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Structural Models vs Random Walk: The Case of the Lira/$ Exchange Rate

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After a presentation of the existing structural models of exchange-rate determination we show that their out-of-sample predictive performance of the lira/$ exchange rate is inferior to that of the simple random walk model. Only by moving away from these single-equation, semi-reduced form models towards suitable economy-wide macroeconometric models can one hope to beat the random walk. Following this course, we show that the Mark V version of our continuous time macroeconometric model of the Italian economy outperforms both the existing structural models and the random-walk process, in out-of-sample forecasting tests concerning the lira/$ exchange rate.

INTRODUCTION

The studies of Meese and Rogoff (1983a, 1983b, 1985) showed that the structural models of exchange rate determination could not outperform a random-walk process in out-of-sample forecasting tests, even when their forecasts were based on actual realized values of future explanatory variables.

These studies have given rise to a body of literature which extends the forecasting comparison to more sophisticated structural models (such as generalized portfolio balance models and models with time-varying coefficients) and clarifies the relative performance of structural and random-walk models. This literature has been recently surveyed by Isard (1987), so that we can limit ourselves to mention a few additional works. Blundell-Wignall (1984), by using a more complex portfolio model specified as a set of simultaneous equations (rather than as a reduced-form model) finds a better forecasting performance for the Deutsch mark effective rate than the benchmark random-walk model, though on the basis of a forecasting period of only the third and fourth quarters of 1981. Finn (1986) finds that a rational expectations monetary model for the US-UK exchange rate forecasts as well as a random walk model. Somanath (1986) examines the DM/$ exchange rate and finds that the introduction of a lagged adjustment term can contribute towards better performance of structural models. Finally, Boothe and Glassman (1987) suggest the use of error correction models (ECM), which in their opinion are best suited for theories that postulate long-run relationships such as, for example, the long-run proportionality between the exchange rate and relative money stocks in the monetary models.

The possible reasons for the failure of the structural models are surveyed by Isard (1987) who—when coming to the "lessons" part of his paper—suggests that one should move away from single-equation, semi-reduced form models in the hope that "models that simultaneously take account of a complete system of

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macroeconomic relationships will be able to improve on the single-equation, semi-reduced form models in capturing the associations between exchange rates, interest rate differentials, and other variables" (p. 15). The point that single-equation econometric modelling is inferior to multiple-equation modelling is, of course, well known, and has been made by many authors in different contexts (Pagan, 1987). This is exactly what we have been advocating for a number of years in the context of exchange-rate determination (Gandolfo, 1976, p. 6; 1979, p. 102; 1981, p. 171;1986, pp. II.402–403). It should be added that not every macroeconometric model is ontologically suitable for this purpose. Since the exchange rate is just one of the endogenous variables of an economy-wide model, its determination occurs in conjunction with the determination of the others, in a general (dis)equilibrium setting in which stocks and flows, real and financial variables, etc., all interact. Thus, economy-wide macroeconometric models that embody a partial view of exchange-rate determination (in the sense that they, like the single-equation models, take account only of some factors), are unlikely to fulfil our hopes.

In this paper, after a brief presentation of all the existing structural models, we compare their predictive performance with the results of a random-walk model in out-of-sample predictions of the lira/$ exchange rate; the result is a dismal failure of the structural models, which are consistently outperformed by the random walk model. We then show that by using (a system of 24 stochastic nonlinear differential equations which we call) the Mark V version of the continuous time macroeconometric model of the Italian economy developed by Gandolfo and Padoan, it is possible to outperform all the existing structural models of the exchange rate and, of course, the random-walk process, in out-of-sample forecasting tests concerning the lira/$ exchange rate.

THE STANDARD STRUCTURAL MODELS AND THEIR OUT OF SAMPLE PERFORMANCE

Specification

The modern structural approach (also called the *asset market* approach) takes the exchange rate as the relative price of two moneys (the monetary approach) or as the relative price of bonds (the portfolio approach). The two views differ as regards the assumptions made on the substitutability between domestic and foreign bonds, given the common hypothesis of perfect capital mobility. The monetary approach assumes perfect substitutability between domestic and foreign bonds, so that asset holders are indifferent between them and bond supplies become irrelevant; this, in turn, implies *uncovered interest parity*. Conversely, in the portfolio approach domestic and foreign bonds are imperfect substitutes, and their supplies become relevant; this, in turn, implies the presence of a risk coefficient or *risk premium*. In the simplest version of the monetary approach, PPP (Purchasing Power Parity) is taken to hold instantaneously thanks to perfect price flexibility. More sophisticated versions, though accepting the validity of PPP in the long run, acknowledge that in the short run there may be deviations from it due to price stickiness; thus the proportionality between the exchange rate and the relative money supplies becomes a long-run relationship, that may be iolated in the short run. As regards the portfolio approach, instead of introducing the stocks of the various assets, which cannot be easily observed, the usual procedure is to express the risk premium in terms of such easily observable variables as the cumulative imbalances in the trade accounts of the home country and the rest of the world, and the cumulative imbalance in the capital movements account.

Lack of space precludes a detailed description of these models, but they are well documented and surveyed in the literature (Gandolfo, 1986, Sects 18.8 and A.18.3). Thus we simply recall that the models considered by Meese and Rogoff were the flexible-price (Frenkel-Bilson) monetary model, the sticky-price (Dornbusch-Frankel) monetary model, and the sticky-price (Hooper-Morton) asset model; they did not consider the Hooper-Morton model with risk because it had been rejected by the authors themselves since the coefficient of the term representing risk had the wrong sign, but we shall include this version because we are considering a different period and a different currency.

The quasi-reduced forms of the four models can be subsumed under the following general specification:

$$e_t = a_0 + a_1(m - m_f)_t + a_2(y - y_f)_t + a_3(i_s - i_{sf})_t + a_4(i_L - i_{Lf})_t + a_5(\overline{CA} - \overline{CA}_f)_t + a_6\overline{K}_t + u_t, \quad (1)$$

where the subscript *f* denotes the foreign country, *t* is time, and

e = logarithm of the spot exchange rate (price of foreign currency)
m = logarithm of the money supply
y = logarithm of real income
i_s = short-term interest rate
i_L = long-term interest rate
$\overline{CA}$ = cumulated trade balance
$\overline{K}$ = cumulated capital movements balance
u = disturbance term.

The four models are derived as follows:

(FB) Frenkel (1976)–Bilson(1978): $a_1 > 0, a_2 < 0, a_3 > 0, a_4 = a_5 = a_6 = 0$;

(DF) Dornbusch (1976)–Frankel (1979): $a_1 > 0, a_2 < 0, a_3 < 0, a_4 > 0, a_5 = a_6 = 0$;

(HM) Hooper and Morton (1982): $a_1 > 0, a_2 < 0, a_3 < 0, a_4 > 0, a_5 < 0, a_6 = 0$;

(HMR) Hooper and Morton with risk: $a_1 > 0, a_2 < 0, a_3 < 0, a_4 > 0, a_5 < 0, a_6 > 0$.

The FB and DF models are monetary models; the difference between them is that the FB model assumes purchasing power parity (PPP) in both the short and the long run, while the DF model assumes PPP only in the long run and allows for sticky prices in the short run. The Hooper-Morton model draws from both the monetary and the portfolio approach to exchange-rate determination. It follows the DF model but introduces the effects of trade-balance surpluses: a persistent domestic (foreign) trade-balance surplus (deficit) indicates an appreciation of the long-run exchange rate. Though they allow for different coefficients on the domestic and foreign cumulated trade balances, we follow Meese and Rogoff (1983b) and assume that domestic and foreign trade balance surpluses have an effect on the exchange rate of equal magnitude but opposite sign. Finally, the HMR model introduces imperfect asset substitutability with a risk premium, that we approximate by $\overline{K}$ following Dooley and Isard (1983).

Subsequent studies by Somanath (1986) suggest that, contrary to findings by Meese and Rogoff, the introduction of the lagged dependent variable among the explanatory variables improved the forecasting ability of the model, indicating a non-instantaneous adjustment of the actual exchange rate to its equilibrium value as given by the right-hand-side of eq (1). Thus we also tested the lagged version of the four above models, that is

$$e_t = a_0 + a_1(m - m_f)_t + a_2(y - y_f)_t + a_3(i_s - i_{sf})_t + a_4(i_L - i_{Lf})_t + a_5(\overline{CA} - \overline{CA}_f)_t + a_6\overline{K}_t + a_7e_{t-1} + u_t, \quad (2)$$

Finally, Boothe and Glassman (1987) suggested the use of error correction models (ECM), which in their opinion are best suited for theories that postulate long-run relationships such as, for example, the long-run proportionality between the exchange rate and relative money stocks in the monetary models. The basic idea of the ECM formulation is simply that a certain fraction of the disequilibrium is corrected in the following period. The ECM specification of the four models under consideration is

$$\Delta e_t = a_0 + a_1(\Delta m - \Delta m_f)_t + a_2(\Delta y - \Delta y_f)_t + a_3(\Delta i_s - \Delta i_{sf})_t + a_4(\Delta i_L - \Delta i_{Lf})_t + a_5(\Delta\overline{CA} - \Delta\overline{CA}_f)_t + a_6\Delta\overline{K}_t + a_7(e - m + m_f)_{t-1} + a_8e_{t-1} + u_t. \quad (3)$$

The ECM specification is equivalent to the cointegration between the relevant variables (Engle and Granger, 1987); in our case these are the exchange rate and the relative money stocks. A test of $a_8 = 0$ is a test of both the ECM specification and the long-run proportionality; the cointegration between the exchange rate and the relative money supplies can be further tested by running the cointegrating regression of Engle and Granger and applying various unit root tests (e.g. Fuller, 1976; Dickey and Fuller, 1981).

Results

The estimates of the four models in level form, i.e. of eq. (1), have shown the presence of multicollinearity in all models, so that we have followed the standard procedure of using first differences. In this way we have also taken account of the suggestion of Boothe and Glassman (1987), who advocate the use of first differences because in their experience exchange rates seem to be non-stationary, precisely a series integrated of order 1, so that the first differences ought to be white noise (this matter will be investigated below in more depth). The lagged forms (2) and the ECM specification (3) were also estimated.

The period of estimation (with OLS) was from 1960-I to 1984-IV; as will be clear in the next section, the choice of this sample period was dictated by the use of this same period in the estimation of our economy-wide structural model. The out-of-sample period was from 1985-I to 1987-IV; in evaluating the out-of-sample performance the actual values of the explanatory variables including the lagged value of the exchange rate were used, following the procedure established by Meese and Rogoff and followed in all subsequent studies. The lira/$ exchange rate is the end-of-period value as given in line *ae* in Italy's page in *International Financial Statistics* of the IMF. Details of the other data used are available from the authors on request.

Table 1 lists the two statistics commonly used to evaluate and compare predictive accuracy. These are the root mean square error (RMSE) and the mean absolute error (MAE). The root mean square error is the main criterion for comparing the accuracy of forecasts, but the use of MAE is more appropriate when exchange rates follow a non-normal stable Paretian process with infinite variance or when the exchange-rate distribution has fat tails with finite variance (Meese and Rogoff, 1983a, p. 12). It should also be noted that, since we are employing logarithms, these statistics are approximately in percentage terms and unit free, so that they are comparable across horizons and models.

TABLE 1
Out-of-Sample Multi-Step-Ahead Forecasting Performance (%)

Model	RMSE	MAE
FB	6.23	5.12
FB with lag	6.35	5.29
FB with lag and EC	8.61	7.41
DF	6.27	5.23
DF with lag	6.27	5.21
DF with lag and EC	8.71	7.48
HM	7.58	5.69
HM with lag	6.15	7.35
HM with lag and EC	8.82	7.37
HMR	8.29	6.73
HMR with lag	7.55	6.36
HMR with lag and EC	8.77	7.05
RW	4.76	4.00
RW multi-step-ahead	9.26	8.22

The predictions of the models are called multi-step-ahead because the forecasts for all the eleven out-of-sample quarters were obtained by using the same coefficients (those estimated in the given sample period).

The results reported in Table 1 show that all structural models are outperformed by the random walk. The lagged version shows a slight improvement with respect to the basic version of each model (conversely, the ECM version has a worse performance), but remains definitely inferior to the RW. It should however be noted that, according to Schinasi and Swamy (1987), the benchmark for the unlagged models ought to be the multi-step-ahead random walk, namely the predicted value for any point of the forecasting period ($\hat{e}_{T+i}$, where T is the last point of the estimation period, and i denotes the out-of-sample points) is always equal to the value observed in the last period of the estimation sample ($\hat{e}_{T+i} = e_T$). If so, the unlagged models outperform the multi-step-ahead RW; but it should be noted that this is attributable to the fact that the last quarter of 1984 shows the maximum depreciation of the lira vis-á-vis the dollar, so that the multi-step-ahead RW is bound to yield abnormally poor results.

Although we are comparing forecasting performances, we cannot ignore the economic evaluation of the estimation results deriving from the structural models. These are a dismal failure: with the exception of the constant term and of a_5 (where present), no coefficient is significant and a_1 has the wrong sign in most cases. The usual tests of residual serial correlation (DW, h, Godfrey's LM) rejected the presence of this phenomenon, and the LM test rejected the presence of heteroskedasticity in the basic models, with mixed evidence as regards the lagged and ECM versions.

An objection to the multi-step-ahead procedure could be that a forecaster would use the most up-to-date information available at the time of a given forecast. Thus we also used the rolling regression technique (see Meese and Rogoff, 1983a) to re-estimate the parameters of each model every forecast period; this means that to forecast the exchange rate for period $T + i$ we used the models estimated with all the data from the beginning of the sample period (1973-I) to time $T + i - 1$ (the initial estimation period was 1973-I to 1984-IV as before). The results are given in Table 2.

The structural models remain inferior to the RW model as regards the three month and (in most cases) the six month interval, while most of them show a slight improvement over the RW as regards the twelve-month horizon. The significance of this improvement, however, is dubious, and the forecasting

TABLE 2
Out-of-Sample Rolling Regression Forecasting Performance (%)

	Horizon					
	3 Months		6 Months		12 Months	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE
FB	6.22	5.30	6.73	5.64	6.99	5.63
FB lag	4.51	4.03	4.97	3.52	6.42	5.27
FB lag + EC	7.11	5.74	7.16	6.32	8.23	6.86
DF	6.14	5.16	6.74	5.36	6.58	5.34
DF lag	4.49	3.79	4.89	4.02	6.45	5.33
DF lag + EC	7.13	5.78	7.60	6.16	9.13	7.52
HM	6.96	5.61	7.35	5.96	7.27	5.13
HM lag	5.18	4.54	5.90	4.80	7.67	5.51
HM lag + EC	7.36	5.81	7.75	6.19	8.75	6.95
HMR	7.46	5.98	7.84	6.34	7.59	5.46
HMR lag	5.40	4.83	6.08	4.43	7.91	6.16
HMR lag + EC	7.74	6.29	7.89	6.18	9.19	7.68
RW	4.48	4.18	5.29	4.07	7.65	6.05

performance of the structural models remains very poor (indeed, it deteriorates as the length of the horizon increases).

Further tests

The failure of the structural models cannot be attributed to time-varying coefficients: in fact, the Cusum and Cusum Square tests (Brown, Durbin, Evans, 1975) showed the presence of a structural break in the regressions in 1974, but the introduction of a dummy for that period (after which these tests accepted the hypothesis of constancy) did not improve significantly the results. Since the coefficient a_7 in the ECM specification (3) was never significant, this gave evidence against both the ECM specification and the long-run proportionality between the exchange rate and the relative money supplies. The matter was further investigated by examining the nature of the two series concerned through unit-root tests (for a survey of these tests see Engle and Granger, 1987). The Fuller statistic was -2.09 for e and -1.37 for $(m - m_f)$, so that the null of the presence of a unit root could not be rejected (see the table in Fuller, 1976, p. 373). In addition, the Dickey-Fuller test (1981, p. 106) showed that e does not follow a random walk with drift. Finally, both the Dickey-Fuller (.9838) and the augmented Dickey-Fuller (1.874) statistics (see Engle and Granger, 1987, p. 269 for the critical values) led to the acceptance of the hypothesis of non-cointegration between e and $(m - m_f)$: this means that the data do not support the idea that a long-run proportionality exists between the exchange rate and relative money supplies. These results are in line with those of Giovannetti (1987), who also finds that the exchange rates of the lira vis-à-vis the currencies of the major industrial countries are not cointegrated with prices through a simple PPP equation.

The forecasting failure of the structural models examined therefore seems due to inexact theoretical assumptions, in particular to the assumption of a long-run proportionality between the exchange rate and relative money supplies, an assumption that is rejected by the data. This, however, should not be taken as a failure of economic theory and as a point in favour of those who advocate pure time series techniques as opposed to structural models. In our opinion the exchange rate is too complex a phenomenon in a modern advanced economy to be captured by oversimplified structural models, in particular by the single-equation, semi-reduced form models such as those in circulation. The proper course of action, thus, is not to try to improve the performance of these models, but to move away from them towards suitable economy-wide macroeconometric models in which all the associations between the exchange rate and the other variables (both real and financial) of a modern economy can be captured by means of an eclectic approach. We have been advocating this approach for many years, and in the next section we show that it is, indeed, fruitful.

THE OUT-OF-SAMPLE PERFORMANCE OF THE ITALIAN CONTINUOUS TIME MODEL

Specification of the model and exchange-rate determination

The economy-wide macroeconometric model that we have used is the MARK V version of the Gandolfo-Padoan Italian continuous time model, which consists of a simultaneous system of 24 stochastic differential equations. Even a brief description of this model would require an entire paper; fortunately such a paper is published elsewhere (Gandolfo and Padoan, 1987, 1990), so that we only present a list of the equations and variables for the reader's convenience (see the Appendix), and give an idea of how the exchange rate is determined. However, a few words are in order on the fact that the model is not only specified but also estimated in *continuous time.* By using a stochastically equivalent discrete analogue for estimation with the available discrete data (Bergstrom, 1976, 1983, 1984; Gandolfo; 1981; Wymer, 1972 and mimeos), one can actually estimate the parameters of the original continuous model, which can then be used for all subsequent purposes, such as forecasting and simulation. One practical advantage of this is that one can produce forecasts for any time interval, such as one month ahead, one of the intervals considered by Meese and Rogoff, and not only for the time unit inherent in the data. This is particularly important in our case, given that we had to use quarterly data in estimation (most series—in particular national accounting data—are quarterly). Another advantage of the continuous time approach that is relevant in the present context is the estimation of adjustment speeds, on which more below.

Let us now come to the *exchange rate,* which is determined through the balance-of-payments equation (see eq. (24) in Table A1). It should be emphasized that by using the balance-of-payments definition to determine the exchange rate we are *not* necessarily adhering to the traditional or "flow" approach to the exchange rate. We are simply using the fact that the exchange rate is determined in the foreign exchange market, which is reflected in the balance-of-payments equation, under the assumption that this market clears instantaneously (as it actually does, if we include the monetary authorities' demand or supply of foreign exchange as an item in this market; in our model this item is given by eq (16), the monetary authorities' reaction function). In this sense the balance-of-payments equation is an equilibrium condition, not an *ex post* identity. In fact, no theory of exchange-rate determination can be deemed satisfactory if it does not explain how the variables that it considers crucial (whether they are the stocks of assets or the flows of goods or expectations or whatever) actually translate into supply and demand in the foreign exchange market which, together with supplies and demands coming from other sources, determine the exchange rate. All these sources—including the monetary authorities through their reaction function on the foreign exchange market, eq (16)—are present in the balance-of-payments equation. From the formal point of view, since our model is a general (dis)equilibrium model, it is perfectly legitimate (and consistent with any theory of exchange-rate determination) to use the balance-of-payments definition to calculate the exchange rate once one has specified behavioral equations for *all* the items included in the balance of payments.

Of course, one could have followed an alternative approach, namely (i)to drop either the capital movements equation (eq. (12)) or the reserve reaction function (eq. (16)), (ii)to introduce an exchange-rate determination equation, and (iii) to use the balance-of-payments equation (eq. (24)) to determine either the capital movements or the reserve changes residually. From the formal point of view both approaches are mathematically equivalent given the general equilibrium nature of our model. The choice has been made on the grounds that the specification of an exchange-rate determination equation would have induced us to accept one or the other theory, whereas the balance-of-payments equation is more "neutral" once it is properly treated.

A further important consideration should be added. In our opinion, a basic problem in the debate on exchange rate determination is the question of the adjustment speeds in the various markets. If one believes that asset markets adjust instantaneously or, at least have very high adjustment speeds and, in any case, much higher than those of goods markets, then it follows that it is the flows related to asset markets which have immediate effects on the exchange rate, given the much lower speed of the flows related to goods markets. But if this is not true, then the asset market approach is not a correct way of describing the process of exchange-rate determination: no wonder, then, that the resulting structural models perform very poorly. The continuous time approach enables us to determine the adjustment speeds rigorously (whichever the length of the observation interval); therefore, by using the balance-of-payments equation in which all the relevant variables are present and *come from adjustment equations with their specific estimated adjustment speeds,* we do not impose any arbitrary constraint on the data but let them speak for themselves.

Results

The model was estimated with quarterly data from 1960-I to 1984-IV (such a long sample period was necessary to have sufficient data for FIML estimation, which is required by the continuous time approach); this left enough out-of-sample data (from 1985-I to 1987-III) for the examination of the predictive performance; thus both the sample period used for estimation and the out-of-sample period used for the forecasting experiments were the same as those used for the standard structural models. Although the issue dealt with here is the forecasting performance of the model vs that of a random-walk process, we cannot leave the extremely good estimation results unnoticed. Good forecasting results from a model which is not able to "explain" (in the sense that the postulated theoretical relationships among the variables turn out to be inconsistent with the data) may be satisfactory for a pure forecaster but not for an economist. The Carter-Nagar system R^2_w statistic, which is a measure of the overall goodness of fit of the model (Carter and Nagar, 1977) is 0.8331, which suggests a very high confidence level (the associated χ^2 statistic is 7189 with 73 df) that the model as a whole is consistent with the data. Further, 66 out of 72 estimated parameters are significantly different from zero at least at the 5% level on asymptotic tests and 61 are also significant at the 1% level. All parameters have the theoretically correct sign.

After obtaining the parameter estimates according to the procedure explained in Gandolfo (1981, sect. 3.3.2), these values were substituted back into the original system of nonlinear differential equations which was then used for both in-sample and out-of-sample forecasting. In discrete-time models it is usual to make the distinction between (a) single-period (or "static") forecasts and (b) multi-period (or "dynamic") forecasts. The former are those obtained by letting the lagged endogenous variables take on their actual observed values; the latter are those obtained by letting the lagged endogenous variables take on the values forecast by the model for the previous period(s). The equivalent distinction in continuous time models is made according to whether the solution of the differential equation system is (a) recomputed each period, or (b) computed once and for all. In case (a) the differential equation system is re-initialized and solved n times (if one wants forecasts for n periods), each time using the observed values of the endogenous variables in period t as initial values in the solution, which is then employed to obtain forecasts for period $t + 1$. This is equivalent to the single period forecasts in discrete models. In case (b) the observed values of the endogenous variables for a given starting period are used as initial values in the solution of the differential equation system, which is then employed for the whole forecast period. This is equivalent to the dynamic forecasts in discrete models. Although it is a well known fact that dynamic forecasts are generally less good than static ones because the errors cumulate, we decided to use dynamic forecasts to test the predictive performance of our model, because these are the only ones which can be employed to produce forecasts for a time interval different from that inherent in the data. The starting period was January 1985. As regards the exogenous variables, their *ex post* realized values were used, which is consistent with the procedure followed above. Finally, the basic random-walk model was used as the benchmark, although—as already noted above—some authors suggest that comparing multi-step-ahead predictions of structural models with one-step-ahead predictions of the random-walk model gives the random-walk model an unfair advantage over structural models which do not include a lagged dependent variable; in this case a multi-step-ahead (with or without drift) prediction of the random walk is on a more equal footing with the structural model's predictions (Schinasi and Swamy, 1987). The reason for our choice is that a continuous time model specified as a differential equation system embodies all the relevant dynamics, including the one that discrete-time models try to capture by introducing lagged dependent variables as explanatory variables.

The out-of-sample forecasts of e (not of Δe) were computed for one month, three months, six months, and twelve months horizons starting from the beginning of 1985. The usual statistics (RMSE, MAE) are given in Table 3. The table shows that our model outperforms the random walk process on both RMSE and

TABLE 3

The Out-of-Sample Predictive Performance of MARK V (%)

	Horizon							
	1 Month		3 Months		6 Months		12 Months	
Model	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
MARK V	2.63	1.94	2.62	2.04	2.47	2.06	2.24	2.14
RW	3.37	2.76	5.39	4.66	10.80	10.57	19.76	19.05

MAE measures. A result worthy of note is that the predictive performance of our model as measured by RMSE improves with the lengthening of the time horizon considered, while the performance of the random walk deteriorates. This is in accordance with the idea (Gandolfo, 1981, p. 94) that this category of models (i.e. continuous-time models such as ours), with their more rigorous theoretical basis and emphasis on plausible long-run behavior, are preferable for medium term forecasting.

CONCLUSION

Our study makes it clear that the exchange rate cannot be explained and forecast by simple single-equation semi-reduced form models. This suggests that the proper course of action is to move away from such models towards economy-wide macroeconometric models which are more suitable because they can capture all the associations between the exchange rate and the other variables (both real and financial, both stocks and flows) of a modern economy. Continuous time models are particularly suited for this purpose.

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APPENDIX

TABLE A1. Equations of the Model

Private Consumption

(1) $$D \log C = \alpha_1 \log (\hat{C}/C) + \alpha_2 \log (M/M_d),$$

where

(1.1) $$\hat{C} = \gamma_1 e^{\beta_1 D\log Y}(P/PMGS_f \cdot E)^{\beta_2}(Y - T/P),\ \beta_1 \gtrless 0,\ \beta_2 \gtreqless 0;\ M_d = \gamma_2 e^{-\beta_3 i_{TIT}} P^{\beta_4} Y^{\beta_5},\ \beta_3 \gtrless 0,$$

Rate of Growth in Fixed Capital Stock

(2) $$Dk = \alpha_3[\alpha' \log (\hat{K}/K) - k] + \alpha_4 Da,$$

where

(2.1) $$\hat{K} = \gamma_3 \tilde{Y}, \qquad \gamma_3 = \kappa/u,$$

Expected Output

(3) $$D \log \tilde{Y} = \eta \log (\tilde{Y}/Y),$$

Imports

(4) $$D \log MGS = \alpha_5 \log (\hat{MGS}/MGS) + \alpha_6 \log (\hat{V}/V) + \alpha_7 PCC,$$

where

(4.1) $$\hat{MGS} = \gamma_4 P^{\beta_6}(PMGS_f \cdot E)^{-\beta_7} Y^{\beta_8}, \qquad \hat{V} = \gamma_5 \tilde{Y},$$

Exports

(5) $$D \log XGS = \alpha_8 \log (\hat{XGS}/XGS) - \alpha_9 Da,$$

where

(5.1) $$\hat{XGS} = \gamma_6(PXGS/PF_f \cdot E)^{-\beta_9} YF^{\beta_{10}}(\gamma_3 Y/K)^{-\beta_{11}},$$

Output

(6) $$D \log Y = \alpha_{10} \log (\tilde{Y}/Y) + \alpha_{11} \log (\hat{V}/V),$$

Price of Output

(7) $$D \log P = \alpha_{12} \log (\hat{P}/P) + \alpha_{13} Dm + \alpha_{14} \log (M/M_d),$$

where

(7.1) $$\hat{P} = \gamma_7(PMGS_f \cdot E)^{\beta_{12}} W^{\beta_{13}} PROD^{-\beta_{14}},$$

Price of Exports

(8) $$D \log PXGS = \alpha_{15} \log (\widehat{PXGS}/PXGS),$$

where

(8.1) $$\widehat{PXGS} = \gamma_8 P^{\beta_{15}}(PF_f \cdot E)^{\beta_{16}},$$

Money Wage Rate

(9) $$D \log W = \alpha_{16} \log (\hat{W}/W),$$

where

(9.1) $$\hat{W} = \gamma_9 P^{\beta_{17}} e^{\lambda_4 t},$$

Interest Rate

(10) $$Di_{TIT} = \alpha_{17} \log (M_d/M) + \alpha_{18}[i_f + \log (FR/E) - i_{TIT}] + \alpha_{19} D \log E + \alpha_{20} Dr + \alpha_{21} Dh,$$

Bank Advances

(11) $$D \log A = \alpha_{22} \log (\hat{A}/A) + \alpha_{23} Dk, \qquad \alpha_{23} \gtrless 0,$$

where

(11.1) $$\hat{A} = \gamma_{10} e^{\beta_{18} i_{TIT}} M, \qquad \beta_{18} \gtrless 0,$$

Net Foreign Assets

(12) $$D \log NFA = \alpha_{24} \log (\hat{NFA}/NFA) + \alpha_{25} \log (PMGS_f \cdot E \cdot MGS/PXGS \cdot XGS), \qquad \alpha_{25} < 0,$$

where

(12.1) $$\hat{NFA} = \gamma_{11} e^{\beta_{19}[i_f + \log(FR/E) - i_{TIT}]}(PY)^{\beta_{20}}(PF_f \cdot E \cdot YF)^{-\beta_{21}},$$

Monetary Authorities' Reaction Function on Money Supply

(13) $$Dm = \alpha_{26}(\hat{m} - m) + \delta_3 Dh + \delta_4 Dr,$$

(13.1) $$\hat{m} = m^* + \{\delta_1[D \log (PY) - (\rho_P + \rho_Y)] + \delta_2 Di_{TIT}\}, \qquad \delta_1 \gtrless 0, \qquad \delta_2 \gtrless 0,$$

Taxes

(14) $$D \log T = \alpha_{27} \log (\hat{T}/T),$$

where

(14.1) $$\hat{T} = \gamma_{14}(PY)^{\beta_{22}},$$

Public Expenditure

(15) $$D \log G = \alpha_{28} \log (\gamma_{13} Y/G) + \alpha_{29} D \log Y, \qquad \alpha_{29} \gtrless 0,$$

Monetary Authorities' Reaction Function on International Reserves

(16) $$D \log R = b\delta_5 \log (E_c/E) + (1 - b)\delta_6 \log (\hat{E}/E) - \delta_7 D \log E + \delta_8 \log (\hat{R}/R),$$

where

$$\hat{E} = PXGS/\gamma_{14}PF_f, \quad \hat{R} = \gamma_{15}PMGS_f \cdot E \cdot MGS, \quad b = \begin{cases} 1 \text{ under fixed exchange rates,} \\ 0 \text{ under floating exchange rates} \end{cases}$$

Inventories

(17) $$DV = Y + MGS - C - DK - XGS - G,$$

Fixed Capital Stock

(18) $$D \log K = k,$$

Rate of Growth in Money Supply

(19) $$m = D \log M,$$

Public Sector's Borrowing Requirement

(20) $$DH = PG - T,$$

Rate of Growth in International Reserves

(21) $$r = D \log R,$$

Rate of Growth in Bank Advances

(22) $$a = D \log A,$$

Rate of Growth in H

(23) $$h = D \log H,$$

Balance of Payments

(24) $$PXGS \cdot XGS - PMGS_f \cdot E \cdot MGS + (UT_a - UT_p) - DNFA - DR = 0.$$

TABLE A2
Variables of the Model

Endogenous

A = nominal stock of bank advances
a = proportional rate of growth of A
C = private consumption expenditure in real terms
E = lira-dollar spot exchange rate
G = public expenditure in real terms
H = public sector borrowing requirement
h = proportional rate of change of H
i_{TIT} = domestic nominal interest rate
K = stock of fixed capital in real terms
k = proportional rate of change of K
M = nominal stock of money (M2)
m = proportional rate of change of M
MGS = imports of goods and services in real terms
NFA = nominal stock of net foreign assets
P = domestic price level
$PXGS$ = export price level
R = nominal stock of international reserves
r = proportional rate of change of R
T = nominal taxes
V = stock of inventories in real terms
W = money wage rate
XGS = exports of goods and services in real terms
Y = real net domestic product and income
$\tilde{Y}$ = expected real net domestic product and income

Exogenous

E_c = official lira-dollar parity under fixed exchange rates
FR = forward exchange rate
i_f = foreign nominal interest rate
PF_f = foreign competitor's export price level (in foreign currency)
$PMGS_f$ = import price level (in foreign currency)
$PROD$ = labour productivity
t = time
$(UT_a - UT_p)$ = net unilateral transfers, in nominal terms
YF = real world income