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Used Capital: Implications for Isoquants, Production Functions, and Shepard's Lemma

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I. INTRODUCTION

This paper will show that aggregate production functions will typically not be "regular" when the quantity of capital services provided by a "used" capital item is proportional to its rent. They will be concave but not strictly quasi-concave. Isoquants will have linear segments that coincide with isocost lines. In turn, this observation implies that factor inputs are not determined by output and factor prices alone.

The assumption of a strictly quasi-concave production function is central to Shepard's lemma and neoclassical investment functions. This paper will show that when the capital factor includes used capital, production functions are not strictly quasi-concave. Because of the abundance of used capital in the real world, the results shown in this paper raise doubts about the applicability of received theory. A different paradigm than underlies most theoretical and empirical writings on production is implicit in the argument of this paper.

It has been known for a long time that rigorous factor aggregation is possible only under implausible conditions [Solow 1956, Fisher 1965, Sato 1975, Usher 1981]. That there are severe problems with the concept of capital emerged from the Cambridge capital controversies [Harcourt 1972; Bliss 1975]. However, factor intensity reversal occurs only under extremely rare conditions and hence is not a fatal problem for practical research. However, aging capital is universal; and this will be shown to be a severe problem for the dominant paradigm.

This paper is organized as follows. Shepard's lemma and neoclassical investment functions and textbook treatments of non-regular production functions are discussed in Section 2. The argument that used capital containing production functions typically have isoquants containing linear segments and that the production function is not strictly quasi-concave is presented in Section 3. In Section 4 various related issues including the differentiability of cost functions are discussed.

II. IMPORTANCE OF STRICT QUASI-CONCAVITY

Shephard's Lemma

An active area in production research has been the use of factor share equations derived from cost functions to study substitution among different inputs and the nature of technical change.[1] These have typically been built around Shephard's lemma which holds that the partial derivative of the cost function with respect to factor price gives the cost minimizing input for that factor.

The derivation of Shephard's lemma and relations between a factor's income share and the

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prices of other factors depends on the cost function being differentiable at the relevant points [McFadden 1978, p. 14; Shephard 1970, pp. 170–171]. Because of the tradition in economics of using well behaved functions whose derivatives exist, many economists believe such functions are typical and do not ask whether functions actually have derivatives before trying to estimate them or to use theories that depend on their existence. This appears to be relevant for many production function studies using cost functions and Shephard's lemma to study substitutability. The applicability of Shepard's lemma will be questioned for studying production functions containing used capital goods. Authors using cost function derivatives do not seem to have considered the possibility that the nature of the physical world and his aggregate variables' construction together assure that the assumptions underlying the estimation procedures are mutually inconsistent.[2]

Frequently, use of a translog cost function (or other flexible form) is justified as being a second order approximation to any twice differentiable cost function, thus making it unnecessary to justify the choice of functional form. If the true cost function (if a meaningful approximation to one exists for the aggregate variables used) is nondifferentiable, as this paper will argue, this argument becomes irrelevant.

Empirical studies frequently fail to exhibit concavity over the entire region (Barnett and Lee, 1985). One possible explanation is that the real world function being approximated is not strictly quasi-concave and even minor data errors or random fluctuations cause the fitted function to exhibit convexity over some part of the fitted function.

Investment Theory

Another area for which the argument of this paper is important is neo-classical investment theory which is dependent on the existence of an optimal and unique quantity of capital for any set of factor prices. Investment is then a function of the difference between the existing stock and this optimum stock (Jorgenson [1967, p. 141]). If production functions are not strictly quasi-concave and unique capital inputs do not exist, the neoclassical investment theory appears inapplicable. As will be argued, a given set of factor prices is consistent with using a large quantity of new capital or a small quantity of old capital (the two cases may involve machines of the same design with the old machines requiring more maintenance).

The Textbook Treatment

That production functions can be non-regular is well known. Standard linear programming production models [Dorfman et al., 1958] permit linear sections and often result in production at corner points where relevant derivatives are not defined.

Although linear sections are possible in standard theory, the usual impression is that they are rare enough to assume away. For instance, one textbook says "your intuition no doubt informs you that an isoquant cannot be linear under most circumstances for such would suggest perfect substitutability of labor for capital in production." [Call and Holahan, 1980, p. 161]. Another text argues that "Thus, as labor is substituted for capital, the marginal product of capital increases." [Ferguson and Maurice, 1978, p. 190]. Another [Pappas and Brigham, 1979, p. 220] states "In the typical case the marginal rate of technical substitution is not constant but diminishes as the amount of substitution increases." Textbooks appear to be implicitly assuming new capital and excluding deteriorated or obsolete goods. To use more labor and less capital, new machines can be replaced with labor intensive old machines.

The minimum quantity of rent defined fixed capital required to produce any item is usually small if old or obsolete capital goods on the margin of production are used. Thus, Samuelson's textbook [1970, p. 517] errs in saying: "Both labor and capital are needed in production: take away all capital, or alternatively all labor, and you will be left with negligible total product." Ferguson [1969, p. 51] reaches a similar conclusion: "The more reasonable assumption seems to be that a positive usage of all inputs simultaneously is required to produce a positive output." At least as far as fixed capital is concerned, both forget the possibility of using zero rent old capital goods.

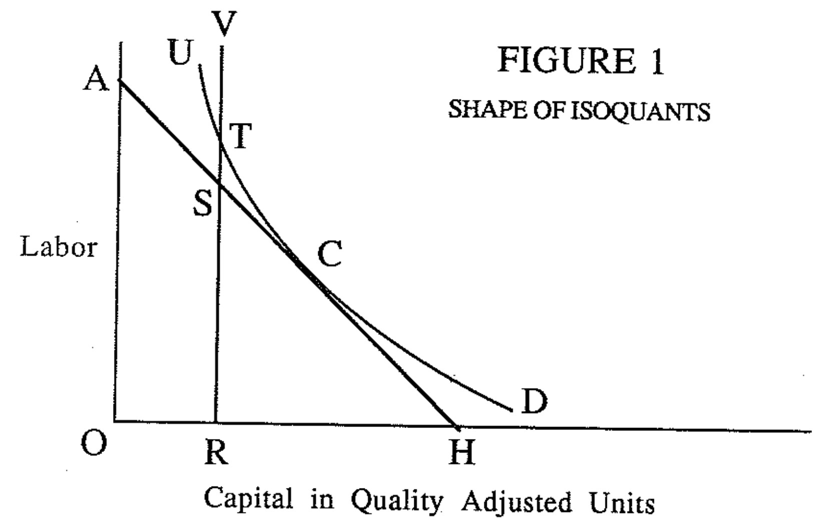

Figure 1. Shape of Isoquants

The textbook treatment of capital-labor substitution can be seen in Figure 1. A typical curved isoquant ATCD is shown. It shows different machine designs with each representing the minimum input of *new* capital for the given input of the variable factor. It is curved with no straight line segments. The isocost line ASCH has a single point of tangency to the isoquant at C. Firms will choose that machine design which minimizes total production costs at the existing set of prices. If factor prices shift, the new equilibrium point can be found by drawing the new isocost line and locating the new point of tangency. *New* machines will be designed to have their factor inputs at this point of tangency. The word new is underlined because in a world of durable capital this seems to be implied (see below).

Is this a realistic model of the options open to the typical firm? If the firm is constrained to use only new capital goods, and to vary capital input by changing machine design or by changing the ratio of number of machines to other factors then it is. However, such an explicit constraint is seldom, if ever, imposed.

Inspection shows that the capital stock consists of both old and new goods; and that at any particular time, most machines and structures are old. While they remain useful for their intended purposes, they perform them less efficiently. This deterioration usually appears as increased inputs of one or more cooperating factors (often a composite factor called maintenance) while the inputs of other factors remain the same. For instance, a truck may require increased maintenance while still requiring a single driver. The problem is how to incorporate the rather obvious fact that firms have the option of utilizing deteriorated and obsolete goods into the received theory.

III. USED CAPITAL AND ITS IMPLICATIONS FOR RECEIVED THEORY

How Much Capital Does an Old Machine Represent

The first question to be addressed is how to measure the stock of capital when there are both new and old machines. This, of course, is part of the capital aggregation problem that has received so much attention. The usual solution is to weight different vintages by their rents or quasi-rents, an extension of the general procedure of weighting by prices. Since the competitive model is usually assumed, this is equivalent to weighting by marginal product (which is sometimes referred to as efficiency [Jorgenson 1974]—a most unfortunate usage since it suggests physical efficiency).

Of course, in actual application the aggregation may involve quality weighting of physical quantities; but the major aggregate production function researchers appear to believe their aggregation methods do have the property that all units of each aggregate factor have the same service price and (with cost minimization) the same marginal products.[3] The most common procedure for capital combines different vintages by weights that are believed to be good approximations of marginal products and rents, often exponential functions of time [Jorgenson 1974 and note 3].

Now that we have a definition (a quality weighted aggregate in which quality is proportional to rent), which includes deteriorated as well as new capital, let us ask whether the conventional curved isoquant applies to firms with access to used machines and structures.

Representing Used Capital Goods

Let us consider how used capital goods would be shown in Figure 1. To understand the argument, consider the typical case in which the good's output remains constant as it ages. To place the old machines on Figure 1, one must understand what determines their rents since the quantity of capital input they provide is assumed proportional to these rents.

A machine's rent plus the cost of purchasing the other factors, here only labor, will just exhaust the value of the machine's output with constant returns to scale. If the rent exceeded the value of the output minus the costs of associated factors, no one would choose to rent the machine, and the machine's owner would lower the rent asked. If the rent were lower than this, numerous firms would seek to rent the machine and its rent would be bid up. Thus, if

R = the rent of the machine
w = the wage rate
L = the labor input
Q = value of the output of the machine.

then $R = Q - wL$. With the quantity of capital defined to be proportional to the rent that would be charged in a competitive market, the capital input $(K) = aR = aQ - a\,wL$ (where a is the constant of proportionality by which the rent is multiplied to give the units of capital input). This simple linear equation, derived from the standard theory of rent, identifies the quantity of capital employed with specified labor inputs.

If Q is the value of output from the new machine represented by point C, the above specifies a straight line through C (which represents use of a new machine) and sloping upwards and to the left. This is the straight line ASC of Figure 1. If used machines with the same output actually are available to the firm, the locus of combinations of rent defined capital and labor open to the firm are on line ASC. This line lies below the exante isoquant ATCD. For most points on the ex-ante isoquant (representing production of output Q with a particular capital input), there will be an old machine which produces the same output with the same capital input but less labor input. Thus, the corresponding points on the *ex-ante* isoquant cannot be on the *isoquant proper* since they do not represent minimum usages of labor for the specified capital inputs.

If the firm has access to at least one point on line ASCH other than C, it follows that there will be at least two points on this straight line that are also on the isoquant. This will prevent the isoquant from being curved, a key part of the regularity assumptions usually made in neoclassical theory.

Which points on the straight line ASCH the firm has the option of operating on depends on history and the availability of used machines. Notice that once the possibility of using old capital is recognized, the production function depends not only on technology but on the economy's history and which used machines actually exist to be used.

If machines of design C have been introduced too recently for used versions to exist, the only point on the line will be C. However, it is very likely that there are used versions of at least one other design available. The services of these older machines will be priced to lie along the straight line ASCH. If the older designs actually use less labor than the newer designs, the older machines could lie to the right of the point C. Only by an extreme coincidence would the older designs just happen to fall exactly on point C. (This would occur where these designs used the same quantity of labor but cost more to make. Such old designs would involve more reproduction cost capital but provide the same services measured by rent.)

An Example of Rent Calculation

Observation shows that the optimum output (capacity) for a typical machine declines little, if any, with age. Thus, such behavior will be assumed here. The machine will be assumed to have an output of 100,000 units per yer valued at $100,000 at all ages. In the first year, 80% of the cost is represented by capital services and the remainder consists of one man-year of labor, $20,000. As the machine ages the maintenance increases by one man-year per year until the machine is abandoned. The Table below shows the factor inputs for the machine at successive ages. The capital input is measured in the usual way, as a quality adjusted input where the quality is taken as proportional to the rent (and hence marginal product) under perfect competition. A unit of capital services is defined to be that amount of capital services provided by a capital good renting for $10,000 per year. In constructing capital aggregates, different items (and vintages) are given weights proportional to their rents.

The Table provides a typical example of how the rent for deteriorated capital goods is

TABLE 1
Example of Capital Determination

Year	Labor Input	Labor Value	Value of Output	Rent of Machine	Capital Input
0	1	$ 20,000	$100,000	$80,000	8
1	2	40,000	100,000	60,000	6
2	3	60,000	100,000	40,000	4
3	4	80,000	100,000	20,000	2
4	5	100,000	100,000	0	0

determined, and from the rent the quality weighted units of capital. At any given time firms in the industry have a choice of using machines of vintages from 0 to 5 years. A plot of the minimum cost options open to a firm for producing a specified output quantity, say 100,000 units would show a point for each vintage. Because capital quantity is proportional to rent, and because of the equilibrium conditions for rent, the various points given above all fall on a straight line whose formula is:

$10,000 X Capital Input + $20,000 X Labor Input = $100,000. This formula yields an isocost line since it shows the tradeoff between capital and labor subject to the constraint that the total cost be $100,000. This isocost line represents the minimum cost the firm can incur for the given output. The above points are also on the isoquant since each one shows an input combination open to a firm free to hire labor and rent capital in a competitive market, and none of these points is dominated by another input combination open to the firm. The same argument could be made if the quantity of capital services considered to be provided by a particular vintage was defined as proportional to that vintage's marginal product.

Both Capacity and Rent Changing Over Time

Complications arise from the possibility of the machine's optimal output changing over time. If the capacity of a machine declines with age, more machines will be required for the same output. It is thus necessary to multiply the number of old machines, and the per machine rent, by the reciprocal of the capacity to discover the factor input required to produce with old machines the same output that could be produced with one new machine. Let K be the quantity of rent measured capital represented by new machines of the design used at point C, c being the output of the machine as a fraction of the original machine's output and k its rent as a fraction of the original rent. It then takes $1/c$ machines to have the same output as the machine used at point C, and the total rent of old machines with the same output as the machines used at point C is Kk/c. For the point representing the old machines to use exactly K capital, it is necessary that k/c be 1. This requires that $k = c$, or that the percentage decline in rent with time must exactly equal the output decline for all ages of capital. This condition will be met only by the most extreme coincidence. Observation of the real world shows that capital goods' optimum output (capacity) changes little with time although their rent does decline with time, often due to increased maintenance costs.

Summary of Argument on Concavity

A strictly quasi-concave production function logically implies that only a single set of factor proportions will be consistent with any set of factor prices. The converse is also true. If the same output can be produced with multiple sets of factor proportions at equal costs, there must be more than one point lying on both the minimum cost isocost line and the isoquant. If machines rent for different prices as they age, and the optimum output does not change in exact proportion, there will be different factor proportions depending on the age of the machines used. If this happens the isoquant cannot be curved at all points and the production function cannot be strictly quasi-concave.

Since used machines renting for less than new machines but producing the same output, or an output that has decreased by a different percentage than the rent has decreased by, are inconsistent with the neoclassical theory of strictly quasi-concave production functions, observation of such machines constitutes powerful evidence against the theory.

IV. SOME RELATED ISSUES

The Continuity of the Isoquant

Another regularity assumption frequently made is that both the isoquant and the related production function are continuous. This regularity assumption may be met if the isoquant has a linear section from the zero capital axis to point C. However, while the normal situation is for there to be some old capital goods available to the firm, the firm may not have the option of using machines representing all levels of capital input to the left of C. Old capital goods usually have some scrap value. If they have scrap value, it will pay to scrap them if a normal return on the scrap value is less than their rent. A minimum rent may also be set by the rent the good could earn in some other use, perhaps a different industry.

Thus, there is likely to be some minimum value for the capital input obtainable through use of old capital goods. In the diagram, the vertical line passing through S represents the minimum rent consistent with using old, deteriorated capital goods. Should the firm wish to employ a smaller capital input than represented by S, it will not be able to do so by using old goods. There will be no unscrapped goods on line AS. Thus, the firm wishing to use small quantities of capital goods will have to use new capital goods produced to a non-capital intensive design. These will be to the left of T on the ex-ante isoquant UTCD. Between S and the point just to the left of T, the isoquant has a discontinuity because the method of production shifts from using old, deteriorated or obsolete capital goods on the linear segment of the isoquant to using new goods designed for non-capital intensive production.

As an example consider road construction. Between S and C, the firm uses deteriorated or obsolete machines. However, even old machines have an alternative use as a source of spare parts. The only way the road building firm can use less capital than at S (where machines at the margin of scrapping are used) is to shift to new machines designed to use little capital (perhaps new wheelbarrows instead of old steamshovels). This creates a discontinuity in the production function.

The Definition of the Quantity of Capital Services

The above argument involves a particular definition of capital services, a weighted average of the number of different capital goods with the weights for any particular good proportional to the price for its services in a perfectly competitive market. This appears to be the standard definition in the literature.

Some readers may be uncomfortable with the implications of weighting older capital goods by a measure of quality or efficiency proportional to their rents. Before proposing an alternative procedure, they should remember that any weighting scheme not proportional to rents will cause the marginal product (and price) to depend on the particular item selected. For instance, if the weights given to old goods are not proportional to their prices, the marginal product (and price) of a unit of services from old capital will differ from the marginal product (and price) of a unit of new capital. This will make any theoretical constructs depending on the law of one price or equal marginal products for old and new capital goods impermissible. The usual growth accounting formulas are among those logically depending on factors having a single price and marginal product [Miller 1983c, 1985]. Of course, any time one is weighting different ages of capital goods by their rents one is indirectly adjusting for obsolescence, a procedure which presents logical problems in measuring technical progress [Miller 1983a, 1983b]. Elsewhere, the author (1985) has criticized Denison's growth accounting for using a definition of capital quantity for which old and new goods had different marginal products in an argument which required they have the same marginal products.

Similar effects can occur with other heterogeneous factors such as land and labor when quality differences imply differences in output per physical unit used or differences in input of other factors per unit of output.

Implications of Deterioration for Other Inputs

One of the basic conclusions of the argument presented so far is that factor inputs are *not* uniquely determined by prices and outputs if old capital goods are present. This is an observation about the world and does not depend on the exact aggregation procedure used to define capital. Suppose the alternative factor to capital is labor (here representing all variable factors). Older machines require more maintenance labor. The very fact that old labor intensive machines are in use insures that labor input does not depend only on factor prices and output (as textbook theory suggests) and hence that variable inputs are *not* determined by only factor prices and outputs.

The Multiple Factor Case

The above graphical arguments relate to two factors inputs. The problem is more general. A definition will be useful here. A factor is a variable in an aggregate production function, such as land, labor or capital. Subfactors are inputs aggregated with other inputs of the same type to produce quantitative measures of factor inputs. Different capital vintages, land of different fertilities, and labor of different qualities are all subfactors.

For any aggregate factor composed of several subfactors whose weights are proportional to prices of subfactor services, there will usually be a combination of cooperating factors whose cost when added to the subfactor's rent will equal the minimum achievable cost—being neither above nor below it. The reader may imagine the factor called labor in Figure 1 as being a composite of cooperating variable factors (the mix giving the lowest cost). While fitted production or cost functions can be tested for concavity, the absence of strict concavity when its presence would be a logical result of the inputs' construction merely casts doubt on the data's quality.

One special case should be noted. Many machines deteriorate with use and require increasing maintenance (a combination of labor, spare parts and purchased services) to remain usable. When maintenance rises to the point where replacing the machine is cheaper, it is done. Until then the rent adjusts to keep it employed. This can be depicted as a tradeoff between the use of capital and maintenance to produce the services of maintained capital. Isoquants showing maintenance versus unmaintained capital services will have a linear segment. Then, the services of maintained capital (produced by unmaintained capital plus maintenance) and other factors can be plausibly depicted by curved isoquants of the usual shape.

Implications for the Existence of Isoclines

One consequence of the absence of strict concavity is that there will not be isoclines or a single expansion path connecting points where isocost lines and isoquants are tangent. However, the concept of an expansion path can be replaced with that of an expansion region showing the area of minimum cost production. In Figure 1, this would be the area line SC passes over as the isoquant is moved outwards. If durable capital has a significant scrap value or alternative use,

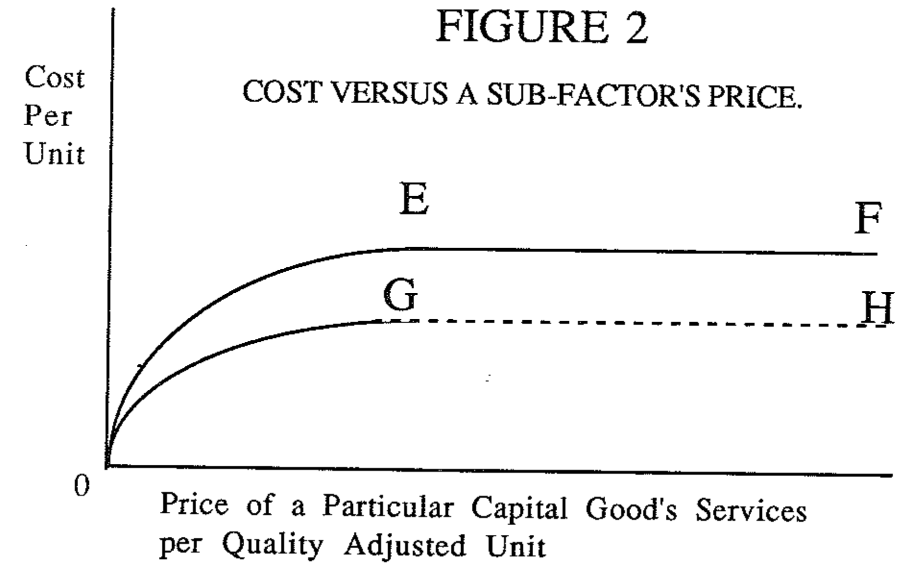

Figure 2. Cost Versus a Sub-factor's Price

there would also be a left hand boundary to the expansion (or lowest cost) region. Likewise, the idea of an isocline line can be replaced with that of a isocline region. Alternatively, one can explicitly define the isocline as a feature of an ex-ante isoquant using new goods.

The Cost Function

The basic problem can also be seen in Figure 2 which shows a graph of unit production cost versus the price of a subfactor, perhaps a particular capital good's services. The unit cost is invariant to the price of the subfactor's services from E, where the curve begins to slope, to point F. The long flat section from E to F arises because a firm always has the option of not using the subfactor if its services are priced higher than a competing subfactor (perhaps a newer or older capital good of the same type). The curve suddenly begins to slope at point E because for all prices below the price at E, the subfactor would be used. At point E the curve is not differentiable. Unfortunately, a firm using a rent measured input will be producing at a point E. If the factor was priced too high to be used at E, it would not be used and its price would decline until it was used. If there is a price at which it would be used (there need not be), it will be at E. Thus, there will be a firm using the input at point E if the item is used at all. It follows that the cost function will not be differentiable where the firm is operating.

The careful reader will note Figure 2 depicts the case where a single capital good's service price is increased. Yet a factor price increase usually affects all items included in the factor aggregate. All units of capital or labor could increase proportionately in price. However, an assumption of this factor price increase pattern does not eliminate the problem. There is a curve of the OEF shape for every subfactor or item in the composite factor. Unfortunately, the slope of section OEF is typically different for the different subfactors. If the price increase affects a subfactor of which many quality weighted physical units are used (new capital goods, high quality land, skilled labor) the cost function partial derivative is higher than if production is with a subfactor of which fewer units are used (old machines, low quality land, unskilled labor).

Obviously if 100 units of a quality adjusted factor are being used per unit of output, a small increase in the price of the factor raises total cost more than if a process is being used requiring only 10 units of the same factor (which is quite consistent with both firms achieving minimum costs) and the factor price goes up by the same percentage. For an infinitesimal percentage increase in the factor price, the effect on total costs would be ten times as great as in the former case. Indeed if point E represents new capital, a proportional price increase is consistent with any increase in total costs between the slope of the line OE—just to the left of E and zero. If an unique derivative of a cost function with respect to an aggregate factor price cannot be unambiguously defined, it obviously cannot give the optimum factor input as predicted by Shepard's lemma. The possible range in total cost change per unit factor price change is wide (being zero at the lower limit) and can say virtually nothing about optimum factor inputs. In general, if fixed capital is an input, the least capital intensive input choice will involve very old machines whose rent equals a normal return on scrap value (which is usually very low). This non-capital intensive choice will usually use only a small fraction of the capital input required with new capital goods. At most, the factor share for at least one input combination achieving minimum cost will equal or exceed the ratio of an infinitesimal cost change to the factor price change causing the cost change.

An Uncertainty Principle

In the presence of deterioration caused heterogeneity, a well defined marginal product (a partial derivative of the production function with respect to the aggregate factor) requires aggregating by rent. However, to do so prevents having a unique capital input or a well defined derivative for the cost function. There is a fundamental uncertainty principle here that prohibits both derivatives from being well defined.

The Characterization of Techniques Solution

The above analysis has shown how multiple sets of factor proportions achieve minimum cost (i.e. a linear segment for an isoquant). Without additional restrictions, factor proportions cannot be determined from factor prices alone. This naturally suggests imposing additional restrictions to make factor proportions determinate. There are several possible restrictions that might be used.

One is to pick a well defined point on a production function's linear segment and study the factor proportions there. For instance, one might study the most capital intensive of all minimum cost solutions. For Figure 1 that would be point C at one end of the linear segment. This corresponds to the case where the firm uses new capital (and less maintenance).

An advantage of studying new machines' capital input is that purchase prices provide a natural capital quantity measure. Since dollars are homogeneous, heterogeneous capital aggregation is no longer a problem. With reproduction cost capital, factor proportions are well defined and of intrinsic interest. This procedure defines an unique capital intensity for a technique, and an unique ratio of capital to any other inputs whose ratio to output doesn't change with time. These often include operating labor, materials and parts, and energy.

V. CONCLUSIONS

There are major problems in the current use of aggregates with weights proportional to rents and service prices in production functions. The most important theoretical problem is that factor inputs are not uniquely determined by output and relative factor prices. Shephard's lemma does not hold. One solution for this problem is to abandon the neo-classical aggregate production function and study the factor inputs of different techniques. This requires that the capital input be defined not by its ability to produce income but by its ability to produce output with other factors (capacity in investment studies) or by reproduction cost.

NOTES

1. Jorgenson and Fraumeni [1981]; Moroney and Trapani [1981], Berndt and Wood [1979], and numerous papers in Berndt and Field [1981]).
2. Nerlove [1963], Berndt and Wood [1975, 1979], Halvorsen and Ford [1979], Moroney and Toevs [1979], Mohr [1980], Klotz et al [1980], Nadiri and Bitros [1980], Moroney and Trapani [1981], Anderson [1981], Norsworthy and Harper [1981], Brown and Christensen [1981], Berndt et al [1981].
3. Gollop and Jorgenson [1980], Christensen et al. [1980], U.S. Dept. of Labor [1983, p. 38 and p. 77].

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