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Money in The Production Function: An Alternative Test Procedure

Keith Christian Jensen, Shyam Kamath and Robert Bennett*

INTRODUCTION

A major controversy in neoclassical production and monetary theory relates to the inclusion of real money balances as an input in the production function. In attempting to integrate the theory of money with the pure theory of production, a number of alternative approaches have been suggested but the most direct link has been attempted by theoretical and empirical economists who have included real money balances in the neoclassical production function. Friedman (1959) was perhaps the first to make the suggestion that real balances be considered as a productive factor input but it was subsequent work by Johnson (1969) which rationalized the conception of money as an explicit input in the microanalysis of the neoclassical production function. Related work by Bailey (1971), Levhari and Patinkin (1968), McGregor and Walters (1973), Nadiri (1969) and Saving (1971) included real balances as an input in the aggregate production function, which is specified analogously to the microeconomic production function.

The empirical debate over the inclusion of real money balances in the aggregate production function was initiated by Sinai and Stokes' seminal 1972 paper. In that paper, they estimated an aggregate Cobb-Douglas production function for the U.S. private domestic sector and produced results showing that real money balances have a significant effect as an input in the production function. A number of papers were written modifying and criticizing the Sinai and Stokes hypothesis but, for all intents and purposes Sinai and Stokes (1975, 1980, 1981) seem to have effectively replied to every one of the criticisms (the major criticisms and the rebuttals are discussed below).

However, we believe that the supposed settlement of the debate in favor of Sinai and Stokes is attributable to the manner in which the hypothesis was tested using conventional test procedures. In this paper we develop an alternative test procedure which is logically more comprehensive and rigorous than conventional procedures and which imply that Sinai and Stokes' claim of success for their original hypothesis is brought into question. We show that all the evidence produced in support of the Sinai-Stokes hypothesis cannot be sustained on theoretical and empirical considerations. We believe the alternative test procedure and the results obtained will shed new light on the discussion initiated by Sinai and Stokes.

The debate over whether real money balances belong in the aggregate production function has essentially been confined to the matter of misspecification and appropriate testing procedures. The confirmation tests used (values for t-statistics, $\bar{R}^2$, standard errors, etc.) including the more sophisticated specification tests (such as the BAMSET, RAMSET, etc.),

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are however insufficient for rigorous testing of the theory. In the second section, we briefly summarize the debate over real money balances as an omitted variable in the aggregate production function. In the third section, we discuss problems that arise in conventional test procedures in the context of the money-in-the-production-function debate. The fourth section develops the alternative procedure which constitutes a logically complete and rigorous extension of our conventional procedures. The fifth section applies the alternative test procedure to the debate. A summary and some conclusions are presented in the sixth section.

The Money-in-the-Production-Function Debate

Sinai and Stokes (1972) directly tested the hypothesis that real balances are a factor input into the neoclassical production function.[1] The rationale for including real balances in the production function was the increased efficiency of monetary economies as compared with a barter economy because "... in a monetary economy, productive efficiency may increase as labor and capital services, released from the special tasks required in a barter economy, are used in production" (1972, p. 290). They formulated a Cobb-Douglas production function which included real money balances as the third factor of production in addition to capital and labor as follows:

(1) $$Q = Ae^{\lambda T}L^{\alpha}K^{\beta}m^{\gamma}u$$

where Q = output
L = labor
K = capital
m = real money balances
T = time trend
A = efficiency parameter
λ = rate of disembodied or neutral technical change
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Without insisting that this equation is the "correct specification," their purpose being "simply to examine the potential significance of real money balances in the production function" (p. 291), they estimated the following log-linear transformation of (1)

(2) $$\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln m + \lambda T + u$$

Using ordinary least squares, they found real money balances, alternatively measured as the common monetary aggregates M1, M2 and M3 divided by a price index, to be significant, adding slightly to the goodness of fit ($\bar{R}^2$), but exhibiting a substantial degree of autocorrelation in the estimated equation. They corrected for this by applying a second-order GLS correction, but failed to check for multi-collinearity in the regression. On the basis of this test they concluded that real money balances are an important input in the neoclassical production function.

A number of criticisms of the Sinai and Stokes findings were subsequently published, mainly in the *Review of Economics and Statistics*. The criticisms can be grouped into the following broad categories: (a) Criticisms regarding the autoregressive transformation used by Sinai and Stokes in correcting their OLS estimates. These include the two comments by Prais

(1975a, 1975b). The theoretical thrust of these two criticisms was that the nature of the relationship actually modeled was not a production function but a mongrel relationship since the transformation used resulted in the change in *real value of money* balances and not the *level* of such balances being used as an argument in the function. Consequently, Prais argued that the Sinai-Stokes results were the outcome of "a misuse of econometric techniques" (1975a, p. 243). In their rebuttal, Sinai and Stokes (1975, 1977) argued that the criticisms were invalid because of a lack of understanding on Prais' part of the type of correction actually used. They cited Khan and Kouri's (1975) and Butterfield's (1975) results using simultaneous equation models as vindicating their results and effectively dealing with Prais' suggestion that simultaneous behavioral functions be considered. (b) Comments regarding the effects of simultaneity on the Sinai-Stokes estimates. These include the comments by Butterfield (1975) and Khan and Kouri (1975) mentioned above, the article by Short (1979), and a related note by Finnerty (1980). The first three comments were intended to provide further "confirming evidence" regarding the Sinai-Stokes hypothesis in a simultaneous equation framework while the last provided a purely theoretical argument for including real cash balances. (c) Criticisms regarding specification errors in Sinai and Stokes' formulation. Included here are the comments by Ben-Zion and Ruttan (1975), Boyes and Kavanaugh (1979) and Niccoli (1975). The impact of these criticisms was that other "appropriate" approaches (the "induced innovation" approach (Ben-Zion and Ruttan), the C.E.S. functional form (Boyes and Kavanaugh) and the "investment" approach (Niccoli)) were more supportive of the data and hence Sinai and Stokes' interpretation of the results were highly questionable. Sinai and Stokes (1975, 1980) replied to these criticisms by arguing that the inclusion of variables suggested under the alternative approaches were themselves highly questionable both on theoretical and empirical (e.g. unstable coefficients) grounds. They showed that "more powerful" specification tests (e.g. BAMSET, RAMSET etc.) revealed the statistical deficiencies of alternative specifications, particularly the Boyes and Kavanaugh (1980) results.

What is significant about the debate is that it essentially focused on the question of misspecification and whether appropriate empirical definitions and testing procedures were adopted. More importantly, both the original Sinai-Stokes contribution and the criticisms were presented in the conventional "confirmationist" mold.[1] The tests conducted by the different protagonists were aimed at finding confirming instances of their version of the theory or an alternative theory. There was no attempt to deal with the logical problems of "confirmatory" testing. The confirmation tests used (values for t-statistics, DW statistics, $\bar{R}^2$ etc.) including the more sophisticated specification tests (such as the BAMSET, RAMSET etc. tests) however are insufficient for the rigorous testing of the Sinai-Stokes hypothesis. It is not possible to be sure that it is not one or more auxilliary hypotheses which are responsible for anomalous test results, rather than the particular hypothesis under consideration. This "Duhem-Quine problem" has been recognized in the economics literature and several authors Archibald (1959), Boland (1977), Cross (1982) and Roll (1977)) have noted that the models examined in empirical tests must of necessity involve a conjunction of several statements. Boland (1977) points out that models can be broken down into three parts:

PART (A):

A_1
A_2 — The "Core" Behavioral Assumptions of the Theory
A_3 . . . e.g. $Q = f(L, K, m)$
A_n

PART (B):

B_1		
B_2		The Simplifying Assumptions of the Theory
B_3	. . .	e.g. $Q = Ae^{\lambda T}L^{\alpha}K^{\beta}m^{\gamma}$ and $\alpha + \beta + \gamma \geq 1$
B^n		

PART (C):

C_1		
C_2		The Testing Conventions i.e. procedural specifications and rules of evidence
C_3	. . .	e.g. the evidence constitutes a confirming instance of model if $\bar{R}^2 \geq .90$ and t-values and D.W. 2, etc.

Utilizing Boland's framework in relation to the Sinai and Stokes test does not put their core Part (A) hypothesis ("money matters") at stake. It boils down only to a "confirmation" of real money balances in a specific form of a production function (Cobb-Douglas) at Part (B). At best their test relates to Parts (B) and (C).[2] There is however an alternative test procedure developed by Robert Bennett (1981) which allows us to focus on the "core" statements of a theory while testing a model of the theory.

An Alternative Test Procedure

The Bennett procedure involves empirically testing a model of a theory (i.e. of Parts (A), (B) and (C)) along with logical counterexamples of the theory's "core." These counterexamples, being statements logically denied by the theory (e.g. "there is at least one Q which does not change when m changes in $Q = f(L, K, m)$"), are then specified in model form and tested using similar, or if possible, exactly the same methods and (confirmation) criteria as were used to test the model of the theory. A counterexample, if found to be supported or confirmed, would provide logical grounds for seriously questioning the original theory. To the extent that a valid counterexample at the "core" (Part (A)) can be found, contingent on the acceptance of the conventions and criteria in Parts (B) and (C), this procedure provides a means of getting around the Duhem-Quine thesis.

The outcome of testing both a model of the theory and a counterexample of the theory constitutes a logically complete and rigorous extension of our standard testing procedures. In traditional testing we have only two possible outcomes, either a substantially "GOOD-FIT" or a "BAD-FIT." Since the Bennett Test Procedure is a conjunction of statistically estimating both a model of the theory and a counterexample of the theory the number of possible outcomes is as follows:

TABLE I
The Bennett Table

Case	Model (theory)	Counterexample (~theory)	Comments
I	Good-Fit	Bad-Fit	(corroborations)
II	Good-Fit	Good-Fit	(???)
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Cases I and II demonstrate that while using confirmation criteria alone, we can run into two contradictory situations. Case I is what is implicitly assumed in conventional testing when a model is a statistical GOOD-FIT. However, by not testing on "the other side of the street" (a counterexample of the theory), the possibility of Case II is overlooked. A Case II test result should raise questions about the test procedures, the ancillary assumptions and rules of evidence as well as the "core" behavioral assumptions. Because of the logical nature of the counterexample, the empirical ambiguity of Case II leaves us no choice but to reexamine both the articulation of our theoretical concepts and the sophistication of our testing methods. Since the counterexample is by construction at the "core" behavioral assumptions level (Part (A)), a GOOD-FIT for the counterexample when Parts (B) and (C) are placed beyond question seriously brings into question the behavioral theory.

If we were to consider testing only the model without the counterexample, Cases III and IV provide us with the traditional "rejection" of a model or theory. Yet, here again, our traditional "rejection" is based on the assumption that the logic of the outcome was of the Case III type. When our statistical methods are beyond question, the outcome of Case III would be considered a refutation of the theory that is, the "core" assumptions. Case IV is also important because this case would encourage us to look for ways of improving our theories and testing conventions. Modifying the rules of evidence until a Case I, II, or III result is obtained is not a recommended procedure. Adjusting the rules of evidence after the fact could have an unpredictable effect on the likelihood of errors analogous to the Type I and Type II errors of hypothesis testing. The rules of evidence should, at least ideally, reflect our understanding of the influencing factors on observed reality not accounted for in the tests. Case IV results should not therefore be simply discarded.

The alternative approach is superior to testing procedures designed to find confirming instances of the theory (alone) for the reason that it is logically possible to use a confirming instance of the "core" counterexample to argue for (but not prove) the falsity of the theory while on the other hand, it is not logically possible to use a confirming instance of the theory's prediction to argue for the truth of the theory. Such an approach would seem to provide an adequate basis for a Popperian approach to econometric methodology as has been prescribed by Blaug (1980), Hendry (1980, 1983) and others.

The test procedure is also superior because, in contrast to the case in conventional testing where failure to find a confirming instance of an observational model of the theory cannot be treated as negative evidence (i.e. as a basis for criticizing a theory), inappropriate testing conventions are not likely to lead to deceptive results. It is more likely that a confirming instance of a model of the theory will not be found because of inappropriate testing conventions, than for the same reason, a confirming instance of a model of the counterexample will be found.

However, some qualifications need to be made regarding the alternative procedure. It is still not possible to *prove* that the theory is false by using the alternative procedure because the problem of induction remains. Specifically, there are two unsolvable aspects of the problem of induction that make the test results only a contingent basis for arguing that the theory under examination is false. In the first place, there is no method available by which one can establish the truth of an observation statement. And secondly, it is not possible to prove that an observation statement can, or cannot be considered as a confirming instance of a model of a counterexample, even if the universal statement to which the counterexample corresponds is false and the real world is such that refuting evidence is there to be observed. It is for this reason that model building and testing conventions become important. On the other hand, a major are however insufficient for rigorous testing of the theory. In the second section, we briefly summarize the debate over real money balances as an omitted variable in the aggregate production function. In the third section, we discuss problems that arise in conventional test procedures in the context of the money-in-the-production-function debate. The fourth section develops the alternative procedure which constitutes a logically complete and rigorous extension of our conventional procedures. The fifth section applies the alternative test procedure to the debate. A summary and some conclusions are presented in the sixth section.

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Results of the Alternative Test Procedure

In applying the Bennett Test Procedure to the Sinai and Stokes hypothesis we chose to look at their Cobb-Douglas specification:[3]

(3) $$Q = Ae^{\lambda T}L^{\alpha}K^{\beta}m^{\gamma}u$$

with the usual interpretation on the variables. Utilizing the same Christensen and Jorgenson-Friedman and Schwartz (1969, 1970) data set we replicated closely the Sinai and Stokes results in the log-linear form:

(4) $$\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln m + \lambda T + u$$

In deriving a counterexample of the core hypothesis we were in effect looking for a hypothesis which contained real money balances empirically, but in a way that would contradict the hypothesis that real balances mattered in the production function.[4] Since Sinai and Stokes tried to fit a Cobb-Douglas production function, our counterexample is also developed in a Cobb-Douglas form. This is deflated by the real value of money balances,

(5) $$Q/m = Ae^{\lambda T}(L/m)^{\alpha}(K/m)^{\beta}u$$

so as to maximize the number of features the conventions of the two tests have in common. Transforming (or deflating) the variables where $Q^* = (Q/m)$, $L^* = (L/m)$, and $K^* = (K/m)$, we estimated (5) in log-linear form as follows:

(6) $$\ln (Q^*) = \ln A + \alpha \ln (L^*) + \beta \ln (K^*) + \lambda T + u$$

A money-deflated production function of the type hypothesized in (5) would be denied by Sinai and Stokes' original hypothesis since, if it passed the conventional tests, it would demonstrate a zero influence for money. Equation (3) would imply that it is money-deflated values of L and K that matter as the "true" underlying relationship.[5] Confirmation of equation (5) using the same Part (B) and Part (C) conventions and criteria would provide sufficient grounds to criticize the inclusion of the real value of money balances in the production function.[6]

The reason equation (3) constitutes a valid "core" counterexample can be seen from the results that Sinai and Stokes obtained. Their conclusion can be summarized in terms of the following values of the parameters of their model:

$$\alpha + \beta + \gamma > 1, \quad \text{and} \quad \gamma \neq 0$$

If, in fact, our counterexample, deflated by the real value of money balances, were to provide a GOOD-FIT, given the confirmation criteria, it would imply that:

$$\alpha + \beta = 1, \quad \text{and} \quad \gamma = 0$$

In such a case, it could be said that a change in m does not affect Q and thus money does not matter in the Cobb-Douglas production function. This would seriously put into question the Sinai and Stokes hypothesis. On the other hand, if the counterexample "failed" and the

prediction of the model of the hypothesis advanced by Sinai and Stokes passed the confirmation tests, the Sinai and Stokes hypothesis would be provisionally "corroborated" (a Case I result).

It could be argued that our "core counterexample" can be mechanically transformed to a restricted version of Sinai and Stokes' equation (2):

(7) $$\ln Q = \ln A + \alpha \ln L + \beta \ln K + (1 - \alpha - \beta) \ln m + \lambda T$$

While this is a mechanical manipulation of our equation (4) it has some positive features. If in fact $(1 - \alpha - \beta)$ were to be shown to be insignificantly different from zero, then it would clearly indicate the validity of our counterexample which is an existential statement of the type "there is at least one Q which does not change when m changes in $Q = f(L, K, m)$."

In order to abstract from the "simplifying" assumptions (Part (B) and the "procedural conventions" (Part (C)), the original Sinai and Stokes model and the two counterexamples were run using the same procedural specification (an unconstrained Cobb-Douglas form in the case of (6), but different procedural specification in the case of (7) where the parameters on labor and capital are restricted) and the same testing conventions. To be completely consistent with Sinai and Stokes we corrected for autocorrelation by applying a second-order GLS procedure. The result of the test was that the counterexample was a GOOD-FIT for each of the three monetary aggregates used—M1, M2 and M3—for both the unrestricted and restricted versions. In terms of the Bennett Table these results fall in the Case II category for M1 and M2, which brings into serious question the strength of Sinai and Stokes' claim of success for their hypothesis. In the case of M3, which Stanley Fischer (1974) pointed out should be the most significant of the monetary aggregates in the "delivered production function" we, in fact, have a Case III result, a refutation of the original Sinai and Stokes model against which the counterexamples provide a GOOD-FIT, while the original model results in a BAD-FIT. Our results are summarized in Tables 2a and 2b.

TABLE 2a
Summary of Regression Results

MODEL/COUNTEREXAMPLE	PARAMETER	VALUE (t-value)	PARAMETER	VALUE (t-value)	PARAMETER	VALUE (t-value)
SINAI-STOKES	lnA	-2.63668 (-4.74892)	lnA	-2.83583 (-4.07366)	lnA	-2.82794 (-3.39682)
	lnL	1.05685 (9.27616)	lnL	1.09903 (10.81230)	lnL	1.12805 (10.90760)
	lnK	0.44441 (4.06193)	lnK	0.38193 (4.07979)	lnK	0.35850 (3.80172)
	lnM1	0.09351 (1.84667)	lnM2	0.13864 (1.79402)	lnM3	0.12232 (1.34626)
	T	0.00369 (1.05184)	T	0.00388 (1.02912)	T	0.00461 (1.12523)
	$\bar{R}^2$	0.99229	$\bar{R}^2$	0.99035	$\bar{R}^2$	0.9896
	D.W.	1.91	D.W.	1.93	D.W.	1.91
	S.E.E.	0.021	S.E.E.	0.021	S.E.E.	0.021

TABLE 2a *continued*

MODEL/COUNTEREXAMPLE	PARAMETER	VALUE (t-value)	PARAMETER	VALUE (t-value)	PARAMETER	VALUE (t-value)
SINAI-STOKES						
COUNTEREXAMPLE-Unrestricted	lnA	0.07855	lnA	0.11651	lnA	0.14104
		(0.77302)		(1.22560)		(1.49535)
	lnL*	0.85233	lnL*	0.84714	lnL*	0.86277
		(6.67268)		(7.12233)		(7.55379)
	lnK*	0.18999	lnK*	0.22490	lnK*	0.23865
		(1.67781)		(1.95776)		(2.12600)
	T	0.01744	T	0.01704	T	0.01675
		(8.76191)		(8.55462)		(8.63630)
	$\bar{R}^2$	0.95146	$\bar{R}^2$	0.93652	$\bar{R}^2$	0.94688
	D.W.	2.27	D.W.	2.21	D.W.	2.14
	S.E.E.	0.025	S.E.E.	0.024	S.E.E.	0.024
COUNTEREXAMPLE-Restricted	lnA	0.07873	lnA	0.11640	lnA	0.14119
		(0.77469)		(1.22483)		(1.49629)
	lnL	0.85214	lnL	0.84725	lnL	0.86264
		(6.67198)		(7.12421)		(7.55075)
	lnK	0.19017	lnK	0.22479	lnK	0.23880
		(1.67898)		(1.94743)		(2.12650)
	lnM1	-0.04231	lnM2	-0.07204	lnM3	-0.10144
		(-0.82035)		(-1.12484)		(-1.59201)
	T	0.10744	T	0.01705	T	0.01675
		(8.75493)		(8.55932)		(8.63090)
	$\bar{R}^2$	0.96892	$\bar{R}^2$	0.96954	$\bar{R}^2$	0.97278
	D.W.	2.27	D.W.	2.21	D.W.	2.14
	S.E.E.	0.024	S.E.E.	0.024	S.E.E.	0.024

TABLE 2b

Summary of Alternative Test Results**

	MODEL (THEORY) SINAI & STOKES	COUNTEREXAMPLE (~ THEORY) UNRESTRICTED	RESTRICTED
Using:			
M1	GOOD-FIT	GOOD-FIT	GOOD-FIT (M1 not confirmed)
M2	GOOD-FIT	GOOD-FIT	GOOD-FIT (M2 not confirmed)
M3	BAD-FIT	GOOD-FIT	GOOD-FIT (M3 not confirmed)

SUMMARY AND CONCLUSIONS

In this paper we examine the debate over the inclusion of real money balances in the neoclassical aggregate production function and propose and apply an alternative test procedure to rigorously test the original Sinai and Stokes (1972) hypothesis. The alternative procedure provides a logically complete extension of existing conventional procedures by identifying four possible types of test outcomes obtained by testing both the theory under consideration and a valid counterexample of the theory.

We apply the alternative test procedure to the Sinai and Stokes hypothesis by developing a counterexample of a "money-deflated" production function. We test both a restricted and an unrestricted version (equations (4) and (5) respectively) of our counterexample and our results indicate that both the original Sinai and Stokes formulation and the counterexample pass (as far as possible) identical confirmation tests. These results put into question Sinai and Stokes' claim of success for their original hypothesis.

It is important to note that we test only for the Cobb-Douglas specification originally used by Sinai and Stokes. Other specifications (e.g. translog specifications, simultaneous equation specifications etc. used by Short (1971)) would require the similar development of valid counterexamples and reapplication of the test procedure. Since the debate over the inclusion of real money balances as a factor input in the aggregate production function has important implications for the theory of production and the explanation of total factor productivity, for understanding the relationship between the real and monetary sectors, and for monetary growth theory, it is important to rigorously test other model specifications of the theory. Such tests may provide grounds for provisionally "corroborating" these other specifications thus salvaging some element of the money-in-the-production-function hypothesis. However, it must be recognized here that even one instance (e.g. a particular Cobb-Douglas specification) of a Case II, III or IV result would constitute damaging evidence because of the requirement that every theory has at least one universal "all or none" statement incorporated in it. A GOOD-FIT result for the counterexample provides logical grounds for criticizing the original "core" hypothesis. Case II, III and IV type results would provide grounds for proceeding to other, possibly more fruitful channels of inquiry in monetary and production theory (cf. Moroney (1972)). This would provide a means of sustaining "progressive problem shifts" (cf. Lakatos (1970)) in empirical monetary economics.

FOOTNOTES

1. Such an approach is based on the philosophical viewpoint of logical positivism. See Blang (1980), Boland (1982) and Caldwell (1982) for discussions of the features and limitations of logical positivism. Boland (1982) also provides a critical discussion of the methodology of "conventionalism" adopted by most economists today.
2. Often in analytic papers mathematical economists are concerned with the consistency of their theory at the (A) and (B) levels, and econometricians are concerned with the consistency at the (B) and (C) levels, but what the Duhem-Quine problem points out is that we must have consistency throughout all three levels (A), (B) and (C).
3. The Bennett Test Procedure, it should be noted, is only applicable to models of a theory (i.e. the whole conjunction (A), (B), and (C)). In the case of the other functional forms that have been examined in the Sinai and Stokes literature further counterexamples must be developed and tested against those functional forms.
4. As noted, the test procedure we are using involves examining the Part (A) or "core" assumptions of the theory. In the case of Sinai and Stokes' theory these core assumptions are difficult to identify. There are several possible interpretations of what an assertion such as "real money balances are a factor of

production" could mean; many of them are not refutable (although most of them may be conceivably false). The counterexample we examine, however, is a model of the logical denial of the following refutable interpretation of their theory's core assumptions: "all neoclassical production functions to which inputs and outputs confirm for any finite period of time have real money balances as an argument at every level of inputs and outputs." Accordingly, Sinai and Stokes tested the following model of their theory, "inputs and outputs in the U.S. economy from 1929 to 1967 conformed to the relationship (1), where $\gamma \neq 0$."

5. The counterexample constitutes the logical denial of Sinai and Stokes' hypothesis as we have interpreted it in footnote 4 and is of the form "inputs and outputs in the U.S. economy from 1929 to 1967 conformed to the relationship (3), where $\alpha + \beta = 1$ and $\gamma = 0$." Equation (5) quite clearly represents an existential statement which is equivalent to "there is at least one Q which does not change when m changes in $Q = f(L, K, m)$."
6. There is a correspondence between equation (3) and the alternative approach to the microanalysis of money and production taken by Gabor and Pearce (1958), Lange (1936), Vickers (1968) and others. However, our results should not be interpreted as support for this or any other view. Our counterexample is not a competing theory, but rather it is an assertion which cannot be true if the original theory is not false.

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