the discriminatory situation over the nondiscriminatory situation depends on which group the profit-takers has chosen to discriminate against. Consideration of Figure IV shows that picking on too large a group or too small a group as the target for evasion will mean a failure to improve the position of the profit-takers over the position they could achieve in a nondiscriminatory system. If discrimination had no motive other than increasing the material welfare of the group which sets hiring policy, then whether discrimination in the form of occupational segregation appeared or not would depend on the availability of a well-defined "target" group of the right size. The fact that the target groups vary greatly throughout the world in the proportion of the labor force which they constitute would argue that a dis-

REFERENCES


The University of Chicago, which apparently represents the farthest-out western point of the intellectual territory included in the Eastern Economic Association, is well known as the home of the "Chicago School." To some at least of the members of the alleged school, the existence of the school is a figment of the establishment imagination; and there is some reason to think that even if Chicago itself did not exist, it would be necessary for the Ivy League schools to conjure up a "Chicago School," if only to frighten the little ones into obedience as they huddle around the fireplace on those long cold nights when the Republic-

ans are in power (or only in office), and to comfort their elders with the reassurance that political wisdom and social sophistication have insalubrious economic value.

Be that as it may, many economists have from time to time been puzzled over what type of educational process turns clean-cut college graduates indistinguishable from others of the same ilk and age group into Chicago scholars and doctors. The process is indeed a complex one, and in the view of some former students (even successful ones) not despicable without gross violation of the laws of liberal and openness. But an important part of it is the "Core Examination," in other words the money part of the qualifying examination for proceeding to the Ph.D.; it is so called because—and this may be one of the dis-

tinguishing characteristics of the so-called "Chicago School"—the Chicago department holds as an article of faith (or at least behaves

"as if" it were true) that economics has a hard core of central propositions in price theory and monetary theory, which hard-core the student must have digested before he can be allowed to proceed to acquire his professional doctoral qualifications. This note presents the monetary theory half of the Core Examination held in August 1973, together with notes on the answers provided by the Department members who graded the examination, for the information of readers who may be interested in knowing what goes on in a place that Carl Sandburg once called—of course, thinking of something she entirely—"hog-farter to the world." Readers of The Eastern Economic Journal may find particularly piquant the contrast between the flavor of these questions and answers, and Paul Samuelson's proof presented in the first issue of this Journal, in answer to an M.I.T. graduate micro examination question, that the landowner (capitalist) is never worth the cost of his market-organiz-

ing services to the farmers (working class).1

1 Paul A. Samuelson, "Is the Rent-Collecter Worthier of His Full Hire?" The Eastern Economic Journal, Vol. 1, No. 1 January 1976, pp. 7-20. The Samuelson analysis obviously confuses market efficiency with the transfer of land rent formerly enjoyed (disfiltered) by farmers to a landlord, in order to make its societally starting point. Each rent-collecter and tax collector pays most of the collection on to the owner. The rent collector, as collecter, will earn a wage equal to the value of his services in collecting rent, keeping the books, and so forth; this will be a deduction from the gross rent received by the landlord as owner of the land, a rent in return for which the landlord does nothing except to arrange, possibly through the collector, that the rent is paid to the highest bidder. The Samuelson proof is that the gains from efficient use of labor through land-renting are not worth the transfer of all rent to the landlord—a proposition which stated in this form is not surprising.

3-
Ph.D. Core Examination

Theory of Income, Employment, and the Price Level

Write the following information on the first page of your examination paper:

_____ Your CODE NUMBER and not your name
_____ Name of examination
_____ Date of examination

Write the following on each following page of your exam paper:

_____ Top left: code number
_____ Top right: number of page

WRITE ONLY ON ONE SIDE OF EACH PAGE. WRITE IN BLACK INK.

When you fold your paper at the end of the exam, write your code number on the back, and indicate the total number of pages.

I. (60 points).

Indicate whether you believe each of the following statements to be TRUE, FALSE, or UNCERTAIN. In each case write a few sentences explaining your answer. Your grade will be determined by your explanation.

1. The equilibrium interest rate on government bonds is higher, the higher is the equilibrium price-earnings ratio for shares of common stock.

2. A spontaneous rise in the expected rate of inflation will increase all interest rates by the same amount.

3. A balanced-budget multiplier as in excess of unity implies that the marginal value of public-sector output exceeds its marginal cost.

4. The turnover of bank deposits in the Province of Quebec (ratio of checks drawn to deposit balances) is substantially lower than that in the rest of Canada. This means that if the federal government spends money on public works in Quebec, the multiplier effect on Quebec incomes is small, because the money rapidly drains out of the Province.

5. If the services of money are an input to the production process and if money has no direct utility yield, then the long-run behavior of the capital-output ratio and per capitis income is independent of monetary policy.

II. (20 points).

A small number of countries do not have a central bank (or similar institution) and hence money created in some other country (call it country A) is used domestically as the circulating medium. In this situation:

(a) what are the determinants of the money supply?

(b) what is the relevance, if any, of the concept of high-powered money?

(c) what determines the internal price level?

(d) how would inflation in country A influence the price level in the country in question? Is your answer the same whether the rest of the world uses fixed or flexible exchange rates?

III. (20 points).

Discuss the welfare costs of inflation and in so doing be certain to cover the following points:

(a) costs of an anticipated vs. an unanticipated inflation;

(b) true social costs vs. those costs that would appear as a reduction in measured GNP;

(c) the relation between the inflation tax revenue and the welfare cost (cost of collection);

(d) the probable distribution of the welfare costs of a fully anticipated inflation between labor and capital.
IV. (10 points).

How would you explain a simultaneous rise in the real wage rate and in the rate of "unemployment" assuming a stable price level?

V. (20 points).

Assume a simple economy with a government, a central bank, and a commercial banking system, but no other financial intermediaries. The currency/deposit ratio is 3, the commercial banks' cash reserve ratio is 2, the central bank holds a ratio of 25 of gold to government debt, the gold reserve is $10 billion, total government debt is $150 billion, total real capital is $160 billion, and the public finances a quarter of its holdings of real capital by personal loans from commercial banks.

(i) Draw up a two-way table or a diagram showing the amounts of assets and liabilities of various kinds held or owned by the central bank, the commercial banks, and the public.

(ii) Assuming that the interest rate on government debt is determined by the equation \( r = \frac{G}{M} \), where \( M \) is the money supply and \( G \) is public debt held by the public, what is the rate of interest?

(iii) What would be the effect on the money supply and the interest rate of a doubling of the price of gold?

(iv) What would be the effect on the money supply and the interest rate of a doubling of the currency/deposit ratio?

VI. (20 points).

(a) Why are government deposits usually excluded from the definition of the money supply?

(b) Should the same rule be applied to deposits owned by non-residents?

(c) Should certificates of deposit be included in the money supply?

VII. (50 points).

If the price level changes at a constant rate, the nominal and real interest rates adjust so that there is equilibrium in the public's holdings of real and nominal assets. Suppose that the rate of change of the price level changes in an unpredictable fashion over time. What changes, if any, would this require in the theory of portfolio balance? What empirical effects would this have? Would variable rates of change in the price level increase the social cost of inflation?
\[ dY = dG + GdY - C(dG + CgdG) - YdG \]
\[ \frac{dY}{dC} = \frac{(1 - C)(1 - \gamma)}{1 - C} = 1 - \gamma \]

For this to exceed unity, \( \gamma \) must be negative, which requires the marginal value of public sector output to be negative.

4. False. The data imply a relatively low transactions velocity of circulation of bank deposits (which might be offset by a relatively high velocity of circulation of currency) but nothing about whether money deposited in banks in the Province "dwindles out." In any case, the multiplier effect of an increase in government spending in the Province on incomes there depends on a matrix multiplier system involving the marginal propensity to save of the Province and the rest of the world and the marginal propensities of each to import from the other. If the Province were a closed system, the high ratio of deposits to transactions might be interpreted to mean a high ratio of money to income held and hence a relatively large offset to the stimulative effect of increased government spending through its induced effects in increasing the demand for money, raising the interest rate, and depressing private spending on consumption and investment. But since the Province is a small part of a large monetary area this effect can be regarded as negligible: the increased money demanded can be obtained by the sale of securities without appreciably affecting interest rates.

Some candidates wrongly distracted themselves that the lower check-to-deposit ratio meant that the banks could hold smaller excess reserves.

5. False. Monetary policy defined as determining the rate of inflation via the ratio of the rate of expansion of the nominal money supply to the exogeneously determined steady-state rate of growth influences the capital-output ratio and income per head in two opposite directions with an ambiguous net effect:

(1) The higher the rate of inflation the higher the alternative opportunity cost of real balances as a factor of production and the lower the output per unit of capital, hence also the lower the amount of saving per unit of capital and the lower the capital per head, ceteris paribus.

(2) The higher the rate of inflation the lower the ratio of real balances to output and hence the lower the real capital gain per unit of output from the growth of real balances; but since saving falls by a fraction \( S \) of the reduction in the capital gain while the amount of saving necessary to keep real balances per head intact falls by the whole amount of the reduction in the capital gain, the amount of savings available for investment in material capital with a given capital per head rises, so that the capital per head would be higher, with a given savings ratio, the greater the rate of inflation.

6. Uncertain. It is not sufficient to argue that since both hypotheses are concerned with wealth as the ultimate determinant of saving they make the same predictions and hence the test would not discriminate between them. It is necessary to pay attention to look at how "permanent income" as a proxy for wealth is actually derived. As a preliminary, though, note that "young people" and "old people" should not be identified with new entrants to the labor force, and the retired respectively, particularly not the latter. The permanent income hypothesis says that a positive proportion of permanent income will be saved at any positive rate of interest, the proportion depending however—though little is made of this in Friedman's book—on "tastes." Permanent income is derived from past experienced income. Hence both younger and older people in the working force should save a positive proportion of permanent income so derived. The life cycle hypothesis would indeed have young people consuming on the basis of their expected future lifetime income (their "wealth") and presumably saving (borrowing) because their current income (and permanent income measured as above) is less than their prospective future income and their "wealth." However, the test would not be definitive, partly because of the difficulty of distinguishing saving in the form of consumers' durables from consumption for young people.

7. Uncertain or false. This is a trick question because it suggests a simple Keynesian effective demand analysis, according to which total spending is lowered (raising inflationary pressure) by cutting old ladies off retirement benefits and so reducing aggregate demand. They will then have to produce anything they consume. However, if the benefits are raised to compensate for the shorter period of receiving them, there is no reason why the old ladies could not borrow against future increments of pension. Furthermore, we have to ask what happens to the money saved on their pensions: if the government imposes and sterilizes it we have a progressive contraction of the money supply, if it returns it to the capital market interest rates falls and if there is some interest-elasticity of demand for money inflationary pressure will be reduced. Alternatively social security taxes may be reduced and what happens is determined by the implicit income-redistribution effect.

On a straight quantity theory approach output increases to the extent that old ladies have to keep working, and this should lower the price level. In the longer run people will get used to the change in the social security retirement age for women and life-time work and consumption should be unaffected, hence no effect on inflationary pressure.

8. True. Since a non-vertical Phillips curve implies that workers accept a reduction in real wages, which they won't if they correctly perceive inflation. One can argue with doubts persuasiveness that correct perception of inflation does not necessarily imply full adjustment to it, making the statement false. One can also appeal to the Veblen argument that adjustment costs (of re-contracting) put labor permanently behind in the sense of accepting a once-over reduction of real wages at the start of inflation, though this would
seem to violate the assumption of correct perception of inflation. Finally, if substitution for money-using is labor-intensive, real wages tend to rise with inflation; but it is not clear that a rise in real wages for this reason—as contrasted with inflation less than anticipated—will increase unemployment.  

9. The answer can be regarded as "uncertain," because the two concepts are applicable to conditions of full employment equilibrium. The marginal efficiency of capital refers to the rate of return on a stock of capital that is kept intact, the marginal efficiency of investment to the return on adding to the stock by net investment. On the assumption that increasing production of investment goods encounters rising marginal costs, the marginal efficiency of capital must exceed the marginal efficiency of investment. In a depression, the capital stock appears excessive, so both the rate of return on the existing stock and on investment are depressed, the same relationship presumably holding except for the possibility that the depression is severe enough for there to be net disinvestment and a cost of production of investment goods lower than it would be if there were full employment and a rate of production of investment goods for replacement higher than the rate prevailing in the depression.  

10. False. If the increased demand for SELA deposits is at the expense of time deposits and SELA’s hold reserves in demand deposits, M₁ rises and M₂ falls (reserve requirements being greater on demand than on time deposits). If the switch is from demand deposits into SELA deposits backed by a fractional reserve in demand deposits at banks, both M₁ and M₂ will tend to rise.  

11. The answer is "false," the reason depending on the sophistication of the analysis. The standard unitary balanced-budget multiplier theorem assumes either a perfectly elastic supply of money, or a perfectly interest-elastic demand for it; otherwise the multiplier process is restricted by the effects of rising interest rates. But in general, given that both taxes and expenditures may fall on capital assets rather than current expenditure and production flows, there is no presumption that the balanced-budget multiplier will be exactly unity, or even positive.  

12. False. An excess stock supply of money can be matched by an excess stock demand for other assets, with no excess of a deficiency in demand in the flow markets, but the equalization process will reduce the yields on these other assets and so affect the flow markets in an inflationary manner. This is in fact Keynesian and Curley-Grimes explanation of how monetary policy influences the economy—through a stock disequilibrium influencing the terms of substitution among existing assets and thus the terms of substitution between present and future uses of goods flows.  

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whole. The case of a country that uses another's currency as its own simply dramatizes the case by excluding both central bank exchange reserve policy and the possibility of changing the exchange rate.  

(a) The money supply must be equal in the long run to the quantity of money demanded, and its determinants are the determinants of that quantity: the price level, real income, the returns on alternative assets, and the returns on money-holding (since commercial banks are not subject to central bank regulation and presumably have relatively more freedom to compete than in a central banking system.)  

(b) High-powered money has no relevance to money supply determination. It is "relevant" (though in ways (1) commercial banks have to hold it, or liquid assets denominated in it i.e., securities of country A, as reserves (fixing their reserve ratio by purely commercial considerations) and this limits the extent to which they can lend to domestic borrowers. (2) The currency in circulation and in bank reserves is an interest-free loan to the government of country A, which derives seigniorage from it.  

(c) The internal price level is determined by the world market prices of traded goods, the prices of non-traded goods (which will be related to the prices of traded goods via factor prices and relative productivity in producing the two types of goods), and the weights used in the price index.  

d) Inflation in country A will raise prices in the small country directly via the prices of goods traded between them, and indirectly via the inflationary impulse given to the prices of third countries' goods expressed in country A's economy by country A's inflation. Whether the rest of the world is on flexible or fixed exchange rates should make no difference in the long run, since the rest must maintain equilibrium by either inflating their domestic currency prices or revolting their currencies against A's currency, though if they maintain fixed rates on A's currency the balance-of-payments deficit will occur in the process of restoration of equilibrium may cause it to adopt less inflationary policies. In the short run, if the rest of the world is on fixed rates against A, the small country may be able to experience less inflation than if the rest had flexible rates on A during the period until prices in the rest catch up on A's prices, by switching imports from A's goods to rest-of-world goods. Note that the small country by definition cannot have a floating rate against A itself; some candidates cited in assuming that it could and contrasted fixed and floating rates for the small country against A's currency.  

III (a) An unanticipated inflation has only redistributive effects (i) from holders to issuers of money and (ii) from creditors to debtors. These may involve welfare gains or welfare losses according to some system for weighting of the welfare of groups of the population but there is no scientific basis for establishing such weights. An anticipated inflation, in the assumption that currency bears no interest and that bank deposit interest is subject to a maximum, involves a redistribution towards the issuers of money (the government, plus the banks)
but not otherwise from creditors to debtors (some money interest rates will be adjusted to offset the anticipated inflation). It also involves a true welfare cost due to substitution of other ways of conducting transactions for the normal use of money (substitution of less for more money-intensive ways of doing business). In order to avoid the "inflation tax" on the holding of money. In addition, if certain prices are controlled, possibly with the support of rationing (e.g., rents, urban transport rates, the exchange rate) there will be further welfare costs due to distortions of resource allocation. And, if inflation is anticipated but the rate anticipated is uncertain, there will be welfare costs associated with this. Note that if interest rates on deposits are controlled, there will be resource waste as banks compete for deposits by excessive establishment of branches, and that if lending rates are controlled and banks favour certain types of lenders there will be resource waste here also.

(b) The redistribution welfare gains or losses from unanticipated and anticipated inflation will not appear in measured GNP, though they may be considerable from changes in its composition. Some but not all of the social welfare costs of anticipated inflation will appear in measured GNP. For example, if money is an input into production inflation will reduce its use as such and therefore measured output. Economisation on cash holding will reduce measured as well as "true" output if performed by firms, or by households at the expense of labor time, but not if performed by households at the expense of leisure time or household production of utility. Some candidates pointed out, though this is rather remote from the question, that underestimation of the rate of inflation may reduce unemployment and raise measured GNP, even though it involves a welfare loss by comparison with the "natural rate" of unemployment.

c) Candidates did not all realize that "collection cost" is a special term introduced by Bailey for describing the welfare cost of inflation, and is not to be confused with the administrative and other costs of collecting taxes (the correct comparison is either with the welfare costs of other taxes, or a comparison of welfare plus administration costs for both). In the simple case where the real rate of return on material capital is zero or is so low by comparison with the rate of inflation as to be negligible, the relation between inflation tax revenue and welfare ("collection") cost is as shown in the diagram. The curve is the demand curve for real balances, \( m \), either absolute or as a ratio to output, as a function of the rate of inflation \( \pi \); \( A \) is the inflation tax revenue and \( B \) the welfare cost, either absolute or as a proportion of national income with inflation at the rate \( \pi \) and real balances \( m \). With a straight-line demand curve the tax revenue is maximized when marginal revenue (not shown) equals zero, which entails \( \pi = 1/2m \), \( m = 1/2m \), and \( B = 1/2A \). With a positive and non-negligible rate of return on alternative assets, the appropriate diagram is the following. The inflation tax revenue is \( A \), the welfare cost \( B + D \). However, the government in the absence of inflation is deriving seigniorage equal to \( C + D \) and the public bearing a welfare cost of \( E \). (The proper starting point for measuring the inflation tax in this case is really not price stability but a rate of deflation equal to the real rate of interest.) The net inflation tax revenue is \( A - B \). This will be negative if the interest-elasticity of demand for money exceeds unity at the rate of interest \( r \) (which may be assumed as empirically certain). The maximum inflation tax revenue occurs when \( 1 + r \) is such that the elasticity of demand for real balances is unity, and again with a straight line demand curve the welfare cost (by comparison now with the socially optimal rate of price deflation) is half the tax revenue.

The foregoing assume that all money is provided by government and yields the inflation tax. Actually the government collects inflation tax on currency plus government debt held at artificially low interest rates by the banking system. The welfare cost therefore applies to a multiple of the base of the inflation tax, and will be correspondingly larger than the foregoing argument implies.

(d) "Labor" and "capital" can be interpreted either as factors or as their owners. The effects of inflation on the distribution of income between owners of capital and of labor will be altered if the groups hold real balances in different proportions than factor incomes are divided. One would argue that owners of capital are rich and get economies of scale in holding transactions balances, also can substitute other earning assets for money more easily than workers. The effects on functional distribution will favour capital or labor according as money-substitution is capital-intensive or labor-intensive compared with the average of economic activity, or as money is more or less substitutable for capital or for labor.

IV. [L. Teber]

An increase in the demand for labor would increase both the number employed and the number of unemployed if the elasticity of supply of labor is positive. Also a rise in the minimum wage rate would increase both the real wage rate and unemployment.
V. (i) 

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<td>Gold</td>
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Explanation: The central bank has 10 gold, hold 10 in 20 = 40 government debt, total base money is 50.

The public holds 10 deposits requiring 2 reserves for each 3 currency, so 50 base money means 30 currency, 20 bank reserves, 100 bank deposits.

The banks have 50 deposits, hold 20 cash and 30 earning assets; 40 of these are loans to public (½ of 100 real capital) and 40 are government debt, leaving 40 government debt of 20 to be held by the public.

(ii) \[ i = \frac{C}{2M} = \frac{40}{200} = 15.4\% \]

(iii) A doubling of the gold price with everything else unchanged, would double all monetary magnitudes (including the money value of real capital and of loans issued against it), since the price level would presumably double. This would imply central bank holdings of 50 public debt and commercial bank holdings of 50 public debt, an excess of 40 over the total, matched by a deficiency of 40 held by the public. The simplest solution would be for the banks to induce the public to borrow another 40 against real capital, public holdings of government debt would be zero and so would the interest rate. An alternative assumption that breaks the central bank’s reserve ratio of 20 percent would be that the 10 extra gold is matched by 0 extra currency and 4 extra bank reserves (20 extra deposits), the money supply increasing to 120 and bank holdings of public debt rising and the public’s holdings falling by 16, making the interest rate \[ i = \frac{24}{312} = 0.077\% \]. Alternatively, the central bank could place the revaluation profit in an inactive account (adding 10 to its gold assets and 10 to its liabilities, the latter being described as “gold revaluation profit account.” Or again, the central bank could sterilize the gold revaluation profit by selling 10 of public debt to the public, increasing its holdings of government debt to 50 and increasing the interest rate to 20 = 19.2%. (Full analysis of this case would require taking account of the rise in the interest rate on the value of real capital and public borrowing from banks.)

Deposits = \( D = \frac{B}{C + r} \), where \( B \) is the monetary base, \( C \) is currency/deposits ratio, \( r \) is reserve ratio, with \( C = 6, r = 2, B = 50, D = 62.5 \).

VI. (a) Government deposits are held at the central bank and at commercial banks. There are two reasons for not including them in the money supply.

(i) The central bank is the agency for the conduct of monetary policy, but the government can influence the privately held money supply by manipulation of the size and location of government deposits. For example it can accumulate deposits at the central bank instead of using them to repurchase public bonds, thereby contracting the monetary base in the same way as an open market sale of securities by the central bank would do. Also, by shifting deposits from commercial banks to the central bank and vice versa it can contract or expand the monetary base of the private sector just as an increase or decrease in the private currency-to-deposits ratio would do.

(ii) (This reason is implicit in the foregoing.) The government’s spending and saving behavior, unlike that of the private sector, is not influenced by the size of its cash balances. Since it can always have the central bank create money for it if necessary, its objective can be to minimize its idle balances.

(b) The same rule is applied to deposits of the central bank owned by foreign central banks, which constitute claims on the country’s international reserves, but foreign central banks and foreign private deposits at commercial banks are counted as part of the country’s money supply. It can be argued that they should be excluded from the money supply because their holders (i) can always adjust the quantity by converting the balances into or out of their own currency, (ii) either will not be influenced in their behavior by changes in the domestic money supply, or if they are, the resulting changes in their behavior will have little impact on the domestic economy. On the other hand, it can be argued that such balances are easily transferable to domestic holders, and that resident deposit holders have the same possibilities of conversion into or out of foreign currencies as foreigners, so that it is simplest to aggregate domestic and foreign deposits into one money supply aggregate.

(c) For those who did not know, certificates of deposit are a special form of short-term security issued by banks and represent deposits that will not be withdrawn from the bank until the due date. They bear interest, like time deposits, but are not subject to the same interest ceiling. Since they are not subject to checking privileges, and cannot be cashed before maturity at a fixed value in terms of money, they obviously do not belong in \( M_1 \) (currency plus demand deposits). It could be argued, on the temporary abode of
purchasing power" concept of money used by Friedman, that they belong with $M_2$ because their holders can manage the maturities to coincide with their cash needs.

VII. (L. Toluer) The key words here are "escalator clauses." Would there be an additional problem with inflation if the rate of change of the price level behaved like a random variable drawn from some distribution if all contracts in nominal terms were converted into contracts in real terms by the use of escalator clauses?

Scale Economies and Non-labor Returns at the Optimum Population*

PAUL A. SAMUELSON

What the landlord can collect in rent was proved to be less than the extra product society achieves by the improved allocation of labor to land that results from a shift from path-shoveling squatterism to a regime of private property in land. Once you understand it, this proposition will hardly cause surprise; for, as Frank Ramsey liked to quote from William Blake, "Truth cannot be expressed so as to be understood without being believed."

My earlier case of a labor-land model can clarify some issues in the theory of optimum population.

Constant Return to Scale

If we adhere to the dogma that production functions on all lands are everywhere homogeneous of the first degree, the highest per capita real income is achieved when there has been enough enthusiasm of labors, footsteps, or conceptions to reduce the labor supply to the point of total enthusiasm of the landowner's rent. In examples where labor's marginal productivity curve falls from the beginning, and where some labor is essential if there is to be any product, the optimal labor supply is infinitesimal, with each infinitesimal of labor getting all the (maximized per capita) product and with land rent zero. No doubt an infinitesimal labor supply will seem absurd: Adam without Eve; or Adam's grin without even his rib. But that only points out the absurdity of assuming first-degree homogeneity at very small, even quantum-physics, scales.

First-degree homogeneity will seem a bit less absurd in cases where initial small, but finite, amounts of labor, L, do not find it optimal to spread over all the (best) land available. In this case, the marginal-product-of-labor curve begins with an initial horizontal branch. Any point on this represents optimum population size, and all involve zero land rent. Squatterism would involve no pushing and shoving, and even if private property in land were permitted, land ownership would not be worth assuming (much like the modern cases where squatters allocate their no longer worthless status).

*All these simple properties of the constant-returns case would hardly need restating were it not for a Knightian error that still permeates the literature. In Risk, Uncertainty, and Profit, Knight asserted three stages for a two-input first-degree-homogeneous neoclassical pro-

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