The Effect of Forecasting in a Simple Stochastic Macroeconomic Model

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This paper investigates the effect on the magnitude of fluctuations in a simple stochastic macroeconomic model of the announcement of forecasts when some (or all) members of the private sector believe the forecasts and base their behavior on them. In the model presented, perfectly accurate forecasting is necessarily destabilizing; however, if the forecaster does not predict the stochastic term (accordingly putting it equal to zero) but otherwise forecasts accurately, then the announcement of the forecasts is stabilizing.1

In the model income consists of two components, an induced component, A, and a stochastic component, B. We measure all variables as deviations from equilibrium. B is assumed to be non-autocorrelated and to have mean zero and variance \( \sigma^2 \). Denoting time periods by subscripts involving \( t \) then income, \( Y_t \), is given by

\[
Y_t = A_t + B_t
\]

(1)

In the absence of any forecast it is assumed that the private sector bases its spending decision on the income of the previous period so that

\[
A_t = aY_{t-1}
\]

where \( 0 < a < 1 \) which ensures that the system is stable. Substituting for \( A_t \) from (2) into (1) gives

\[
Y_t = aY_{t-1} + B_t
\]

(3)

This is a linear first order stochastic difference equation. We use the variance of \( Y \) as our measure of the magnitude of fluctuations. The variance of \( Y \), \( \text{Var}(Y) \), is given by

\[
\text{Var}(Y_t) = a^2 \frac{\sigma^2}{1 - a^2}
\]

(4)

where the subscript denotes the number of the equation: determining income.5

The variance of \( X \) in the linear first order stochastic difference equation

\[
X_t = \alpha + \beta X_{t-1} + \epsilon_t
\]

where \( \alpha \) and \( \beta \) are constants and \( \epsilon_t \) is a non-autocorrelated random variable with mean zero and variance \( \sigma^2 \) is

\[
\text{Var}(X_t) = \sigma^2 \frac{1}{\beta^2}
\]

(5)

and

\[
\text{Var}(\epsilon_t) = \sigma^2
\]

(6)

where \( |\beta| < 1 \) [1, p. 1068]. Equation (6) gives the asymptotic variance and it is this that will be used throughout the paper.

We now introduce forecasting into the model. The forecast level of income in period \( t \) is denoted by \( Y^*_{t} \). It is assumed that a proportion \( g \) of the private sector believe the forecast and base their spending on it, with \( 0 < g < 1 \); the remainder of the private sector continue to base their expenditure plans on \( Y_{t-1} \). Then now

\[
A_t = a(gY^*_t + (1 - g)Y_{t-1})
\]

(7)

If \( g = 0 \) nobody acts on the forecast and (7) reduces to (2) which clearly provides an alternative interpretation of (5). If \( g = 1 \) then everyone believes and acts on the forecasts.

If forecasting is perfectly accurate \( 6 \) we have

\[
Y_t = Y^*_t
\]

and hence

\[
A_t = a(gY^*_t + (1 - g)Y_{t-1})
\]

(8)

Substituting for \( A_t \) from (7) in (1) and rearranging gives

\[
Y_t = a(1 - g)gY^*_{t-1} + B_t - a(1 - g)
\]

(9)

The variance of the disturbance term is \( \sigma^2(1 - a)g^2 \) and the variance of \( Y \) is given by

\[
\text{Var}(Y_t) = \sigma^2 \frac{1}{(1 - a)g^2}/(1 - a)g^2 = \frac{\sigma^2}{1 - (a + 2ag)}
\]

(10)

We may easily show that the announcement of the forecast increases the variance of income. The variance is increased if \( \text{Var}(Y_t)/\text{Var}(X_t) > 1 \). We have

\[
\text{Var}(Y_t) = \frac{1 + a}{1 + a - 2ag} > 1
\]

(11)

for \( 0 < a < 1 \) and \( 0 < g \leq 1 \). Then in our model announcement of the forecast necessarily increases the variance of \( Y \) unless no one believes the forecast. The larger is \( g \) the more the variance is increased. If \( g = 1 \) then the variance is increased to \( (1 + a)(1 - a) \) times the variance without forecasting.

We now make a weaker assumption about the forecaster’s abilities. We assume that he can estimate \( a \) and \( g \) but that he cannot forecast the stochastic term and accordingly puts \( B_t = 0 \). His forecast of \( Y_t \) is now given by

\[
Y_t = a(gY^*_t + (1 - g)Y_{t-1}) + \epsilon_t
\]

(11)

and hence, substituting for \( Y^*_t \) from (11) in (8)

\[
A_t = a(1 - g)gY_{t-1} + B_t - a(1 - g)
\]

(12)

Substituting for \( A_t \) from (7) in (1) yields

\[
Y_t = a(1 - g)gY_{t-1} + B_t - a(1 - g)
\]

(13)

with variance

\[
\text{Var}(Y_t) = \frac{1}{1 - a} - a\frac{1 - a}{1 - a} = \frac{\sigma^2}{1 - a}
\]

(14)

In this case the announcement of the forecast reduces the variance of \( Y \). This is so for

\[
\text{Var}(Y_t) = \frac{(1 - a)(1 + a)}{(1 + a - 2ag)} > 1
\]

(15)

for \( 0 < a < 1 \) and \( 0 < g \leq 1 \). The model presented in this paper has been an extremely simple one. As such it has been
a quite general one; expenditures have been divided up simply into a component induced by the level of income and a stochastic component. Clearly, a much more complex model could have been presented but the conclusions would have been much less general.

Finally, the mechanism by which the results were obtained should be stressed. When income is a lagged function of past income the multiplier effect is spread over a number of periods so that the multiplier effect of a positive stochastic shock is not worked out before the effects of a negative stochastic shock are felt. Accurate forecasting may shorten the time lag involved so that the multiplier effects of stochastic elements are more concentrated than accentuating the magnitude of fluctuations.

REFERENCES


Book Review


Professor Heilbrun makes a significant contribution to the growing literature on urban economics. The book starts out with an excellent survey of urban growth in the United States. This provides a good perspective for subsequent discussion of urban theories and policy issues.

The theoretical component of the book includes some of the major microeconomic and macroeconomic models pertinent to urban growth. In particular, the treatment of the theories of industrial location, the central place theory and the economic base theory reflect excellent analytical capabilities on the part of the author. Only minor criticisms of his treatment of the central place theory may be offered. Recent developments of the theory should have been incorporated. Especially, the consequences of recent empirical work and links with other theory should have been investigated. Recent empirical research has yielded interesting results on the identification of hierarchies of central place functions and identification of threshold marginal goods. Also, problems of optimal design and utilization of transport and communication networks have been studied. These studies have important implications for the formulation of the central place theory which will meet the requirements of closer correspondence with reality. In particular, empirical work tends to show that the formulation of central place theory free of the assumption of no excess profits and free of the assumption of the shape and character of tributary areas is feasible. A particular value of such reformulation is its relation to other theories such as the theory of retailing, and the theory of consumer spatial behavior.

The problem areas Heilbrun selected for thorough analysis represent some of the most critical areas in urban economics. These include urban land use, urban poverty, urban housing and urban finance. These areas have been explored in remarkable depth. The chapters on urban housing and urban housing policy stand out in clarity of exposition and wealth of details.

The coverage of problem areas, however, suggests some weaknesses in the book. It is unfortunate that Heilbrun did not place sufficient emphasis on the criticality of the urban transportation problem which warrants its treatment in a major chapter rather than as a very brief section in the chapter entitled: "Land-Use Patterns, Transportation, and the Form of the City." The urban environment is another somewhat neglected area, despite its critical importance in the context of urban growth. The chapter on urban public services needs to be considerably improved. The market for public services, identification of the key urban services and adequate economic analysis of the demand for and supply of such services should have been the essential prelude to the discussion of financing such services.

In terms of analytical techniques applicable to urban analysis, more attention should have been given to cost-benefit analysis. The literature on this subject is abundant and still growing.

Finally, one major area of concern is missing: urban health care delivery system. Here, interesting work has been done in recent