For researchers, the continued future heavy utilization of the single equation polynomial distributed lag St. Louis approach should be expanded to include a simultaneous equation approach that incorporates the nominal GNP to money relationship.

References


A Multi-Factor Labor-Managed Firm under Price Uncertainty

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1. Introduction

The behavior of a cooperative seeking to maximize the well being of its members has attracted a great deal of attention in the last couple of decades or so. The pioneering works of Ward (1958), Doar (1966) and Vanek (1969, 1970) have been extended in several directions by Maurice and Ferguson (1972), Meade (1974), Funabook (1976), Detsle (1978) and Steinherr and Thisse (1979) to name but few. The common assumption of all these studies is that the product demand function is perfectly known. Yet, very often, decisions have to be made in an uncertain environment.

The first attempt to incorporate price uncertainty into the model of a cooperative was made by Taub (1974). His thesis proposition is that the effect of price uncertainty on the competitive co-operative firm is identical to that on its capitalist twin. Specifically, when facing uncertain price, both a labor-managed firm and a profit-maximizing firm reduce the level of production—assuming that they are risk-averse. This type of response to uncertainty by a profit maximizing firm is dealt with by Sandmo (1971) and Leland (1972). Recently, Taub's proposition has been revised independently by Muxondo (1979) and Paroush and Kahana (1980). These studies reach a somewhat counter-intuitive conclusion, namely that under a fluctuating price a risk-averse labor-managed firm produces more and utilizes more labor than under a stable price.

Thus, Sandmo's result does not apply to the cooperative firm. Consequently, the well known "Ward's effect" might break down. Hawawini and Michel (1979) present a diagramatic exposition of this somewhat surprising result. Muxondo's and Paroush and Kahana's models address the case in which one input is a decision variable, i.e. the cooperative's production function depends only on the labor input or on the number of its members. Hence, the conclusion of these studies applies only to the short run, when capital and other inputs are fixed.

The aim of the present paper is to extend the previous model to the multi-inputs case in much the same manner as Donnat (1966) extends the work of Ward (1958) and as Bar-Ilan and Ulrich (1974) extend Sandmo's (1971) model.

Our conclusion is that it is impossible to derive unambiguous general results for the long run case. Yet, we specify conditions under which the behavior of the cooperative firm under price-uncertainty in the long-run parallels with that of the short run.

Although recent works such as Hey's and Saclings' (1980) have derived traditional results under certainty by simpler means, we prefer Ward's work as our point of departure to the world of uncertainty.

Section II presents the model of a multi-factor cooperative which produces in a competitive market in which price is uncertain. The optimum policy of such a labor-managed firm

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firm is compared to that of its certainty-equivalent twin. In Section III a comparative statics analysis with respect to both the fixed cost and the parameters of the density function is carried out. The concluding Section IV provides a short summary as well as suggestions for further research.

II. The Basic Model and a Comparison of the Solutions

Assume a firm which produces a homogeneous product \( y \) with two factor inputs: labor \( -L \) which is treated throughout as a continuous variable, and capital \( -k \). The production function \( y = f(L, k) \) is assumed to be neo-classical.

Under price uncertainty, the labor-managed firm is assumed to maximize the Von-Neumann-Morgenstern expected utility of net income per worker, i.e. \( \max E u(w) \) with respect to \( L \) and \( k \), where \( w = (pL - rF)/L \) with \( r \), a fixed unit price of capital, \( F \) is the total fixed cost, and \( p \) is a random unit price of the final product such that \( p = \tilde{p} + \gamma \theta \) where \( E \tilde{p} = 0 \), \( \gamma > 0 \), \( \tilde{p} \) and \( \gamma \tilde{p} \) are the elasticities of factors \( L \) and \( k \) with respect to a change in \( \gamma \). The first order conditions for maximization are:

\[
\frac{\partial u}{\partial L} = u'\tilde{y}\tilde{y} = 0
\]

where \( \tilde{y} = \tilde{p}L - rL + F \) and the second order conditions are:

\[
A_{11} = \tilde{u}\tilde{y}\tilde{y} = 0
\]

where \( \tilde{u} = \frac{\partial^{2}u}{\partial L^{2}} \)

\[
A_{22} = \tilde{u}\tilde{y}\tilde{y} = 0
\]

where \( \tilde{u} = \frac{\partial^{2}u}{\partial L\partial k} \).

Assume the existence of a unique positive regular solution, \((L^{\ast}, k^{\ast})\), for the maximization problem. Denote the output yielded by \((L^{\ast}, k^{\ast})\) as \( y^{\ast} \) i.e., \( y^{\ast} = f(L^{\ast}, k^{\ast}) \) and by \((L^{\ast}, \tilde{k})\) the solution for the certainty equivalent maximization problem i.e., \( y^{\ast} = f(L^{\ast}, \tilde{k}) \).

It can be shown that, if labor and capital are competing inputs, the labor managed firm employs more members under price uncertainty than it does under full information.

\[
\frac{\partial L}{\partial \tilde{p}} > 0, \quad \frac{\partial \tilde{L}}{\partial \tilde{p}} > 0
\]

\[
\frac{\partial L}{\partial \tilde{p}} > 0, \quad \frac{\partial \tilde{L}}{\partial \tilde{p}} = 0
\]

\[
\frac{\partial L}{\partial \tilde{p}} < 0, \quad \frac{\partial \tilde{L}}{\partial \tilde{p}} < 0
\]

Theorem I: If \( f_{L} > 0 \) then \( L^{\ast} > L^{\ast} \) and \( k^{\ast} < k^{\ast} \).

Proof: By substituting \( p = \tilde{p} + \gamma \theta \) into (3) and (4) one can find that the equations:

\[
\tilde{y} = \tilde{p} - f_{L} - f_{k} + F
\]

\[
\tilde{y} = \tilde{p} - f_{L} - f_{k} + F
\]

It is also assumed that the utility function displays monotonicity and continuity over the relevant domain, i.e. \( (1) u' > 0, (2) u'' < 0 \). The first order conditions for maximization are:

\[
\frac{\partial u}{\partial L} = u'\tilde{y}\tilde{y} = 0
\]

where \( \tilde{y} = \tilde{p}L - rL + F \) and the second order conditions are:

\[
A_{11} = \tilde{u}\tilde{y}\tilde{y} = 0
\]

where \( \tilde{u} = \frac{\partial^{2}u}{\partial L^{2}} \)

\[
A_{22} = \tilde{u}\tilde{y}\tilde{y} = 0
\]

where \( \tilde{u} = \frac{\partial^{2}u}{\partial L\partial k} \).

Theorem I: If \( f_{L} > 0 \) then \( L^{\ast} > L^{\ast} \) and \( k^{\ast} < k^{\ast} \).

Proof: As in Theorem I it suffices to specify the conditions under which

\[
\frac{\partial y^{\ast}}{\partial \tilde{p}} < 0, \quad \frac{\partial y^{\ast}}{\partial \tilde{p}} > 0
\]

and \( \frac{\partial y^{\ast}}{\partial F} > 0 \).

Comparative statics analysis shows (see Ward (1958) or Donnar (1966)) that

\[
\frac{\partial y^{\ast}}{\partial \tilde{p}} = \frac{f_{L}(f_{L} - f_{L} - f_{L})/\tilde{p} \Delta}{f_{L}(f_{L} - f_{L} - f_{L})/\tilde{p} \Delta}
\]

\[
\frac{\partial y^{\ast}}{\partial F} = \frac{f_{L}(f_{L} - f_{L} - f_{L})/\tilde{p} \Delta}{f_{L}(f_{L} - f_{L} - f_{L})/\tilde{p} \Delta}
\]

\[
\frac{\partial y^{\ast}}{\partial F} = \frac{f_{L}(f_{L} - f_{L} - f_{L})/\tilde{p} \Delta}{f_{L}(f_{L} - f_{L} - f_{L})/\tilde{p} \Delta}
\]

where \( \Delta = f_{L} - f_{L} \).

Solving the following auxiliary problem: minimize \( pL + yk \) subject to \( f(L, k) = y^{\ast} \) where \( y^{\ast} \) is constant one can verify that

\[
f_{L}f_{L} - f_{L}f_{L} = D \frac{\partial y}{\partial y} \frac{\partial y}{\partial y}
\]

where \( \lambda = \text{the Lagrange multiplier} \) and \( D \) is the determinant of the bordered Hessian of this auxiliary problem which is assumed to be positive so as to ensure the existence of the second order conditions. Substitute (15) and (16) into (12), (13) and (14) to obtain

\[
\frac{\partial y^{\ast}}{\partial \tilde{p}} = \frac{\partial y^{\ast}}{\partial \tilde{p}} = \frac{\partial y^{\ast}}{\partial \tilde{p}} = \frac{\partial y^{\ast}}{\partial \tilde{p}}
\]

where \( \alpha_{L} = f_{L}/y \)

and \( \alpha_{L} = f_{L}/y \)
\[ \frac{\delta y^*}{\delta \alpha} = (e_t - e_t)\delta k/s_t \delta y^* \]

\[ \frac{\delta y^*}{\delta F} = e_t \delta k/s_t \delta y^* \]

Combine the two equations in (9) to find that

\[(c^*k^*) F = p_f(1 - a_t - a_t) > 0 \text{ and therefore } 1 - a_t - a_t > 0. \]

Consequently, \( e_t \approx e_t \approx 0 \) is a sufficient condition for the inequalities in (11) to hold and, therefore, the proposition of Theorem 2 is proved.

Following Sandmo (1971), it is assumed that the complete the analysis with a comparative statics with respect to \( \beta \) and \( \gamma \)—the parameters of the density function, and with respect to \( F \).

III. Comparative Statics

Following Sandmo (1971), this section makes an intensive use of the decreasing absolute risk-aversion assumption. More specifically, assume that

\[ -u'(w^1)/u'(w^1) \leq -u'(w^2)/u'(w^2) \quad (17) \]

if and only if \( w^1 \leq w^2 \).

As before, denote by \( y^*, e^*, e^*, k^* \) and by \( y^*, e^*, k^* \) the optimal values of output and labor and capital inputs of the labor-managed firm under price uncertainty and under price stability, respectively. Denote by \( y^*, e^*, k^* \) and by \( y^*, e^*, k^* \) the corresponding optimal values for a profit maximizing firm or the capitalist twin. Paroush and Kahana (1980) show that under (17) \( \delta e^* / \delta e^* > 0 \) and \( \delta e^* / \delta e^* > 0 \) as well as \( e^* - e^* > 0 \) and \( e^* - e^* > 0 \). Here we show that this result is preserved even in the long run, provided that labor and capital are competing factors.

Theorem 3: Under (17), \( \delta e^* / \delta e^* > 0 \) and \( \delta e^* / \delta e^* > 0 \).

Proof: Batsa and Ullah (1974) show that under (17) \( \delta e^*/\delta e^* > 0 \) and \( \delta e^*/\delta e^* > 0 \). Lemma 2 and Lemma 3 in the Appendix states respectively that under (17) \( \delta e^*/\delta e^* < 0 \) and \( \delta e^*/\delta e^* < 0 \). Thus, Theorem 3 immediately follows from these results.

Theorem 4: Under (17), \( \delta e^*/\delta e^* > 0 \) and \( \delta e^*/\delta e^* > 0 \) where \( e_t = e_t \delta k / (F + F) \).

Proof: Batsa and Ullah (1974) show that under (17) \( \delta e^*/\delta e^* > 0 \) and \( \delta e^*/\delta e^* > 0 \). Lemma 4 and Lemma 5 in the Appendix state respectively that under (17) \( \delta e^*/\delta e^* < 0 \) and \( \delta e^*/\delta e^* < 0 \).

We complete the analysis with comparative statics with respect to the fixed cost \( F \).

Theorem 5: Under (17), \( \delta k^*/\delta F > 0 \) and \( \delta k^*/\delta F > 0 \).

The proof of Theorem 7 is given in the Appendix through Lemmas 6 and 7.

IV. Summary and Conclusions

This paper attempts to determine whether the perverse behavior of the labor-managed firm under price uncertainty in the short run holds in the long run where inputs other than labor are under the firm's control.

It turns out that all the results that hold for the short run (see Murzendo (1979) and Paroush and Kahana (1980)) can be readily generalized to the long run provided that labor and capital are competing inputs or that the product produced by the cooperative is an important user of labor. Moreover, perverse behavior is not ruled out in other cases, although ambiguous results cannot be obtained. Further research is called to explore the case in which the target function of the labor-managed firm is not just profit per member but includes, for example, some expected compensation for possible layoffs. A subjective function which includes the expected payment to the members who have to be laid off when price is unexpectedly above a certain value is a case in point.

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Solve the system (A.2.) for a partial change in \( \beta \) to get

\[ \delta \beta^* / \delta \beta = (A_{12} H_1 - A_{12} H_3) / A_{12} \]

Sandmo (1971, p. 68-69) shows that under (17) \( E_a < 0 \) and \( E_a > 0 \) and since \( f_c \xi - f < 0 \) at the optimum then \( H_1 > 0 \) and \( H_1 < 0 \). By Lemma 1, \( A_{12} < 0 \) so that \( \delta \beta^* / \delta \beta > 0 \) and \( \delta \beta^* / \delta \beta < 0 \).

Lemma 5: Under (17), \( \delta \beta^* / \delta \beta > 0 \). \( \delta \beta^* / \delta \beta < 0 \).

Proof: Solve the system (A.2.) for a partial change in \( \gamma \) to get

\[ \delta \gamma^* / \delta \gamma = (E_{a} H_3) / A_{12} \]

Since \( E_{a} H_3 > 0 \), \( E_{a} H_3 < 0 \), \( E_{a} H_3 < 0 \) \( f_c \xi - f < 0 \) then \( E_{a} H_3 > 0 \) and \( E_{a} H_3 < 0 \). By Lemma 1, \( A_{12} < 0 \) so that the two terms on the R.H.S. of (A.5) and (A.6) have the same sign as stated in the proposition.

Q.E.D.

Lemma 4: Under (17) and if \( e_t - e_t > 0 \) then \( \delta e^*/\delta e^* > 0 \).

Proof: Substitute the values of \( \delta k^*/\delta k^* \) and \( \delta e^*/\delta e^* \) from (A.3.) and (A.4.) in \( \delta e^*/\delta e^* = \delta (\delta e^*/\delta e^*) + \delta (\delta e^*/\delta e^*) \) to get

\[ \delta e^*/\delta e^* = \delta (\delta e^*/\delta e^*) + \delta (\delta e^*/\delta e^*) \]

where \( A_{11} = A_{11} \) and \( A_{12} = A_{12} \) are defined in (5)-(7).

By arranging terms, (3) and (4) can be rewritten as

\[ -f_c \xi - f < 0 \]

Divide (3) by (4) and multiply by \( f_c k \) to get

\[ f_c k \]
Substitute (A 8) and the values of $A_{11}$, $A_{12}$, and $A_{22}$ to find that

$$f_{A_{12}} - f_{A_{12}} = (E_2 + 2E_1) + \frac{2E_1p}{f_{A_{12}} - f_{A_{12}}} \frac{\partial E_2}{\partial f_{A_{12}}}$$

The first two terms on the R.H.S. of (A 15) are positive and so is the third term if $E_2 - w_{A_3} > 0$.

Q.E.D.

**Lemma 6:** Under (17), $\partial E_2/\partial f_{A_{12}} > 0$ if $f_{A_{12}} \geqslant 0$.

**Proof:** By (A 2).

$$\partial E_2/\partial f_{A_{12}} = (F_1 - F_2)/A$$

Q.E.D.

Use (A 8) to find also that

$$H_1 = H_2 + \frac{r + F_1 - F_2}{tr}$$

Substitute (A 8) and (A 10) and (A 15) into (A 7) to get

$$\partial y^*/\partial f_{A_{12}} = H_2 + \frac{r + F_1 - F_2}{tr} - \frac{D_2 + \frac{r + F_1 - F_2}{tr}}{A^2}$$

The first two terms on the R.H.S. of (A 12) are negative (see Lemma 1-2) and so is the third term if $t_2 - w_{A_3} > 0$.

Q.E.D.

**Lemma 7:** Under (17), $\partial y^*/\partial F_1 > 0$ if $t_2 \geqslant t_1$.

**Proof:** Substitute (A 16) into

$$\partial y^*/\partial F_1 = \frac{\partial f_{A_{12}}}{\partial f_{A_{12}}}$$

and $\partial \phi_{A_{12}}^*/\partial F_1$ to find that

$$\partial y^*/\partial F_1 = F_1 - F_2 + \frac{r_2 + F_1 - F_2}{tr}$$

Under (17), $F_1 < 0$ and $F_2 > 0$ and if $F_1 \leqslant 0$ then $A_{12} < 0$ which completes the proof.

Q.E.D.

**References**


