A Note On Some Theorems in the Theory of International Trade

Geoffrey A. Jehle*

The purpose of this note is to provide, for pedagogical purposes a simple, unified set of diagrammatic proofs for several fundamental propositions in the standard, 2 x 2 x 2 Heckscher-Ohlin model of international trade. The Heckscher-Ohlin theorem and its two principal corollaries, the Factor Price Equalization theorem and the Stolper-Samuelson theorem, together constitute, "the central body of international trade theory" (Jones and Neary, 1984). However, proofs of these important theorems in the literature and in textbooks at any level typically proceed along different and often arcane mathematical or diagrammatic lines, and so tend to obscure the essential unity of this central body of the theory. Since these and many other propositions in the literature which address the income distributional aspects of international trade all rely on the same set of relationships between relative product prices, relative factor prices and real factor earnings, they may all be demonstrated by means of a simple diagrammatic device reflecting those basic relationships simultaneously.

Let the usual assumptions of the standard Heckscher-Ohlin trade model obtain. There are two countries (I and II) with perfectly competitive economies, producing the same two final goods (X and Y) with two internally mobile, inelastically supplied and fully employed factors (L and K). Let the production of each good in each country take place in the absence of externalities and under conditions of constant returns to scale (CRS). Let the technologies for producing a given good be identical in both countries, but let capital intensities in production (R_L and R_K) differ between goods. In particular, let good X be capital intensive relative to good Y at all relative prices of labor (w = w). Then the relationships in Figure 1 between relative commodity prices (P = P), relative factor prices, capital intensities in X and Y production, and real returns to labor (w = w) and capital (r) obtain in both countries.

Panels A and B of Figure 1 simply reproduce the familiar "Harrod-Johnson" diagram relating relative product prices, relative factor prices and capital intensities in production, and hardly require any comment or justification. Suffice it to point out that any given pair of CRS technologies gives rise to a set of such relationships in general competitive (product and factor market) equilibrium which is unique and which holds in the range of relative product prices, factor prices, and factor intensities consistent with incomplete specialization in production. If the production functions for X and Y are identical across countries, then the same set of relationships will obtain in each country within ranges consistent with incomplete specialization in both countries.

Figure 1 is completed by appending panels C and D which depict the necessary and corresponding relationships between the capital intensity, R_K, and the marginal productivity of labor and capital, respectively, in the production of good X under conditions of CRS. It is, of course, well-known that when the technology exhibits constant returns to scale the marginal product of each factor depends systemati-}

The validity of the relationships depicted in panels A and B, though not disputable, depends nonetheless on some rather delicate reasoning on the subject of general competitive equilibrium in a constant returns to scale economy satisfying the Strong Factor Intensity Assumption. No such delicacy is involved in panels C and D, however, since they simply reflect properties of the assumed technology which are independent of the competitive structure of the economy. It is, of course, true that we are only able to make inferences about factors’ real earnings from the marginal products of the underlying technology.
when industries are populated with profit maximizing, (factor and product) price-taking firms. However, since just those assumption (and more) are necessary to justify the relations between relative product prices, relative factor prices, and factor intensities in panels A and B, one need not shy away from identifying the horizontal axis in panels C and D with the real wage \( w/p \) and the real rental \( r/p \), measured in units of good \( X \).

The four panels of Figure 1 together summarize the complete set of inter-relationships among product prices, factor prices, and real factor earnings from which virtually all of the fundamental propositions in the standard \( 2 \times 2 \times 2 \) Heckscher-Ohlin trade model can be derived. For the sake of completeness, we begin with Johnson's (1957) demonstration of the relative factor price version of the Heckscher-Ohlin theorem itself, and then proceed to consider its principal corollaries and some of their most important applications.

**HECKSCHER-OHLIN THEOREM:** A country will have a comparative advantage in, and so tend to export, that good which uses its relatively abundant factor relatively intensively.

**DEMONSTRATION:** Let relative factor prices in autarky be given by \( w \) and \( r \), respectively, so that Country I is capital abundant relative to Country II in the factor price sense. Then relative commodity prices in autarky are given by \( P^I \) and \( P^II \), respectively, and Country I has a comparative advantage in good \( X \), the capital intensive commodity.

Since the relationships depicted in Figure 1 derive exclusively from the assumptions of general competitive equilibrium and identical CRS technologies in both countries, and are independent of factor endowments (except insofar as the countries' endowments delimit the relevant ranges of the relations depicted which are consistent with incomplete specialization in both countries), the same framework may be used to analyze the distributional effects of changes in the countries' factor endowments. For instance, Mundell's (1957) well-known result on trade and factor mobility can be clearly seen as an application of the Stolper-Samuelson theorem.

**MUNDELL SYLLOGISM:** Under conditions of perfect factor mobility, an impediment to trade which reduces the relative price of (say) \( X \) in country I will lead to a movement of capital from country I to country II, and a movement of labor from country II to country I. This factor movement will cease only when real returns to each factor are equalized in both countries at (say) \( w/p \) and \( r/p \), in panels C and D, respectively. Any impediment to trade, such as a tariff, subsidy, or transport costs, which causes relative product prices \( p^I \) and \( p^II \) in country I to diverge to (say) \( p^II \) in country II in panel A, will cause the real earnings of capital and labor in the two countries to diverge to \( r/p \) and \( r/p \) in panels C and D. But then relative factor prices must be equalized across countries at \( w/p \) in panel B, and relative product prices must be equalized at \( P^I \) in panel A.

One central proposition of standard trade theory yet to be considered is the Rybczynski theorem. Unfortunately Figure 1, alone, can not be employed to demonstrate that if a country experiences growth in one of its factors at constant product prices, then the output of the industry which uses that factor
intensively must increase more than proportionately and the output of the other industry must decline. However, it is possible to demonstrate that the distributional implications of such factor growth (which arguably were of most central concern in Rybczynski’s original article) and to appreciate them as a special case of a larger class of trade and growth problems originally analyzed by Johnson (1962).

**Rybczynski-Johnson Theorem:** Any type of factor growth which, through its effect on both production and consumption at constant product prices, causes an increase (decrease) in the growing country’s volume of trade, will cause a decrease (increase) in the real return in both countries to the factor used intensively in the growing country’s export industry, and an increase (decrease) in the real return to the other factor in both countries, as long as neither country is a “small” country.

**Demonstration:** Let both countries be initially in free trade equilibrium facing the common set of relative commodity prices, $\tilde{p}^X$, in panel A of Figure 1, and suppose that Country $I$ experiences some type of factor growth. Except in the “maize’s edge” case of a production and consumption bias which leave the volume of trade in Country $I$ unaltered, there must be either an increase in its imports and exports, or a decrease in its imports and exports at constant product prices. The former (latter) case, causing an outward (inward) shift in Country $I$‘s offer curve in Figure 2, will lead to a decline (increase) in the equilibrium relative price of good $X$ faced of both countries from $\tilde{p}^X$ to $\tilde{p}^X$ ($\tilde{p}^X$).

Recalling our original assumption that Country $I$ is the exporter of good $X$, the capital intensive commodity, the line of argument from here will by now be quite transparent, and follows in any event directly from the Stolper-Samuelson and Factor Price Equalization theorems. Any decrease in the relative price of good $X$ in both countries will lead, from panels A, D and C in Figure 1, to a decline in the real return to capital in both countries from $\tilde{p}^X/\tilde{p}_K$ to $\tilde{p}^X/\tilde{p}_K$, and an increase in the real return to labor in both countries from $\tilde{p}^X/\tilde{p}_F$ to $\tilde{p}^X/\tilde{p}_F$. (Any increase in the relative price of good $X$ will, by panels A, C, and D, have the opposite effects on the factors’ earnings in both countries.)

**Building upon the familiar “Harrod-Johnson” diagrams, this note has shown how Figure 1 can be used to simplify and unify the derivation of many fundamental propositions in modern trade theory, and to analyze a wide range of theoretical and policy issues relating to the income distributional implications of international trade.** Since the crucial links in most income distributional arguments within the standard Heckscher-Ohlin model depend on the relations between factor intensities and real factor earnings depicted in panels C and D, Figure 1 also serves to emphasize the extreme sensitivity of many of these arguments to the assumption of identical CRS technologies in both countries.

**Figure 2**

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**Footnotes**

1. Let the production function $X = f(K,I)$ be increasing in $K$ and $I$, convex, and have the property of CRS.

2. The marginal products of capital and labor depend only on the factor intensity, $R_k$, and are given by $MPL_K = \tilde{p}_K(R_k) > 0$ and $MPL_L = \tilde{p}_L(R_k) > 0$.

3. The relations in panel C and D follow from differentiation with respect to $R_k$:

   - $\frac{\partial MPL_K}{\partial R_k} = \tilde{p}^X(R_k) < 0$
   - $\frac{\partial MPL_L}{\partial R_k} = -\tilde{p}^X(R_k) > 0$

4. Since perfect factor mobility means that each unit of a given factor serves the same nominal return regardless of the intensity in which it is employed, the choice of good $X$ as numeraire here and throughout is arbitrary. The relations between factor intensity in $X$ production and real factor earnings measured in terms of $X$ are qualitatively the same as those depicted in Figure 1.

5. Since there is no need here to present a taxonomy of all possibilities, suffice it to say that at constant product prices, certain types of factor growth give rise to certain types of production bias, and certain types of consumption bias give rise to certain types of consumption bias. The instability of any particular combination of production and consumption bias will have implications for the volume of the growing country’s trade as constant product prices.

6. The case of an ultra export bias in production growth and neutral consumption growth first analyzed by Rybczynski is an example of the former.

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**References**


