Eastern Economic Journal, volume x, no. 2 April-June, 1984

LOWE AND THE MARX-FEL'DMAN-DOBB MODEL: STRUCTURAL ANALYSIS OF A GROWING ECONOMY

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I. Adolph Lowe - Six Decades of Structural Analysis of Real Capital Formation

Adolph Lowe's contributions to the theory of the trade cycle were widely discussed among German economists during the period of the Weimar Republic (Krohn 1981). The members of the department for research on trade cycles of the Kiel Institute for World Economics (1926-1933) were internationally recognized as among the most important precursors of the modern theory of employment (Garvy, 1975) and in the development of modern non-neoclassical capital and growth theory. Besides Lowe, who served as Chairman, such distinguished scholars as Fritz Burchardt, Gerhard Colm, Hans Neisser and, for a period of time, Wassily Leontief and Jacob Marshak were also members of this scientific community.

After emigrating, to the United States Lowe developed his ideas and ultimately produced a theory of cyclical growth, which centered on the structural analysis of real capital formation. Investment holds the key role in economic development because real capital is the central channel through which all other determinants, be they technical progress, changes in labor supply or the exploitation of natural resources, influence the secular process of an industrial system. His precursorial role in the construction of models with a strong classical flavour has been widely recognized (see e.g. Clark 1974, 1983, and Walsh and Gram 1980, p. 4), though it has, surprisingly, been neglected by Chng (1980).

Starting with an article entitled 'How is Business Cycle Theory Possible at All?', which was published in German in 1926, Lowe pointed out the necessity for modifying Marx's schema of reproduction, which he later viewed as "the only comprehensive macro-economic model of the industrial process of production established before Keynes" (Lowe 1952, p. 141). This modified version rests on the premise that not all sub-divisions of the productive structure are equally important for the study of particular dynamic processes. Lowe considered Marx's schema sto be especially suited for the study of real capital formation - provided that some defects are corrected. (Lowe 1955, p. 586). Specifically, Lowe considered it necessary to extend the two-sector Marxian model to a three-sector schema, through the splitting up of the equipment goods sector. Thus, one capital goods sector produced the equipment for producing consumer goods and the other produced the equipment for the replacement and expansion of both equipment goods sectors (Lowe, 1926 p. 190).

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This subdivision of the equipment goods sectors is relevant for investigating the structural conditions for steady growth and, even more, to address questions of 'traverse analysis'. In the analysis of transition processes there is, in contrast to the traditional steady state or 'golden age' analysis, no assumption of the proportionate growth of all sectors, so that the problem of structural change is moved to the center of the stage. The decisive problem that the economy confronts upon departing from a steady growth path is the inadequacy of the old capital stock. The necessary adjustment process both requires time and cost, and faces difficulties which arise from disproportions between sectors. During the Soviet industrialization debate in the late twenties Fel'dman formalized the notion that investment-priority for the capital-goods sector was a pre-condition for attaining a high rate of growth. In Lowe's growth analysis the 'machine-tools sector' also plays a key role and altering the proportion of total investment allocated to this sector has great direct and indirect consequences for growth.

The main lesson to be learned from the Lowe-Fel'dman models is that the capacity of the machine tools sector is the decisive constraint which limits the rate of growth in a closed economy. In an economy with a very small machine tools sector it is impossible to move from a lower to a higher rate of investment by heroic sacrifices of current consumption. Structural incapacity to supply enough capital goods will prevent a rise in the saving ratio from being fully transformed into the desired level of investment.[1]

The strategic role of the machine tools sector was also confronted by development planners in the fifties and sixties. Countries like India which lack a self-sufficient machine tools sector, pose a problem which neither Fel'dman nor Lowe discusses. Merhave (1969) has called attention to the problem of 'technological dependence' in cases of structural inability to supply the required capital goods, which must then be imported from other economies. Examination of the structural conditions for steady growth and the adjustments required by dynamic disturbances, must therefore extend to the economy's external balance.

Despite the fact that Lowe's intentions are in many respects the same as Fel'dman's who wanted to include all activities that increase the capacity in an economy to produce output in one sector there is one important difference.[2] No ex post transfer of machines between the two departments possible in the Fel'dman model; there is complete rigidity as in most other two sector models with a fixed coefficient techniques. On the other hand, an essential characteristic of the three-sectoral Lowe model is that it combines transferability with specificity. Occurs an ex post transfer of machines between the two sectors of the equipment goods group during the traverse. Dobb (1960, pp. 48-103) made extensive use of Lowe's 1955 version of the three department scheme not to analyze the process of traverse but to discuss the question of the choice of techniques under planned development.[3]

Our present concern is to make the essential characteristics of a three-sector model explicit with the help of techniques developed by Hicks (1965), Spaventa (1968, 1970) and Harris (1973) for their analysis of the two-sector fixed-coefficient models. We concentrate on comparing economies which are in long-run equilibrium. We focus on what is often called 'comparative dynamics', or, more correctly, in Bliss' (1975, p. 71) terms, the 'comparative statics of semi-stationary growth'.[4] Detailed comparison of various economies growing on a steady-state path with different allocations of resources and different shares in total output of consumer and equipment goods is a necessary prelude for the analysis of the traverse, i.e. the path that connects two dynamic equilibria defined by different rates of change. This will become clear at the end of our analysis. The comparative analysis shows us the terminal equilibrium conditions from a structural point of view.

II. The Lowe Model - a "Tripartite" Schema

Lowe's 'tripartite' division of the economy into three vertically integrated sectors displays a level of aggregation which is, in a sense, a kind of a quarter-way house between Marx and Leontief, whose model is indispensable for the solution of practical problems of comprehensive planning but which may present an obstacle in the analysis of complex dynamic processes. The basic structure of the Lowe model may be described as follows:

Sector 1 produces primary equipment goods or machine tools which are directly used for production of the secondary equipment goods in sector 2 and indirectly for the production of consumption goods in section n. Sector 1 is the only one capable not only of producing machines for other sectors but also for itself, i.e. a self-reproducible sector. That puts sector 1 into a key position for any industrial system and a process of growth, especially during a 'traverse' with its structural change. In Sraffa's terminology sector 1 represents the 'basic system'.

Sector 2 which Lowe gets through the subdivision of the equipment goods industry into two sectors represents the link between the machine tools sector 1 and sector n producing consumer goods. Sector 2 uses the same type of equipment goods as sector 1 does so that shiftability of parts of the capital stock between these two sectors is possible. On the other hand sector 2 produces the secondary equipment goods which are used as inputs only in sector n producing consumer goods which means that the capital stock in the latter is not transferable.

Sector n provides the consumer goods for the laborers working in the three sectors.

In this industrial system in which equipment and labor remain in a strict relation of complementarity we have a hierarchy of sectors 1 2 n, or in popular terminology "machines - tractors - corn". The production of primary equipment goods or machines is the bottleneck which any process of rapid expansion must overcome. The technical methods of production will be represented by a matrix of interindustry coefficients, denoted by $\underline{A}$, and by a column vector of direct labor coefficients, denoted by $\underline{l}$:

$$\underline{A} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & 0 & 0 \\ 0 & a_{2n} & 0 \end{bmatrix} \qquad \underline{l} = \begin{bmatrix} l_1 \\ l_2 \\ l_n \end{bmatrix}$$

with $a_{ji} = F_{ji}/O_i$ and $l_i = L_i/O_i$, where L_i and F_{ji} denote the inputs of total labor and fixed capital goods of type j in the production of O_i goods in sector i.

Since the Cambridge debate we know that the sectoral machine-labor ratios $q_i = a_{ji}/l_i = F_{ji}/L_i$ are of crucial importance. In view of dealing with the very complicated questions of traverse analysis, Lowe works with the assumption Samuelson later made in his well-known article on the surrogate production function, i.e. identical capital-labor ratios in all sectors. It may be noted that, in contrast with the capital-labor ratio the machine-labor ratio, is defined in physical terms and, is thus unaffected by price changes resulting from changes in distribution. For purposes of intersectoral comparison we have to recognize the dimension of these technologically given machine-labor ratios. Since sectors 1 and 2 use the same type of equipment goods and labor is homogeneous we can compare q_1 and q_2 directly. Problems arise with q_n because a direct comparison with q_1 and q_2 makes no sense. But, contrary to Nell's assertion that "the ratios cannot be compared dimensionally unless they are expressed in value terms' (Nell 1976, p. 303) there is a way out of the physical dimension dilemma which results from different types of equipment goods being applied in the production of consumer goods. We only have to reformulate the third machine-labor ratio. Multiplication of q_n by l_2/l_1 yields q_n and thus an indirect machine-labor ratio which has the same physical dimension as q_1 and q_2, i.e. primary equipment goods per unit of labor. As an example, q_n q_1 means that the production system of the economy indirectly uses more primary equipment goods per unit of labor in sector n than in sector 1, though no primary equipment goods are directly used in the production of consumer goods.

This reformulation of the machine-labor ratio of sector n offers great advantage when dealing with the quantity system and the price system because machine-labor ratios play a crucial role in the price and quantity equations.

III. Growth Equilibrium: The Quantity System

We go on now to introduce the relations governing output, employment, and accumulation in the Lowe model. In competitive equilibrium with continuous, full, and efficient utilization of the available inputs, the total stocks of primary and secondary equipment goods, F_1 and F_2, and the total stock of labor L are adjusted to the technique in use and to the level of output of the three commodities, 0_1, 0_2, and 0_n. The output of the equipment goods provides for groww investment consisting of depreciation requirements and net additions to the capital stock. Let g be the net rate of growth of the capital stock or rate of accumulation and d the uniform rate of depreciation.[5] Thus we get the following quantity equations:

$$\text{(I)}\quad \begin{aligned} a_{11}(d+g)F_1 + a_{12}(d+g)F_2 &= F_1 \quad \text{primary equipment goods} \\ a_{2n}0_n &= F_2 \quad \text{secondary equipment goods} \\ l_1(d+g)F_1 + l_2(d+g)F_2 + l_n0_n &= L \quad \text{labor} \end{aligned}$$

Let us normalize the system by considering the ratios of all quantities to total labor employed in the economy

$$\text{(II)}\quad \begin{aligned} a_{11}(d+g)f_1 + a_{12}(d+g)f_2 &= f_1 \\ a_{2n}c &= f_2 \\ l_1(d+g)f_1 + l_2(d+g)f_2 + l_nc &= 1 \end{aligned}$$

Note that the seven technical parameters, the machine input coefficients a_{11}, a_{12}, and a_{2n}, the labor input coefficients l_1, l_2, and l_n, and the depreciation rate d are given, and the system provides us with tree equations in the four quantity variables c,g,f_1 and f_2, where c denotes the output of corn per unit of labor, which also coincides with consumption per worker, and $f_j = F_j/L$ denotes the input of fixed capital of type j per unit of labor. This means that the system of equilibrium relations possesses one degree of

freedom and remains open unless one variable is given a predetermined value. To put it another way: once we know either of the four variables then the other three are fully determined. In order to show the possible equilibrium configurations of the system, we take the growth rate as exogenously given and solve for the rest in terms of it. Expressing all quantity variables as functions of g we get the following equations for the outfit of the economy with consumer and equipment goods, the allocation of resources and the compositon of production. (Hagemann and Jeck, 1981)

Outfit with Consumer and Equipment Goods

$$\text{(1)}\quad c = \frac{1-2_{11}(d+g)}{l_n+(a_{2n}l_2-a_{11}l_n)\ (d+g)+a_{2n}(a_{12}l_1-a_{11}l_2)(d+g)^2} = \frac{N_g}{D_g}$$

with $$\frac{dc}{dg} = 1\ a_{2n} \quad l_2N_g^2 + a_{12}l_1(d+g)(1+N_g) \quad D_g^{-2} < 0$$

$$\text{(2)}\quad F_1 = \frac{a_{12}a_{2n}(d+g)}{D_g}$$

with $$\frac{df_1}{dg} = a_{12}a_{2n}l_n \quad 1-g_n^*(q_2-q_1)l_1^2(d+g)^2 \quad D_g^{-2} \gtreqless 0$$

$$\text{(2a)}\quad df_1/dg > 0 \text{ for } \quad q_1 \geqq q_2$$

$$\text{(3)}\quad \frac{F_{11}}{L}\ (g) = \frac{a_{11}a_{12}a_{2n}(d+g)}{D_g}$$

with $$\frac{d(F_{11}/L)}{dg} = a_{11}a_{12}a_{2n}(d+g) \quad l_n(1+N_g)+a_{2n}l_2(d+g) \quad D_g^{-2} > 0$$

(4) $$\frac{F_{12}}{L}(g) = \frac{a_{12}a_{2n}N_g(d+g)}{D_g}$$

with $$\frac{d(F_{12}/L)}{dg} = a_{12}a_{2n}l_n \quad 1-q_1 l_1(d+g) \quad ^2 -q_2 q_n^* \, l_1^2(d+g)^2 \quad -D_g^{-2} \gtreqless 0$$

(5) $$f_2(g) = a_{2n}\frac{N_g}{D_g} \qquad \text{with} \quad \frac{df_2}{dg} = a_{2n}\frac{dc}{dg} < 0$$

(6) $$\frac{f_1(g)}{f_2} = \frac{a_{12}(d+g)}{N_g} \qquad \text{with} \quad \frac{d(f_1/f_2)}{dg} = a_{12}N_g^{-2} > 0$$

<u>Allocation of Resources</u>

(7) $$\frac{F_{11}}{F_1}(g) = a_{11}(d+g) \qquad \text{with} \quad \frac{d(F_{11}/F_1)}{dg} > 0$$

(8) $$\frac{F_{12}}{F_1}(g) = 1-a_{11}(d+g) \qquad \text{with} \quad \frac{d(F_{12}/F_1)}{dg} < 0$$

(9) $$\frac{F_{11}}{F_{12}}(g) = \frac{a_{11}(d+g)}{1-a_{11}(d+g)} \qquad \text{with} \quad \frac{d(F_{11}/F_{12})}{dg} = a_{11}N_g^{-2} > 0$$

(10) $$\frac{F_{2n}}{F_2}(g) = 1$$

(11) $$\frac{L_1}{L}(g) = \frac{a_{12}a_{2n}l_1(d+g)^2}{D_g} \qquad \text{with} \quad \frac{d(L_1/L)}{dg} = \frac{l_1}{a_{11}}\frac{d(F_{11}/L)}{dg} > 0$$

(12) $$\frac{L_2}{L}(g) = \frac{a_{2n}\, l_2 N_g(d+g)}{} \qquad \text{with} \quad \frac{d(L_2/L)}{dg} = \frac{l_2}{a_{12}}\frac{d(F_{12}/L)}{dg} \gtreqless 0$$

(13) $$\frac{L_1}{L_2}(g) = \frac{a_{12}l_1(d=g)}{l_2 N_g} \qquad \text{with} \quad \frac{d(L_1/L_2)}{dg} = \frac{l_1}{l_2}a_{12}N_g^{-2} > 0$$

(14) $$\frac{L_n}{L}(g) = l_n c \qquad \text{with} \quad \frac{d(L_n/L)}{dg} < 0$$

<u>Composition of Production</u>

(15) $$\frac{O_1}{O_n}(g) = \frac{a_{12}a_{2n}(d+g)^2}{N_g} \quad \text{with} \quad \frac{d(O_1/O_n)}{dg} > 0$$

(16) $$\frac{O_2}{O_n}(g) = a_{2n}(d+g) \qquad \text{with} \quad \frac{d(O_2/O_n)}{dg} > 0$$

(17) $$\frac{O_1}{O_2}(g) = \frac{a_{12}(d+g)}{Ng} \qquad \text{with} \quad \frac{d(O_1/O_2)}{dg} > 0$$

Curves (1)-(17) represent comparisons of economies using the same tchnique but operating under a different growth rate. To each exogenously given rate of growth belongs a special real structure of the economy. The dynamic Traverse between one growth equilibrium and another necessarily would involve a change in the whole quantity structure and - as Lowe (1976) shows - surely would not be carried out in terms of the equilibrium relations (1)-(17), which represent curves on which one cannot move in historical time.

"To move, to change outputs, means investing, because the system cannot change output unless it first changes inputs. Yet to change inputs is to <u>draw on the output of the input-producing sectors.</u> The system itself determines its own capacity for change; the capital-goods sector must provide the goods necessary to change the inputs to produce the new pattern of output. In turn, it is implied that the capital-goods sector must produce different goods, or deliver them to different investors, than it did previously. Moreover, this change will necessarily entail shortages and/or surplus capacity in some parts of the economy, relative to he previous full capacity utilization." (Nell, 1976, p. 308)

Obviously, the size of the output-machine ratio in sector 1 places an important upper limit upon the rate of growth achievable. The maximum rate of growth is given by

$$g_{max} = \frac{1}{a_{11}} = d$$

which is a technological datum given by the technique, i.e. independent not only on relative prices but also of the output-machine ratios of the other two sectors. Of course, g_{max} is a theoretical ceiling which is economically irrelevant, because the whole system would only produce primary equipment goods, i.e. the rlative shares of sectors 2 and n would be zero.

Equation (1) exhibits the well-known, montonically inverse consumption-growth relationship providing an essential framework for Lowe's analysis, for it shows the various possible 'dynamic' equilibria between which traverse may take place. The identity of equations (17) and (6) reflects the fact that with steady growth the composition of investment is the same as the composition of the capital stock. Furthermore, one can see that the economy with the higher growth rate

a) has a greater outfit per capita with primary equipment goods or machine tools which are used in sector 1 (eq. 3)

has a smaller outfit with secondary equipment goods (5),

and a greater relation between stocks of primary and secondary equipment goods (5),

b) uses a higher percentage of its primary equipment goods in the key sector 1 than in sector 2 (7, 9),

uses a higher percentage of its labor force in the production of primary equipment goods (11),

also in relation to the production of secondary equipment goods (13),

and uses a lower percentage of its labor force in the production of consumer goods (14).

c) The composition of production is such that the economy with the higher growth rate produces relatively more equipment goods than consumer goods (15, 16) and relatively more equipment goods of the primary type (17). Equations (6), (9), (13) and (17) are reflecting the fact that the contradiction between consumption and accumulation manifests itself in the proportion between the two sectors producing means of production.

But some open questions remain, which all have to do with the specific Lowe-sector 2. The indeterminateness of equations (4) and (12) reflects the characteristic position of sector 2 operating as the bridge between the basic sector 1 and the consumer goods sector n. The reason for this indeterminateness is evident. Sector 2 participates with lower percentages (8 and 13) in a bigger cake (2a and 14). The higher the growth rate the higher is the weight of sector 1 and the lower the weight of sector n - no doubt about that. But the weight of sector 2, using inputs produced in sector 1 and producing equipment goods used in sector n, is influenced by both these factors with no unique result.

IV. Income Distribution and the Price System

We go on now to introduce the relations governing wages, profits, and relative prices in the Lowe model. Competitive equilibrium in a capitalist economy implies that firms obtain the same rate of profit on the value of invested capital. In any period, the equilibrium conditions for the price system are such that the price of a unit of output

must exactly cover its cost of production, equal to wages which are paid at the end of the period plus gross profits consisting of depreciation and net profit. Thus we get

$$(d+r)\,\underline{A}\,\underline{p} + w\underline{l} = \underline{p}$$

or, choosing the consumption good as the numeraire commodity the following price equations:

$$\text{(III)}\qquad \begin{aligned} a_{11}(d+r)\,p_1 + wl_1 &= p_1 \\ a_{12}(d+r)\,p_1 + wl_2 &= p_2 \\ a_{2n}(d+r)\,p_2 + wl_n &= 1 \end{aligned}$$

Note that like the quantity system the price system gives us three equations in the four price variables w, r, p_1 and p_2, where the prices of primary and secondary equipment goods, p_1 and p_2, are relative prices and like the wage rate w and the rate of profit r expressed in units of the consumption good. The price system may be closed through e.g. a subsistence wage theory of a 'central bank theory of the rate of profits' (see Sraffa 1960, p. 33). The equations yield the following unique relationships between the rate of profit, on the one hand, and the wage-rate and relative prices, on the other, all fully identified by the given coefficents of the technique.

$$\text{(18)}\qquad w = \frac{1-a_{11}\,(d+r)}{I_n+(a_{2n}l_2-a_{11}l_n)(d+r)+a_{2n}(a_{12}l_1-a_{11}l_2)(d+r)^2} = \frac{N_r}{D_r}$$

with $\frac{dw}{dr} < 0$

$$\text{(19)}\qquad p_1 = \frac{l_1}{N_r}$$

with $$\frac{dp_1}{dr} = -\,l_1^2\quad l_n(q_n^* - q_1)+2a_{2n}l_2(q_2-q_1)(d+r)\quad N_r^{-2} \gtreqless 0$$

$$\text{(20)}\qquad p_2 = \frac{l_2+(a_{12}l_1-a_{11}l_2)\,(d+r)}{N_r}$$

with $$\frac{dp_2}{dr} = \frac{l_1 l_2}{\frac{2}{N_r}}\quad l_n\,(q_2-q_n^*)-a_{2n}(q_2-q_1)(d+r)\quad l_2(1+Z_r)+a_{12}l_1(d+r)$$

$$(21)\quad \frac{P_1}{P_2} = \frac{l_1}{l_2+(a_{12}l_1-a_{11}l_2)(d+r)}$$

$$\text{with}\quad \frac{d(P_1/P_2)}{dr} = l_1^2 l_2(q_1-q_2)\,[\,l_2+(a_{12}l_1-a_{11}l_2)(d+r)\,]^{-2} \gtreqless 0$$

$$\text{for } q_1 \gtreqless q_2$$

Curves (18)-(21) represent comparisons of economies using the same technique but operating under different distribution patterns. The prices of production are uniquely determined by technology and the given distribution of net output and are independent of the level and composition of output. The wage-profit relationship (18) turns out to have exactly the same parametric form as the consumption-growth relationship (1), with the rate of profit in the place of the growth rate and the wage rate in the place of consumption per capita, so that the two corresponding curves are the exact replica of each other. But it is a characteristc feature of the Lowe model that in contrast to the common two-sectoral or multi-sectoral (see Pasinetti 1977, pp. 199-208) models duality ceases as soon as we compare the relationships between prices and the rate of profits, $p_1(r)$, $p_2(r)$ and $p_1/p_2(r)$, with the relationships between quantities and the rate of growth, $f_1(g)$, $f_2(g)$, and $f_1/f_2(g)$.

Now we must consider how prices vary with the rate of profit. Looking at system (III) we can see that there are two elements in the price of each commodity which vary in opposite directions with differences in the monotonically inverse w-r-relationship. Since Sraffa we know that the key to the movement of relative prices lies in the inequality of the proportions in which labor and means of production are employed in the various industries. With different sectoral machine-labor ratios restoring a uniform rate of profit in the three sectors requires different relative prices. In the Lowe model the sectoral machine-labor ratios q_1, q_2, and q_n play the crucial role for fixing direction and extent of these price differences associated to different rates of profit.

Looking at Table 1 reminds us that there exists one exception of the general rule that prices depend on distribution. It is the famous Ricardo-Marx-Samuelson case of equal organic compositon of capital which is the Lowe model takes the form $q_1=q_2=q_n$. In this case 6 all three sectors are using directly or indirectly (sector n) the same ratio of primary equipment goods per unit of labor. Prices reduce to

$$p_1=l_1/l_n;\ p_2=l_2/l_n;\ p_1/p_2=l_1/l_2,$$

thus, the labor theory of value holds in this special case which marks a watershed between all other cases.

As long as primary equipment goods are produced with the highest (lowest) ratio of primary equipment goods per unit of labor machines are relatively more (less) expensive than tractors or corn when the rate of profit is higher. The first derivation of equation (19) only becomes indeterminate when q_1 is in the middle position between q_2 and q_n.

Table 1

Case	Constellation	$\frac{dp_1}{dr}$	$\frac{dp_2}{dr}$	$\frac{d(p_1/p_2)}{dr}$
1	$q_1 > q_2 \gtrless q_n^*$	+	+	+
2	$q_1 > q_n^* > q_2$	+	?	+
3	$q_1 = q_n^* > q_2$	+	?	+
4	$q_n^* > q_1 > q_2$	?	?	+
5	$q_1 = q_2 > q_n^*$	+	+	0
6	$q_1 = q_2 = q_n^*$	0	0	0
7	$q_1 = q_2 < q_n^*$	-	-	0
8	$q_1 < q_n^* < q_2$	-	?	-
9	$q_1 = q_n^* < q_2$	-	?	-
10	$q_n^* < q_1 < q_2$	?	?	-
11	$q_1 < q_2 \gtrless q_n^*$	-	-	-

Reminding that open questions in the quantity system only arise in connection with the specific Lowe-sector 2 in considering how p_2 varies with the rate of profit we can now see that numerous open questions exist. They always appear when the corresponding machine-labor ratio q_2 is the lowest (cases 2, 3, 4) or highest (cases 8, 9, 10) one. Clear results emerge as soon as the machine-labor ratio of this sector 2 operating as the bridge between sectors 1 and n is also in the middle position.

Whereas p_1 and p_2 depend on the technical coefficients of the consumer goods sector (only as a consequence of our choosing the consumer good as the numeraire commodity since good n is a non-basic) their relation does not. No open question arises concerning the sign of the first derivation of equation (21). The price relation p_1/p_2 moves directly with the rate of profit if q_1 q_2,[6] and inversely if q_1 q_2, i.e. the relation of the sectoral machine-labor ratios is decisive.

V. Saving, Income Distribution, and Capital Accumulation

Up to now the two systems of price and quantity equations were considered as independent of each other, although only for didactic reasons. As is well known, the postulation of a savings function in connection with he Keynesian equilibrium condition of saving-investment equality constitues a relationship between the rate of profit and the growth rate. There remains one degree of freedom in the whole system: given either the growth rate or the rate of profit, all price and quantity variables as well as the combined price - quantity variables like the capital-output or capital-labor ratio are then simultaneously determined.

Choosing a Kaldorian savings function, net savings per unit of labor equal

$$(22)\quad S/L = s_w w + s_r r(P_1 f_1 + P_2 f_2),$$

where s_w and s_r denote the propensities to save from wages and profits, with $1 \geqq s_r > s_w \geqq 0$.

Net investment per unit of labor is

(23) $$I/L = g(P_1f_1+P_2f_2).$$

Thus we get the following r-g-relationship:

(24) $$r = \frac{g}{s_r} - \frac{s_w w}{s_r(P_1f_1+P_2f_2)}$$

Alternative assumptions concerning saving behavior provide a direct link between income distribution and capital accumulation. When there is a different saving proportion for different categories of income, the r-g relationship depends on the distribution of income as well as on technology. In the special case of "classical" saving behavior with s_w=0 equation (24) reduces to

(24a) $$g(r) - s_r\, r(g) = 0$$

i.e. the growth-profit relation is independent of the technology. Our formulation of equation (24a) reflects the fact that there exists a two-sided relationship between profits and investments. Therefore, with view of 'causality' formula (24a) is more neutral than the famous 'Cambridge equation' $r=g/s_r$ according to which the equilibrium rate of profit is determined by the natural rate of growth divided by the capitalists' propensity to save, independently of anything else. Lowe who rather neglects the distribution aspect prefers to work with he very special case of a 'superclassical' savings function, i.e. s_w=0 and s_r=1, which implies the coincidence of r and g ('golden rule'). The reason for this proceeding lies in the fact that his main field of study is 'traverse anlysis', i.e. "the path that connects two dynamic equilibria defined by different rates of change" (lowe 1976, p. 103), and these different g's are assumed to be exogenously given.

VI. Traverse: Structural Analysis and Adjustment to a Higher Rate of Growth of Labor Supply

Lowe's structural analysis underlies the traverse from one natural rate of growth to another in which the capacity augmenting role of investment is at issue. Since Harrod and Domar we know that the capacity augmenting and theincome generating aspect of investment must fit together. Lowe puts the whole strain of adjustment on the savings ratio.

In general, the natural rate of growth consists of two components, the rate of growth of labor supply and the rate of growth of productivity. We are dealing here only with the type of traverse caused by a once for all rise in the exogenously given rate of growth of labor supply.[7] The adjustment process of an economy which experiences an additional influx of labor leads from an initial steady-state growth path to a new one with a higher rate of growth (Lowe 1976, p. 1238). In the above analysis we have compare economies which use the same technique but operate under a different growth and profit rate. Thus we are prepared for the traverse analysis in this section on the assumption that we know the different outfits with consumer and equipment goods, allocations of resources, and compositions of production in the old and in the new steady

state. Furthermore, we know that the ability of an economy which is reacting to a change in the exogenously given growth rate of labor is limited by the quality and structure of the existing capital stock which has now become partly inappropriate. It is clear that the formation of a complementary addition to real capital is the essential precondition for reaching the final equilibrium in which absorbing the additional increment of labor is absorbed and all new workers are provided with consumption goods. Since the balanced capital stock compositions differ between the final and the initial equilibriums the three sectors are growing at different rates on the adjustment path which can be subdivided into four phases.

1. The key to a higher growth rate lies in increasing the shares of sector 1. The same logic requiring that the system as a whole first has to change inputs before it can change output makes such an increase dependent on the prior expansion of capital stock of this sector. At this place a characteristic property of the Lowe model comes into play, i.e. that sectors 1 and 2 are using the same type of equipment goods so that an ex post transfer of machines between these two sectors is possible. In the case of a rising rate of growth the proportion of machines devoted to making primary equipment goods during the first phase of the adjustment process. Since this intersectoral shift leads to a reduction in the growth rate of output of secondary equipment rate is necessary. Obviously, the cost of increasing this growth rate of consumption in the long run through increassing F_{11}/F_1 are greater the higher is a_{11} relative to $a_{12}a_{2n}$. Shifting a machine from sector 2 to sector 1 increases the output of primary equipment goods by $1/a_{11}$ but reduces the output of secondary equipment goods by $1/a_{12}$ and thereby consumption output in the next period by $1/a_{12}a_{2n}$.

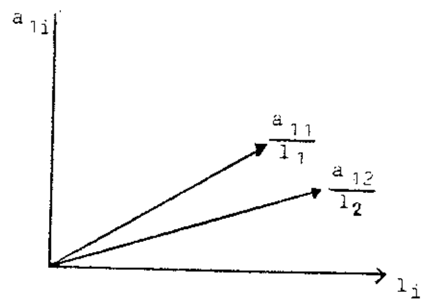
2. Lowe's path criterion of maximum speed of adjustment requires that up to the 'point of maximum expansion from within' all savings have to be invested in the key sector producing primary equipment goods to initiate the process of self-augmentation. At the end of this second phase of the adjustment process the terminally required addition to capital stock F_1 is accomplished[8] and the original level of employment is restored. "Once the labor force operating in the initial dynamic equilibrium has been brought back into employment, aggregate output at the point of maximum expansion from within equals the original output in value terms." (Lowe 1976, p. 130) This does not however, hold for the relative size of the sectors and, thus, for the physical composition of production. The shares of sector 1 have increased.

3. The third phase of the traverse is characterized by the adjustment of the capital stock of sector 2 to the higher rate of growth.

4. In the last phase consumption output adjusts. When the new steady state is reached all sectors will expand with the same rate, the new and higher growth rate of the labor force.

During the traverse there are not only sectoral contortions in quantities but also in prices. Disequilibrium manifests itself in price changes and in non-uniformity of the sectoral profit rates. This divergence of the sectoral profit rates is a strong incentive for the whole adjustment process, which is completed when the rate of profit has reached its new uniform level. From equations (19) to (21) we know that the system of relative prices in the new steady state differs from that in the old one according to the different rate of profit, except in the Lowe-Samuelson case where relative prices are independent of the rate of profit.

The range of Lowe's traverse analysis is limited by the fact that he works throughout with the assumption of identical machine-labor ratios. His statement that perfect substitutability of the capital stocks in the two equipment-goods sectors makes possible "a simple shift of factors between the two subsectors of the capital-good industry" (1976, p. 97) has to be modified as soon as q_1 q_2. Despite the fact that sectors 1 and 2 are using the same type of equipment goods, the transfer of resources between these two sectors a major problem poses which Lowe eliminates through his special assumption. This will become immediately clear from our simple example.



Since q_1 is higher than q_2 a shift of machine tools from sector 2 to sector 1 unequivocally sets free greater amounts of labor relative to primary equipment goods in sector 2 than can be absorbed by using the same machines in sector 1. Thus, the employment problem is aggravated in this case. Working out Lowe's traverse analysis for the non-Samuelson case is thus clearly an important field for future research.

Lowe's instrumental analysis is considerably richer than our rather technocratic structure analysis, which is only preliminary to motor or force analysis. In contrast to Fel'dman and Dobb, force analysis plays a central role in his work because his special interest is to analyze traverse processes in free market systems. His force analysis reveals the significance of a functioning price mechanism (which is in no sense a mechanical apparatus) for an efficient traverse and highlights that, in general, measures of public control become inevitable, especially with investment decisions. Lowes' political economics which conceives the attainment of full employment as inseparable from the respect to ex ante planning of sectoral proportions, thus provides a rationale for planning in a mixed economy which has achieved economic maturity.

Footnotes

[1]Instead of an economy facing this 'Fel'dman constraint' on the investment capacity side there can exist the reverse possibility of an economy facing the 'Preobrazhenski constraint' posed by the consumption side. If, e.g., the initial capacity of the equipment goods industry is sufficient only to replace the depreciated machines growth can only take place as a result of a temporary reduction in the output of consumer goods. If this reduction is impossible because of subsistence reasons then a condition was nearly fulfilled in early Soviet Russia.

[2]Of course, there are some other differences like Fel'dman's Lewis type assumption of an unlimited supply of labor.

[3]For a detailed criticism of Dobb's use of the Lowe model see Hale vi (1981 and 1983). Halevi (1981) also compares Lowe's approach with that of Kalecki.

[4]Our analysis therefore has some affinity to Nell's alternative presentation of Lowe's basic model (1976) although differing in the way of reasoning and in some results. The main point of departure from Nell's interpretation is that in our analysis only good 1 is a basic product whereas Nell's translation of hours of labor into amounts of the consumption good translates also the other two goods into basics. Both interpretations are comptible with Lowe's equilibrium analysis in Part I (especially pp. 37-40) where he points out that "we need not fall back upon the classical hypothesis that labor should be treated as if it were 'produced' by the 'input' of consumer goods" (Lowe 1976, p. 33). However, Nell's interpretation is not comptible with Lowe's traverse analysis where Lowe makes pretty clear that there is a strong hierarchy of sectors in his model containing only one basic product, so that e.g., a pure labor-displacing innovation in the consumer-good sector creates no secondary effects for both equipment goods sectors (Lowe 1976, Chapter 24).

[5]The assumption that the rates of depreciation of the fixed capital goods are given coefficients in the list of parameters of the system implies that the economic lifetime of each capital good is a constant determined independently of distribution. This assumption of 'depreciation by evaporation' or 'radioactive decay' is as essential to Samuelson's demonstration of the existence of a surrogate production function as is the assumption of identical machine-labor ratios in each sector. This 'equal proportions' assumption suffices to make the value of a new machine independent of r and w but it does not suffice to make the wage-profit relationship linear andthe price of an old machine being independent of w and r (see Steedman 1979). An adequate treatment of the economic lifetime of fixed capital goods with different efficiency profiles requires a von Neumann-Sraffa approach. See Hagemann and Kurz (1976) who also show that the return of the same length of the production process and reswitching of techniques are closely linked phenomena. Our analysis therefore has some affinity to Nell's alternative presentation of Lowe's basic model (1976) although differing in the way of reasoning and in some results.

[6]Nell's deviating result that p_1/p_2 depends only on the machine-input coefficients and not on the labor-input coefficients of sectors 1 and 2 rests upon a mistake in the numerator of his formula (Nell 1976, p. 318) for the first derivation of our equation (21) which correctly must read n_a $(a_a n_b - a_b n_a)$ in the Nell notation so that the rlation of a_a/n_a to a_b/n_b and not a_a to a_b decides on the sign of the first derivation of the relative price of the two equipment goods.

[7]This traverse problem was first analyzed by Hicks in Chapter 16 of 'Capital and Growth' (1965) on the basis of a two-sectoral fixed coefficient model whereas in 'Capital and Time' (1973) Hicks analyses the problem of a traverse caused by a change in technology on the basis of a neo-Austrian model. To a detailed comparison of Hicks' more recent traverse analysis with Lowe's analysis of traverses caused by nonneutral innovations in Part IV of his 'Path of Economic Growth' see Hagemann (1983).

[8]Since this process requires time the expansion of the capital stock has to be large enough to absorb not only the permanently increased period flow of labor but also the interim stock of idle labor that accumulates during the construction of the additional machines.

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