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The Money Stock, the Price Level and Real Output: A Trivariate Analysis

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INTRODUCTION

There have been many empirical studies of Granger [1969] causality between money and nominal income for the United States [Sims, 1972; Hsiao, 1981; and Thornton and Batten, 1985]. Most of these studies are concerned with the issue of how changes in one nominal variable, such as the nominal stock of money, affect another nominal variable, such as nominal income. On the whole, empirical evidence indicates the existence of a causal flow from the money supply to nominal income.

Perhaps of even greater importance than examining the causal flow in the money—nominal income relationship is determining whether changes in nominal variables causally affect nominal variables only or whether they can also impact real variables. In other words, do changes in the money supply only affect the price level or do they also affect real variables, such as aggregate real output? The primary objective of this paper is to investigate empirically the Granger [1969] causal ordering between the nominal stock of money and real output.

In order to investigate the effects of monetary changes on real output, it is necessary to establish initially the existence of a causal flow from the money supply to nominal GNP. If such a relationship is found to exist, then the investigation can be extended to the two components of changes in nominal output—changes in prices and changes in real output. Therefore, at the outset, the study involves a bivariate analysis of a causal link between the nominal stock of money and nominal output. Once this relationship is established, then the question of the effects of changes in the money supply on the two components of nominal output—prices and real output—is investigated within the trivariate framework. In this study the technique outlined by Hsiao [1981] is used for selecting the length of the lags of all variables.

The paper is divided into four sections. The first section is devoted to the issue of selecting the "optimal" lag length in casuality testing between money and nominal output. The following section reports the results of bivariate causality tests between money and nominal output. The third part of the paper involves a trivariate analysis which examines the effects of changes in the nominal stock of money on prices and real output. The conclusions of the study are summarized in the final part.

THEORETICAL CONSIDERATIONS AND OPTIMAL LAG SELECTION

Barro [1977, and 1978] investigates the effects of unanticipated monetary changes on unemployment, the price level, and output in the United States. The two studies test the hypothesis that only the unanticipated part of money growth can influence real variables such as the rate of unemployment and real output, while the anticipated part of money growth only leads to corresponding price level changes. Barro finds empirical support for the hypothesis that it is the unanticipated part of changes in the money supply which affects real output. One difficulty associated with this procedure involves obtaining correct

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estimates of unanticipated money growth [Barro, 1977, pp. 105–6]. Another difficulty lies in the choice of lag length in examining the effects of lagged monetary shocks on real output.

Mishkin [1982] examines the issue of the effects of anticipated monetary policy changes on the business cycle. Relying on an arbitrary lag selection in testing, Mishkin finds support for the hypothesis that anticipated monetary changes have an impact on output fluctuations in the economy. Mishkin notes that the lag selection has a considerable effect on the hypothesis testing results [p. 33]. Mishkin's results are supported by the evidence presented by McGee and Stasiak [1985]. Their study indicates that the anticipated money growth and inflation influence the growth of real GNP in the short-run.

Although the effects of anticipated and unanticipated monetary changes on variables such as real GNP have been empirically examined, it may be of interest to investigate the effects of actual observed changes in the money supply on real output. The observed changes in the money stock have two components—an unanticipated part and an anticipated part. Although these components are not observable separately, their sum is observable. If the unanticipated part of the monetary growth affects real output and the anticipated part affects only prices [Barro 1977, and 1978], then the observed changes in the money stock must affect both real output and prices. The main objective of this paper is to examine this proposition within the framework of a trivariate analysis which uses the causality testing technique outlined by Hsiao [1981].

One of the major shortcomings of the studies of the effects of monetary changes on the U.S. economy lies in the lag selection method. Many studies rely on arbitrary selection of the lag structure. Test results obtained through an arbitrary lag selection technique may be unreliable because the distribution of test statistics can be sensitive to lag length [Hsiao, 1981; Thornton and Batten, 1985; Biswas and Saunders, 1986; Saunders, 1988]. The problem is explicitly recognized by Mishkin [1982, p.33]. Consequently, the validity of statistical tests relying on an arbitrary lag selection is doubtful. The present study seeks to remedy this particular problem by using the minimum final prediction error procedure developed by Hsiao [1981] for the lag selection.

For testing Granger [1969] causality Hsiao [1981] suggests a sequential approach which relies on Akaike's [1969a, and b] final prediction error (FPE) criterion. The causality testing method based on the FPE criterion is essentially a search procedure to find the "optimal" lag length.[1] The minimum final prediction error is computed as $(SEE)^2 \cdot (T + K)/T$, where SEE is the standard error of the regression, K indicates the number of parameters, and T is the number of observations.

Hsiao's [1981] procedure has several advantages over other causality testing methods which are based upon an arbitrary lag selection. First, it tests variables with different lag lengths. When the arbitrary lag selection method is used, the variables are always tested with identical lag lengths. Hsiao (1979) points out that in the cases of identical lag length testing, the number of parameters increases with the square of the number of variables. The degrees of freedom are then rapidly diminished. This problem is particularly cumbersome when relatively long lags are required. Second, the miminum FPE procedure offers a lag selection method based upon a statistical criterion rather than on an ad hoc procedure of finding the lag length. Third, the minimum FPE method provides information about the exogeneity and endogeneity of test variables. Additionally, this procedure avoids the conventional selection of the 5 percent or 1 percent levels of significance in causality testing.

Thornton and Batten [1985] discuss various methods of selecting the lag length in causality testing. These methods include the Bayesian estimation criterion suggested by Geweke and Meese [1981], the technique outlined by Pagano and Hartley [1981], and Hsiao's [1981] minimum FPE procedure. When compared to the other two causality testing methods, the minimum FPE procedure performed well in the selection of an appropriate model. Consequently, this method is adopted for the causality tests in this study.

BIVARIATE TEST RESULTS

A well established method of causality testing of the money and nominal income relationship consists of approximating monetary aggregates by three different measures of money (the monetary base, M_1, and M_2) and nominal income by nominal GNP [Sims, 1972]. Therefore, for the purpose of this study, the first step involves establishing a causal flow from the money supply to nominal output. In this study the money supply is approximated by the monetary base while nominal output is measured by nominal GNP.[2]

Seasonally adjusted quarterly U.S. data for nominal GNP (GNP) and the monetary base (B) are used in the bivariate analysis.[3] In the trivariate analysis the quarterly data for real GNP (GNPR), the GNP deflator (GNPD), and the consumer price index (CPI) are used additionally. The sample period under investigation spans the time from 1959-II to 1984-II. All equations are estimated in the first differences of the logarithms of the variables.[4] Hsiao's [1981] causality testing method is implemented in each test equation by identifying the optimal lag structure with the maximum lag length assumed equal to 12. In each case, the criterion of the minimum final prediction error is used to determine the optimal lag selection of all test variables.

The first step (later referred to as step 1) in Hsiao's [1981] procedure involves computing the FPEs of one-dimensional autoregressive processes for two test variables, GNP and B. The minimum FPEs of GNP and B and the number of lags associated with these minimum FPEs are reported in the first part of the Table 1 below as equations (1) and (2). The lag lengths of GNP and B with the smallest FPEs are two and ten respectively. Once the optimal lags of GNP and B are determined, the next step (step 2) involves treating one of the two variables, GNP and B, as the dependent variable and the other as the independent variable. In equation (3) GNP is chosen as the dependent variable and B as the independent (manipulated) variable. In equation (4) B is treated as the dependent variable while GNP is assumed to be the independent variable. The FPE criterion is used to determine the optimal lag of the independent variable while holding the order of the lag operator on the dependent variable (determined in step 1) constant.

Essentially, the FPEs of the dependent (controlled) variable are computed holding the length of its

TABLE 1
Casuality Testing by Calculating Final Prediction Errors (FPEs) of GNP, B, GNPR, CPI, and GNPD*

Equation	Controlled (Dependent) Variable	First Manipulated (Independent) Variable	Second Manipulated (Independent) Variable	FPE × 10^{-4}
I. Bivariate Results				
(1)	GNP (2)			1.0073
(2)	B (10)			0.1695
(3)	GNP (2)	B (1)		0.8586
(4)	B (10)	GNP (1)		0.1710
II. Trivariate Results				
(5)	GNPR (1)			0.9349
(6)	GNPR (1)	CPI (7)		0.8242
(7)	GNPR (1)	CPI (7)	B (1)	0.7795
(8)	GNPR (1)	GNPD (2)		0.8918
(9)	GNPR (1)	GNPD (2)	B (1)	0.8341
(10)	CPI (9)			0.2008
(11)	CPI (9)	GNPR (1)		0.1679
(12)	CPI (9)	GNPR (1)	B (7)	0.1589
(13)	GNPD (3)			0.2085
(14)	GNPD (3)	GNPR (1)		0.2114
(15)	GNPD (3)	GNPR (1)	B (1)	0.1984

*Numbers in parentheses in columns 2, 3, and 4 are lags for minimum FPEs. These lags indicate the number of quarters used for each test variable. The format of reporting is modified from RAM (1984).

lags constant while varying the order of lags of the independent (manipulated) variable from one to 12. The order which results in the smallest FPE is chosen. When GNP is the controlled variable and B is the manipulated variable the optimum lags of these two variables are two and one, respectively. This procedure is repeated by reversing the roles of GNP and B [equation (4)]. Here B is the controlled variable and GNP is the manipulated variable with their optimal lag lengths of ten quarters and one quarter respectively. Overall causality inferences are made on the basis of the comparison of the minimum FPEs of steps 1 and 2.

Equations (3) and (4) are specified in the following manner:

$$\text{GNP}_t = a_0 + \sum_{j=1}^{2} a_j \text{GNP}_{t-j} + b_1 B_{t-1} + U_t \quad (3)$$

$$B_t = \alpha_0 + \sum_{j=1}^{10} \alpha_j \beta_{t-j} + \beta_1 \text{GNP}_{t-1} + V_t. \quad (4)$$

In the above equations, j indicates the number of lags, t stands for the time period, and U_t and V_t are the stochastic terms with all assumed properties.

Adding the lagged monetary base term to the lagged GNP variable [equation (3)] reduces the FPE from 1.0073 to 0.8586 while an inclusion of the lagged GNP variable to the monetary base [equation (4)] increases the FPE from 0.1695 to 0.1710. This implies that a unidirectional causal flow is established from the monetary base to nominal GNP without any feedback.[5] Consequently, using the monetary base as a measure of the money stock, the monetary base is found to be exogenous in the money—nominal output relationship.

Having established a unidirectional causal flow from the monetary base to nominal GNP, it may be of interest to determine the size of the impact of this monetary variable on nominal GNP. The size and the sign of the coefficient of the lagged monetary base term in equation (3) give an indication of the magnitude and the direction of monetary changes on nominal GNP. This coefficient is 0.688, with the corresponding t statistic of 4.327.[6] It indicates a large positive effect of the monetary base on nominal GNP. It also implies that increases in the money supply (as approximated by the monetary base) lead to rapid, positive changes in nominal GNP.

TRIVARIATE ANALYSIS

The bivariate results reported above provide useful information about the causality issue in the money—nominal output relationship. In the case of the monetary base, empirical evidence suggests a unidirectional causal flow from money to nominal output. However, the causality test procedures give no indication to what extent the monetary changes affect the two individual components of nominal output—the price level and real output. Real output can be best approximated by GNPR. As far as the price level measure is concerned, there are two obvious measures of inflation—the GNPD and the CPI. The GNP deflator is also used in computing real total output. Ram (1984) argues that since the GNP deflator and real total output are closely related, there may be some advantage in using a measure of inflation which is constructed independently of the real GNP computation, such as the consumer price index. In this study both measures of inflation, the GNPD and the CPI, are used.

Ram [1984] outlines a framework of a trivariate analysis based on Hsiao's [1981] minimum FPE method. The trivariate analysis suggested by Ram is an extension of Hsiao's bivariate sequential search procedure. For example, for the GNPR equation [equation (7)] with the CPI used to measure inflation, initially GNPR is taken as the only variable of the system. The order of the one-dimensional autoregressive process for GNPR is determined by using the FPE criterion. In this case the lag length is one. So the first explanatory variable is entered in the GNPR equation as GNPR_{t-1}. This specification is illustrated in the above Table as equation (5). Having established that the order of the lag operator on GNPR is one, the lag order of CPI is then determined by using the minimum FPE criterion. The order of the lags of CPI is computed to be seven in this case. This particular specification is illustrated as equation (6). The process is

then continued to determine the lag length of the second manipulated variable B, as reported in equation (7). The optimum lag of B is one. This is the lag which gives the smallest FPE for the entire GNPR equation.

Following the FPE procedure, the real output and the inflation equations are specified within the trivariate analysis framework in the following manner:

$$\text{GNPR}_t = a'_0 + a'_1 \text{GNPR}_{t-1} + \sum_{j=1}^{7} b'_j \text{CPI}_{t-j} + c' B_{t-1} + \epsilon_t \quad (7)$$

$$\text{GNPR}_t = a''_0 + a''_1 \text{GNPR}_{t-1} + \sum_{j=1}^{2} b''_j \text{GNPD}_{t-j} + c'' B_{t-1} + \epsilon'_t \quad (9)$$

$$\text{CPI}_t = \alpha'_0 + \sum_{j=1}^{9} \alpha'_j \text{CPI}_{t-j} + \beta'_1 \text{GNPR}_{t-1} + \sum_{j=1}^{7} \gamma'_j \beta_{t-j} + \xi_t \quad (12)$$

$$\text{GNPD}_t = \alpha''_0 + \sum_{j=1}^{3} \alpha''_j \text{GNPD}_{t-j} + \beta''_1 \text{GNPR}_{t-1} + \gamma'' B_{t-1} + \xi'_t, \quad (15)$$

where ϵ_t, ϵ'_t, ξ_t, and ξ'_t are stochastic disturbance terms. Equations (7) and (9) are the real output equations whereas equations (12) and (15) are the inflation equations. The order of lags associated with different variables in these equations and in other equations are identified by using the minimum FPE procedure as suggested by Hsiao [1981].

The last two rows of the trivariate section of the Table 1 allow inferences to be made about the causal flow from the monetary base to the price level and real output. There appears to be evidence of a causal flow from the monetary base to both components of nominal output: prices and real output. The addition of the lagged monetary base term to the price level equations (12) and (15) reduces the FPEs from 0.1679 and 0.1589 and from 0.2114 to 0.1984, respectively. Similarly, the addition of the lagged monetary term to the real output equations (7) and (9) also reduces the FPEs from 0.8242 to 0.7795 and from 0.8919 to 0.8341. This implies that the impact of the monetary variable on nominal GNP operates through both price level changes and real output changes. Consequently, both prices and real output are affected by changes in the money supply. Furthermore, consistent results are obtained using either measure of inflation.

An indication of the magnitude of the effects of monetary changes on both components of nominal output is given by the values of the lagged coefficients of the monetary base in equations (7), (9), (12), and (15). The coefficients of the lagged monetary base terms in the real output equations (7) and (9) are 0.185 and 0.499 with the corresponding t statistics of 2.581 and 2.923. Both coefficients are significant and their signs are positive. The interpretation of these results is that the monetary base has a positive impact on real output.[7]

OVERALL CONCLUSIONS

This paper investigates initially the causal relationship between the nominal stock of money and nominal output using U.S. quarterly data for the period 1959-II to 1984-II. After this bivariate relationship is examined, the analysis is extended to an understanding of the effects of changes in the money supply on the two components of nominal output—real output and the price level.

The methodology adopted in this study combines Granger's [1969] concept of causality with a sequential search procedure outline by Hsiao [1981] and based on Akaike's [1969a, and b] final prediction error criterion to determine the direction of causality in the money—nominal output relationship. This method not only determines the Granger causal ordering, but it also identifies the order of lags for each variable. Applied to the U.S. quarterly data for the period 1959-II to 1984-II, the bivariate test results indicate a unidirectional causal flow from money (as approximated by the monetary base) to nominal ouput (measured by nominal GNP). Consequently, the monetary base plays an important causal role in the money—nominal output relationship.

The novelty of this study lies not only in the causality testing method relying on the optimal lag selection, but also in its emphasis on establishing a causal flow from the money supply (nominal variable) to the two components of nominal output—the price level and real output (real variable). This part of the investigation is conducted within a trivariate framework. The trivariate analysis indicates that the impact of monetary changes on nominal output operates through both price level and real output changes. Furthermore, it appears that this impact is positive with respect to both prices and real output. This evidence implies that changes in a nominal variable, such as the nominal stock of money, do causally affect real variables, such as an economy's real output.

NOTES

1. Hsiao [1981, pp. 92–3] outlines this procedure in detail.
2. Causality tests involving M_1 and M_2 were also undertaken. Feedback between both of these measures of the money supply and nominal GNP was established. Consequently, these two measures of money are not suitable for the purpose of analyzing the impact of monetary changes on prices and real output.
3. Although all the data used in this study are seasonally adjusted at the source, the lag specifications are sufficiently long to prevent any bias from the source to affect the test results [Sims, 1972].
4. The first differences of logarithms estimation form alleviates some of the problems associated with the nonstationarity of the time-series data.
5. For a detailed description of the causality inferences see Hsiao [1981, pp. 90–3].
6. Space constraints do not permit the inclusion of the estimates of equations (3) and subsequent equations (7), (9), (12), and (15). However, their tabulated results will be furnished upon request.
7. In addition to causality implications described above, it may be of interest to note that equations (9) and (15) provide a better prediction for nominal GNP than is given by equation (3). The prediction for changes in log nominal income equals the sum of the predictions or equations (9) and (15) for changes in log real GNP and changes in log GNP deflator.

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