The Cooperative Firm Under Price Uncertainty

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In this note we extend Ward's [4] model of the cooperative firm operating in a competitive environment by assuming price to be uncertain. The analysis is similar to Sandino's [2] analysis of the competitive firm under price uncertainty.

We assume price, P, is a random variable assuming possible values 0 ≤ P < ∞ with a density function g(P). Since price is random, average profit per worker,

$$v = \frac{PO - WX - R}{X}$$

is also random. Q represents quantity produced and sold; X represents the quantity of labor, each of whom is a member of the cooperative; W represents a unit cost per member of the cooperative; and R represents fixed costs of the cooperative firm. Since profit per member is a random variable, we assume the firm maximizes expected utility of profit per member. Thus,

$$EU[U] = \int_0^\infty U\left(\frac{PO - WX - R}{X}\right) g(P) dP$$

where $E$ is the expectations operator and $U$ is a utility function in the Von Neumann-Morgenstern sense. We assume a continuous and concave utility function with

$$U'(\theta) > 0 \text{ and } U''(\theta) < 0.$$  

The firm is assumed to operate in the short run. Capital is a fixed input and the quantity of labor (members of the cooperative), the variable input. We thus have,

$$Q = f(X).$$

where $f$ defines the production relationship.

We assume a positive but decreasing marginal product, so that

$$f'(X) > 0 \text{ and } f''(X) < 0.$$  

The necessary and sufficient conditions for a maximum to exist are:

$$EU[U]\left(\frac{f'(X)}{f''(X)}\right) - W = R,$$

where $Y = f'(X) - W - s$ is negative by

$$EU[U](Y) + EU[U](Y) < 0.$$  

4. At this point mention should be made of Sandino's relevant remark about the possibility of group preferences not satisfying the transitivity axiom required for the existence of a utility function. We must fall back on his justification of the hypothesis of group preferences as the possibility of one person making the relevant decisions. See Sandino [2], pages 90-92.
assumptions (3) and (4) and the first order condition.

Comparative Statics

The first comparative static result we wish to show is that under the assumption of either increasing or constant absolute risk aversion, increases in the mean price of output will decrease the size (members) of the cooperative. If, however, we assume decreasing absolute risk aversion, it is in general not possible to determine what will happen to the equilibrium size of the cooperative.

Differentiating the first order condition with respect to \( \gamma \) (which is an additive shift that shifts the mean of the distribution* and evaluating at \( \gamma = 1 \), we get:

\[
3X \frac{\partial X}{\partial \gamma} = -EU^{-}[q|Y|X^{-1}] - EU[q|X'| - f] \cdot G
\]

The denominator is negative from (9). The second term in the numerator is positive from the first order condition. The first term in the numerator is positive, zero or negative under the assumption of increasing, constant or decreasing absolute risk aversion respectively. Assuming increasing absolute risk aversion, we have,

\[
\frac{-U'[q]}{U'[q]} \geq \frac{-U'[q]}{U'[q]}
\]

for \( f \geq x + \frac{W}{P} \), \( \gamma \) is the value of average profit when \( P = \frac{x + W}{f} \). Multiplying both sides of (8) by \( U'[q] \), we get

\[
-\frac{U'[q]}{U'[q]} \geq \frac{-U'[q]}{U'[q]}\frac{U'[q]}{X'}Y
\]

Note that inequality (8) holds for \( f \leq x + \frac{W}{P} \), since \( Y \) is then negative. Thus it holds for all \( f \) taking expected values of both sides of (8) we get

\[
EU^{*}[q|Y] \leq EU[q|X'] - f \cdot G
\]

This is true since \( EU'[q|Y] \) is not random and by virtue of the first order condition. Thus result (7) is negative if we assume increasing absolute risk aversion. Under the assumption of constant absolute risk aversion the first term in the numerator is zero and thus result (7) is still negative. However if we assume decreasing absolute risk aversion result (7) cannot be signed since the first term in the numerator is positive.

To determine how the size of the cooperative varies with the fixed costs of the firm we differentiate with respect to \( f \). We, thus, have

\[
\frac{\partial X}{\partial f} = EU^{*}[q|Y|X^{-1}] - EU[q|X'| - f] \cdot G
\]

The denominator and the last term of the numerator are negative. The first term in the numerator is negative, zero or positive under the assumption of increasing, constant or decreasing absolute risk aversion respectively as we have previously shown. Thus increasing the fixed cost of the cooperative will increase its size under the assumption of increasing or constant absolute risk aversion. However, under the assumption of decreasing absolute risk aversion the effect on the size of the cooperative is indeterminate.

In the certainty case variations in \( W \), the cost per member, does not affect the size of the cooperative. However in the uncertainty case it can be shown that the size of the cooperative will vary directly, not vary or vary inversely under the assumption of increasing, constant or decreasing absolute risk aversion.

respectively. Differentiating (5) with respect to \( W \) we get

\[
\frac{\partial X}{\partial W} = EU^{*}[q|Y] \cdot f \cdot G
\]

The sign of the numerator depends on our assumption about absolute risk aversion as we have shown previously while the denominator is negative. Thus, we can make an unequivocal statement about the sign of (12).

When we introduce a proportional profit tax with a full loss offset provision the size of the cooperative may depend on the tax rate which is not the case with price certainty. The result is exactly the same as Saudino's and, therefore, will not be repeated here.

REFERENCES


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* See Saudino, ibid., page 61.