Eastern Economic Journal, Volume XVI, No. 1, January–March 1990

Penalty Schedules and The Optimal Speed Limit

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In the absence of restrictions some drivers choose speeds exceeding what would be socially optimal, since they capture the benefits of increased speed; the cost, however, such as damage to third parties, is borne by others. Given this externality, imposing a speed limit may increase social welfare. Inducing individual drivers to comply with the speed limit requires enforcing mechanisms, such as policing and penalties for violations. While the policing mechanism has been discussed by Lee [12], little attention has been devoted to analyzing driver response to a given penalty structure. This is surprising given that the broader problem of criminal behavior in response to punishment has been dealt with in the literature on the economics of crime, pioneered by Becker [2].

This paper incorporates the penalty schedule associated with the enforcement of a speed limit into the analysis of speed choice. We show that certain penalty schedules may induce some drivers to increase speed, relative to their unrestricted choice. A similar phenomenon has been documented by Block and Heineke [4] and Dickens [8] in the context of rational criminal behavior. Dickens [8], for example, shows that increasing the severity of punishment may increase the crime rate, a conjecture proposed by Akerlof and Dickens [1]. However, if the penalty schedule is well structured it is possible to induce all drivers to obey the speed limit. This raises the question of the socially optimal speed limit. Lee [12] suggests that the limit be set as low as possible. In contrast, our analysis indicates that when total compliance is desired, a specific socially optimal speed limit may be identified.

Early studies of the optimal speed limit and the related question of individual speed choice focused on determining the benefits and costs of the 55 mph limit. Castle [5] and Forester, McNown and Singell (FMS) [9] conclude that the costs outweigh the benefits, while Clotfelter and Hahn [6] and Miller [13] conclude the opposite.[1] In a natural extension, Jondrow, Bowes, and Levy [10] developed and tested a model of the optimal speed limit. These papers, however, did not discuss the effects of policing and penalty schedules. Lee [12] explicitly includes policing costs and concludes that the speed limit should be set as low as is politically feasible. The penalty schedule is, however, taken as given in this moldel. This paper shows that the structure of the penalty schedule is essential for understanding driver behavior.[2]

This model is based on the assumption that "speed kills"; that is, the probability of suffering a fatal accident is solely a function of the driver's speed. Recently, Lave [11] provided empirical support for the conjecture that "variance kills, not speed." He found that the cross-sectional dispersion of speeds, not the average speed, is positively correlated with the fatality rate. Although it seems paradoxical, our assumption is consistent with Lave's [11] findings. Under a plausible functional form for the probability of a fatal accident, we show that the average speed will not affect the fatality rate, even if driver speed is the only variable determining the probability of an accident.

THE MODEL

Let each driver be risk-averse with a subjective utility function, U(T, F), where $T = d/v$ represents the time required to travel a distance d at speed v.[3] Let L be the speed limit and $F = F(v - L)$ the monetary fine imposed on a caught violator. The violator also loses an amount of time ΔT as a result of the ticketing process. Utility decreases with both T and F. The probability of an accident, a(v), increases with

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speed. The probability of being stopped and fined, S, given that $v > L$, increases with the amount of policing, P, but is assumed independent of the driver's speed. To simplify the analysis assume that only fatal accidents occur, that the utility of dying is zero, and that having an accident and getting a ticket are independent events.[4] Given these assumptions, a driver's expected utility when restricted by the speed limit, $EU_r(v)$, is given by

$$EU_r(v) = U(T(v), 0)[1 - a(v)][1 - S] + U(T(v) + \Delta T, F(v - L))[1 - a(v)]S \tag{1}$$

Notice that (1) reduces to the unrestricted objective function, $EU_u(v) = U(T(v), 0)\,[1 - a(v)]$, whenever $S = 0$. Thus, in our model the speed limit affects driver behavior only when policing is present, ruling out voluntary and costless compliance with the speed limit.

The privately optimal speed for each driver is found by differentiating (1) with respect to v which, after rearranging, results in:[5]

$$\begin{aligned}&\{U_1(T, 0)T'(v)[1 - a(v)] - U(T, 0)a'(v)\} + \{[U_1(T + \Delta T, F) - U_1(T, 0)]T'(v)[1 - a(v)]\\ &\quad + [U(T, 0) - U(T + \Delta T, F)]a'(v) + U_2(T + \Delta T, F)F'(v - L)[1 - a(v)]\}S = 0\end{aligned} \tag{2}$$

The first term in braces in (2) is the first order condition for the unrestricted case. Since the driver must also consider the effects of the speed limit, contained in the second term in braces, the privately optimal speed in the presence of a speed limit differs from the unrestricted optimum. Although it may seem intuitive that speed should be reduced when violators face potential penalties, this is not necessarily the case. For speed to decrease relative to the unrestricted optimum, the second term in braces in (2) must be negative. Let us analyze the sign of the three components of this term. Assuming that the utility of time saved by driving faster increases at a decreasing rate,[6] then $[U_1(T + \Delta T, F) - U_1(T, O)] < 0$ and since $T'(v) = -d/v^2 < 0$ the first component is positive. The second component is also positive since $a'(v) > 0$, and utility decreases in both arguments. Finally, if fines increase with speed, i.e., $F'(v - L) > 0$, the third component is negative. Thus, the sign of the second term in braces in (2) is indeterminate and we cannot say *a priori* if drivers will increase or decrease their unrestricted choice of speed when confronted with a speed limit.

A corollary is that a constant or decreasing fine schedule may induce drivers whose unrestricted optimal speed is greater than the speed limit to increase their speed once the limit is imposed. Even with a strictly increasing fine schedule some drivers may increase their speed if the marginal penalty is too low. This shows that policing is necessary, but not sufficient, for speed reduction. Intuitively, drivers increase speed when confronted with a low marginal fine because a caught violator will suffer a loss of expected utility from additional travel time and fines. In an *ex-ante* sense this loss can be overcome by increasing speed.

We have assumed so far that the driver's optimum speed is found at the point where the first order condition (2) equals zero. While this provides a local optimum, v_r, the global optimum may occur at the speed limit itself. This is possible because the driver's expected utility function has a discontinuity at $v = L$ whenever the fine, $F(v - L)$, contains a fixed component. Figure 1 shows how this can occur.

In Figure 1, $EU_u(v)$ is the expected utility for the unrestricted driver, and v_u is the privately optimum speed. With a speed limit restriction the driver optimizes $EU_r(v)$, which has two local optima, $v = L$ and $v = v_r$. The driver's choice will be L when $EU_u(L) > EU_r(v_r)$. This condition suggests that all drivers may be induced to obey the speed limit by creating a high loss of expected utility from exceeding the speed limit even slightly. In practice, this may be achieved through a high ΔT, and/or a high fixed component in the fine schedule.

The question of the socially optimal speed limit assuming that all drivers rationally comply with the speed limit is addressed below.

THE SOCIALLY OPTIMAL SPEED LIMIT UNDER TOTAL COMPLIANCE

Let the function $W(L, P)$ represent net aggregate social welfare from imposing a speed limit, L, given that P policing units are consumed. Let the unrestricted expected utility function be given by $EU_u(v, v_u)$,

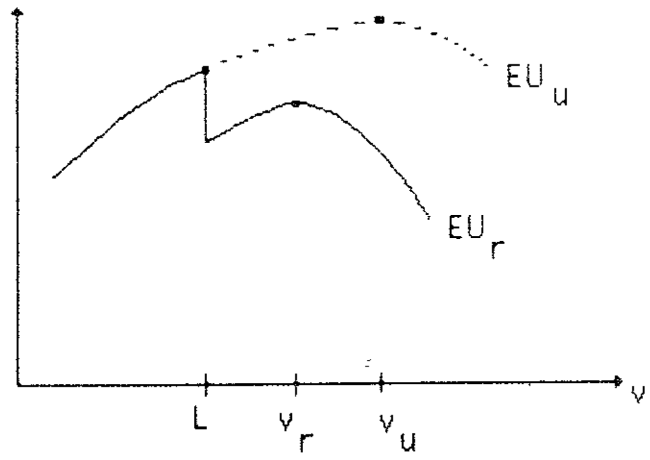

Figure 1. Unrestricted and Restricted Expected Utility Functions

where speed v is the driver's choice variable, and v_u is a driver-specific parameter. $EU_u(v, v_u)$ reaches a unique maximum at $v = v_u$, where the unrestricted optimum speeds $v_u \in [v_m, v_M]$ have a distribution function $H(v_u)$. The external cost of driving at speed v is $C(v)$. Each policing unit costs k dollars.

Social welfare is the sum of all the benefits and costs across drivers. Under total compliance, all drivers whose unrestricted optimum speed exceeds the limit will drive at a speed of L. Thus, social welfare is

$$W(L, P) = \int_{v_m}^{L} [EU_u(v_u, v_u) - C(v_u)]\, dH(v_u) + \int_{L}^{v_M} [EU_u(L, v_u) - C(L)]\, dH(v_u) - kP \tag{3}$$

Using Leibniz' rule, we differentiate (3) and simplify, to get

$$W_1(L, P) = \int_{L}^{v_M} [EU_{u1}(L, v_u) - C'(L)]\, dH(v_u) \tag{4}$$

$$W_2(L, P) = -k \tag{5}$$

Notice from (4) that unless all drivers have the same unrestricted optimal speed v_u, it is impossible to have $EU_{u1}(L, v_u) - C'(L) = 0$ for all drivers with $v_u \in (L, v_M]$. Consequently, the optimal speed limit under total compliance, L_T, depends on the particular distribution function for drivers, $H(v_u)$. Notice also from (5) that zero policing is optimal. Realistically, (5) suggests that it is socially preferable to impose very high fines, rather than to increase policing, to induce total compliance with the speed limit. This is reasonable since policing is socially costly whereas the penalty schedule, even though high, will never be applied under total compliance.

AN EXAMPLE

Consider a continuum of drivers with unrestricted privately optimal speeds uniformly distributed across the population, $v_u \sim U[v_m, v_M]$. Assume that the individual benefit and external cost functions for each driver are given by $EU_u(v, v_u) = e - b(v - v_u)^2$ and $C(v) = cv^2$, respectively. Total policing costs are kP. The penalty schedule is $F(v - L) = M + m(v - L)$ for $v > L$, with $M, m > 0$, and $F(v - L) = 0$ for $v \leq L$. To keep the example simple, assume that $\Delta T = 0$.

In this example all drivers obtain the same expected utility, e, if there is no speed limit since they optimize $EU_u(v, v_u)$ and their chosen speed is v_u. Once a speed limit is introduced each driver optimizes the restricted function $EU_r(v) = EU_u(v, v_u) - S(P)F(v - L)$, where $S(P) = tP$ is the probability of being caught, and the parameter t represents the efficiency of the monitoring technology.

To find the socially optimal speed limit we evaluate (4) using the functions in this example. Integrating and setting to zero gives

$$(v_M - L)[-(b + 2c)L + bv_M] = 0$$

Thus, the socially optimal speed limit under total compliance, L_T, is[7]

$$L_T = [b/(b + 2c)]v_M \tag{6}$$

From (6) it can be seen that $L_T < v_M$ or, equivalently, social welfare increases by imposing a speed limit that forces at least some drivers to reduce their speed. Note that (6) also allows for the possibility that $L_T < v_m$. That is, it may be socially optimal to induce *every* driver to reduce speed, as Lee [12] has suggested. In our model, however, this is not a foregone conclusion, but depends on the values of v_m and of the parameters b and c.

As (5) indicates, zero policing is optimal. More realistically, we must choose the lowest value of P for a given penalty schedule, consistent with total compliance. Total compliance is achieved by having $EU_u(L_T, v_u) > EU_r(v_r)$ for all drivers. Solving this inequality for P gives the minimum policing required for individual driver compliance

$$P > [b/(tF)][(v_u - L) + (v_u - v_r)](v_r - L) \tag{7}$$

From (7) it is seen that the policing required decreases as the total penalty, F, the monitoring technology, t, and the speed limit, L, increase. Since fines will never be applied with total compliance but some costly policing will still be required, the socially optimal course of action is to establish fines as high as possible.

IDENTIFYING THE KILLER: VARIANCE VS. SPEED

It has been assumed that driver speed is positively correlated with the probability of a fatal accident; the higher the kinetic energy of a colliding body, the greater the likelihood that it suffers serious damage. Some have argued that perhaps it is not speed per se, but rather the dispersion of speeds on a given highway that kills. (Clotfelter and Hahn [6, p. 283]). The rationale for this alternative view is that speed dispersion is an index of the number of takeovers, and that overtaking is the most probable situation leading to a highway accident.

Lave [11] tested the determinants of the fatality rate empirically using aggregate data for each state. His results appear to support the proposition that "variance kills, not speed." Lave's regression is of the form

$$FR = m_0 + m_1\mu + m_2\sigma + \epsilon \tag{8}$$

where μ is the average speed for a given state, σ is the standard deviation of speeds for that state, and FR is the state's fatality rate, defined as the number of fatalities per 100 million miles traveled. The results indicate that m_1 is statistically insignificant, whereas m_2 is significantly positive. Lave [11, p. 1162] concludes that "... there is no discernible effect of speed on the fatality rate." This section undertakes to reconcile Lave's [11] empirical results with the principle that driver speed is the fundamental determinant of the probability of a highway fatality, as assumed throughout this paper.

Suppose that the probability of a fatal accident for an individual driving at speed v, per 100 million miles driven, is a(v). If speed has distribution function H(v), $v \in [v_m, v_M]$, the fatality rate is given by

$$FR = \int_{v_m}^{v_M} a(v)\, dH(v) \tag{9}$$

Using Taylor's theorem we have

$$a(v) = a(0) + a'(0)v + a''(0)v^2/2 + R(v) \tag{10}$$

where $R(v) = a'''(0)v^3/3! + a^{iv}(0)v^4/4! + \ldots$ is the remainder term.

The average speed and the speed variance may be expressed as

$$\mu = \int_{v_m}^{v_M} v\, dH(v)$$

and

$$\sigma^2 = E(v^2) - \mu^2$$

Substituting these expressions in (9) gives

$$FR = a(0) + a'(0)\mu + a''(0)\mu^2/2 + a''(0)\sigma^2/2 + \int_{v_m}^{v_M} R(v)\, dH(v) \tag{11}$$

Equation (11) shows that even if the only determinant of the probability of a fatal accident is the driver's speed, the fatality rate will generally be a function of the average speed and the speed variance.

Although (11) suggests that (8) is misspecified, Lave's assertion that speed dispersion helps explain the fatality rate is supported by our model. It is still not clear, though, why Lave's results do not show a role for the average speed, although (11) indicates it may be a relevant variable. This apparent anomaly may be explained by assuming that the probability of a fatal accident is proportional to the kinetic energy of the driver, that is $a(v) = kv^2$. If this is the case, then $a(0) = a'(0) = 0$, $a''(0) = 2k$, and $R(v) = 0$. Thus, the coefficient of the average speed might be zero in a regression of (8), as Lave found.

CONCLUSION

This paper has focused on the structure of the penalty schedule as a crucial element in any speed limit implementation. The penalty for caught violators is both monetary and in terms of additional travel time. If inadequately formulated, a penalty schedule may induce drivers to increase their speed relative to their unrestricted choice. It is always possible, however, to design a penalty schedule such that drivers will reduce their speed. It is also possible to induce total compliance with the speed limit.

It has been showed that if the probability of suffering a fatal accident is solely a function of the driver's speed, the fatality rate will be a function of the average speed and of the variance of the speed distribution.

NOTES

1. Miller [13] refined the data used by FMS and found that in at least two of the four cases studied in that paper, the benefits of the 55 mph limit outweigh the costs.
2. Although we focus on individual motorists, Beilock [3] has shown that many accidents are caused by truckers who are forced to speed because of the tight schedules imposed by their employers. This raises the welfare issue of whether the fine should be paid by the driver or the employer.
3. For a discussion of some of the limitations of the subjective utility model, see Schoemaker [14].
4. Obviously, accidents may cause various degrees of injuries, and we could think of introducing an "intensity of accident" variable into the analysis. This suggests the use of varying liability penalties as an alternative to fines for regulating speed. See de Meza [7] for a discussion of optimal liability laws.
5. We use U_1, U_2 to denote the partial derivative of U with respect to its first and second arguments, respectively. If there is only one argument, as in $F(v - L)$ and a(v), the first derivative is denoted by a "prime" (′).
6. That is, we assume that the marginal utility of leisure is decreasing. This assumption, however, is not essential to the analysis.
7. The first order condition is also satisfied at $L = v_M$, but results in a minimum.

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