The Cooperative Firm Under Price Uncertainty

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In this note we extend Ward's [4] model of the cooperative firm operating in a competitive environment by assuming price to be uncertain. The analysis is similar to Sandino's [2] analysis of the competitive firm under price uncertainty.

We assume price, \( P \), is a random variable assuming possible values \( 0 \leq P < \infty \) with a density function \( g(P) \). Since price is random, average profit per worker,

\[
\sigma = \frac{P - WX - R}{X}
\]

is also random. \( Q \) represents quantity produced and sold, \( X \) represents the quantity of labor, each of whom is a member of the cooperative, \( W \) represents a unit cost per member of the cooperative, and \( R \) represents fixed costs of the cooperative firm. Since profit per member is a random variable, we assume the firm maximizes expected utility of profit per member. Thus,

\[
E(U)[\sigma] = \int_0^\infty U\left(\frac{P - WX - R}{X}\right) g(P) dP
\]

where \( E \) is the expectations operator and \( U \) is a utility function in the Von Neumann-Morgenstern sense.\(^3\) We assume a continuous and concave utility function with

\[
U'(q) > 0 \quad \text{and} \quad U''(q) < 0.
\]

The firm is assumed to operate in the short run. Capital is the fixed input and the quantity of labor (members of the cooperative), the variable input. We thus have,

\[
Q = f(X),
\]

where \( f \) defines the production relationship. We assume a positive but decreasing marginal product, so that

\[
f'(X) > 0 \quad \text{and} \quad f''(X) < 0.
\]

The necessary and sufficient conditions for a maximum to exist are:

\[
E(U)[\sigma(X)] = \frac{f'(X) - WX - sX}{X^2} = 0
\]

and

\[
G = E(U)[Y]^2 + E(U)[f'(X) - Y] < 0
\]

where \( Y = f'(X) - WX - s \). \( G \) is negative by

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3. To be exact, Ward does not call his Yugoslavian firm a cooperative firm.
4. See the discussion between Bernheim [1] and Sandino [3] concerning the relationship between fixed costs and fixed capital.

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