A Note About The Interest Rate and The Revenue Function

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The concern of this note is to reexamine the generally accepted notion that

"... the revenue function provides a very general tool for modeling production so long as there are no distortions. (Dixit and Norman, 1960, p. 160). In particular, and contrary to popular belief, there is no need to restrict the analysis to the case where final goods are produced directly by primary factors. There may be many patterns of goods being used as inputs to the production of other goods."

Whether or not there is, indeed, a "popular belief" of the kind described by Dixit and Norman (who do not tell us where it is to be found), it is our contention such a belief is ill-founded. Yet it is no less true that the presence of produced inputs can cause certain problems for the use of the revenue function, as that is normally presented, when a positive rate of interest is paid on the value of those inputs. (And economic systems with a uniform and constant interest rate have, of course, been widely studied, e.g., for example, Malinvaud [1953], Mirrlees [1969], Starrett [1970], Gale and Rockwell [1975]).

The usual derivation of the revenue function is based on the claim that "Production decisions will maximize total profit... the problem will be to... maximize the value of net output" (ibid p. 31). But, if entrepreneurs have to pay a positive rate of interest on the capital advanced for the purchase of produced inputs, the maximisation of profit will not maximize the value of net output; rather it will maximize that value minus the value of total interest payments. What follows from this for the properties of the "revenue" function?

The Linear Programming Case

Consider first a linear programming representation of a competitive economy, in which the technical possibilities are shown by an output matrix, \( B \geq 0 \), a produced input matrix, \( A \geq 0 \), and a primary input matrix, \( E \geq 0 \). The vectors \( p > 0 \) and \( c > 0 \) represent a given commodity price vector and a given primary input endowment vector, respectively. \( r > 0 \) is a given interest rate. An activity vector, \( x \), and a competitive primary input price vector, \( w \), are to be chosen to solve:

\[
\begin{align*}
V(p, r, c) &= \max_{x} p^{T} [B - (1 + r)A]x \\
U(p, r, c) &= \min_{w} \forall w \geq p^{T} [B - (1 + r)A]x \\
\text{s.t.} &\quad \exists x \leq c \\
\text{s.t.} &\quad x \geq 0
\end{align*}
\]

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Since there are always feasible solutions, there are always optimal solutions and \( V = U \) are uniquely determined.

It is readily seen that \( V(p, r, e) \) is continuous, non-decreasing, linearly homogeneous and convex in \( p \); decreasing in \( r \); non-decreasing, linearly homogeneous and concave in \( e \). Moreover

\[
\begin{align*}
\Delta V &= \Delta y \left[ (\delta V / \delta y) - r \Delta y \right] \\
\Delta V &= -\Delta y \Delta x \\
\Delta V &= \Delta y \Delta e \\
\Delta V &= \Delta x \Delta e
\end{align*}
\]

where \( \Delta s \) is the increment of \( s \) and \( M^i \) is the \( j \)th row of matrix \( M \). Therefore, where \( V(p, r, e) \) is differentiable (i.e., where \( s \) and \( e \) are uniquely determined),

\[
\frac{\partial V}{\partial y} = y - r k
\]

where \( y_j \) is the net output of \( j \) and \( k_j \) is the capital stock of \( j \);

\[
\frac{\partial V}{\partial w} = -pk
\]

where \( k \) is the capital stock vector;

\[
\frac{\partial V}{\partial \Delta w} = w_1
\]

It will be noted at once, from (1), that \( \partial V / \partial p = y_j \) if and only if either the interest rate is zero or commodity \( j \) is not used as a produced input.

It can, of course, also be seen from the original statement of the LP problem that while

\[
\Delta w \Delta e = 0,
\]

from the Min problem, the Max problem yields only

\[
\Delta p (\Delta y - r k) = 0
\]

or

\[
\Delta p \Delta y = r \Delta p \Delta k
\]

If the interest rate is zero, or if there are no produced inputs, we can be sure, from (5), that \( \Delta p \Delta y = 0; \) but in general this is not known. Hence the presence of produced inputs does matter for the use of the revenue function when combined with the presence of a positive interest rate.

The Continuous Case

Consider the problem

\[
V(p, r, e) = \text{Max} \left[ p y - (1 + r) p k \right], \text{subject to } f(q, k, e) = 0,
\]

where the notation is as above, except that \( q \) now appears as the vector of gross outputs. (The constraint \( f(q, k, e) = 0 \) has 'normal' properties.) First order conditions are naturally \( \frac{\partial V}{\partial p} = y_j - L(\Delta f / \Delta q_j), \frac{\partial V}{\partial r} = -L(\Delta f / \Delta k) \) and \( f(q, k, e) = 0 \), where \( L \) is a Lagrangean multiplier. If these can be solved for \( q, k, L \) we may write the maximum value of \( V \) as

\[
V(p, r, e) = [p q (p, r, e) - (1 + r) p k(p, r, e)]
\]

From (6),

\[
\frac{\partial V}{\partial p} = [q_j - (1 + r) k_j] + \sum_i p_i \left( \frac{\partial q_i}{\partial p_j} - (1 + r) \frac{\partial k_i}{\partial p_j} \right)
\]

\[
= (q - r k) + L \sum (\frac{\partial q_j}{\partial p_j} \Delta f / \Delta q_j + \frac{\partial q_j}{\partial p_i} \Delta f / \Delta q_i)
\]

\[
= (q - r k)
\]

In obtaining (7) we have, of course, simply applied the envelope theorem; more important is the fact that (7) in effect reproduces (1), above, but now for the continuous case. Even when \( V(p, r, e) \) is differentiable everywhere, with respect to \( p_i \), \( \partial V / \partial p_i \) is not identical to the net output of \( j \), unless either the interest rate is zero or \( j \) is not used as a produced input.

The same kind of argument as was used to derive (1) also shows that

\[
\frac{\partial V}{\partial r} = -(pk)
\]

which reproduces (2), above, but now for the continuous case. Also, it is clear from the 'normal' properties of \( f(q, k, e) \) that

\[
\frac{\partial ^2 V}{\partial p \partial e} = w_1
\]

It follows at once from (3) that

\[
\frac{\partial V}{\partial e} = \left( \frac{\partial V}{\partial e} \right) / \Delta e = \frac{\partial ^2 V}{\partial p \partial e} < 0
\]

from the concavity of \( V \) with respect to \( e \). But (7) yields only

\[
\frac{\partial y_j}{\partial p_j} - r \frac{\partial k_j}{\partial p_j} = (\Delta p)^2 \frac{\partial y_j}{\partial p_j} > 0
\]

from the convexity of \( V \) with respect to \( p \). Hence

\[
\frac{\partial y_j}{\partial p_j} > r \frac{\partial k_j}{\partial p_j}
\]

But, \( \frac{\partial y_j}{\partial p_j} > 0 \) is not ensured when \( r > 0 \) and \( j \) is used as a capital good.

We note finally the effects of a positive interest rate on Samuelson's (1935) 'reciprocity conditions.' (Compare Gram, 1985.) From (1)' and (3), the equality of \( \partial V / \partial \Delta e \) and \( \partial V / \partial \Delta p \) implies that

\[
\frac{\partial ^2 V}{\partial p \partial e} = \left( \frac{\partial V}{\partial e} \right) / \Delta e = \frac{\partial ^2 V}{\partial p \partial e}
\]

it is not generally true that \( \frac{\partial V}{\partial e} = \left( \frac{\partial V}{\partial p} \right) / \Delta e \). Similarly, from (7), the symmetry of the matrix \( V \) yields

\[
\frac{\partial y_j}{\partial p_k} = \frac{\partial y_i}{\partial p_k} + r (\frac{\partial k_j}{\partial p_i} - \frac{\partial k_i}{\partial p_j})
\]

therefore \( \frac{\partial y_j}{\partial p_k} = (\Delta p_j / \Delta p_k) \) if and only if either \( r = 0 \) or the matrix \( \left[ \frac{\partial k_i}{\partial p_j} \right] \) is symmetric; note that the second condition holds if there are no produced inputs, or by a fluke. Only in a system having either a zero rate of interest, or no produced inputs, can we be sure that all the 'reciprocity conditions' will always hold.

Unless the presence of a positive interest rate is to be described as the presence of 'a distortion'—and why should it be?—it would seem that the above quote from Dixit and
Norman was insufficiently cautious. We have seen that some 'revenue function' results are affected by the presence of produced inputs if a positive rate of interest is paid on their value. And such a rate of interest is usually paid.

REFERENCES


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Industrial Conflict, the Quality Of Worklife, and the Productivity Slowdown in U.S. Manufacturing

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Production is human activity. Yet most studies of the productivity slowdown have treated production as a technical relationship between purchased inputs and final outputs [exceptions include Christiansen, 1982; Flaherty, 1985; Gordon, 1981; Kendrick and Grossman, 1980; Naples 1986; Norsworthy and Zabala 1985; Weiskopf, Bowles and Gordon, 1983]. This article explores social as well as technical determinants of the growth of production-worker productivity in U.S. manufacturing. In particular, workplace conflict and industrial accidents are identified as factors affecting the growth of labor per labor-hour and therefore productivity.

Initially productivity is decomposed into two components: labor efficiency, and the ratio of effort to hours hired. Technical determinants of productivity are enumerated. Explanatory variables related to the social relations of production are then developed. Econometric results and their implications follow.

A SOCIAL-RELATIONS APPROACH TO PRODUCTIVITY GROWTH

Labor productivity is by definition the ratio of output to labor-hours. Productivity analyses tend to assume labor services per labor-hour are fixed by employment contracts. But companies actually contract for workers' potential—their skills, job experience and general capacity to do work. Management must then prevail on employees to perform the desired services. Whether the existing labor-management conflict over the work process derives from human nature and the moral hazard of shirking [Lazear, 1981], or from the structure of capitalism which gives rise to alienated labor [Bowles, 1985; Gintis, 1976; Marglin, 1974], remains a subject of debate.

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