women. In a view that seems typical of those interviewed, one woman lamented, "I believed that if I was happy, my children would be happy. I believed my mind was too good to stay at home. I believed all the time." Perhaps, this phenomenon may help to explain why the youth variable was such a dominant variable in Fosler's results for 1970 but not in 1980 (see footnote 1).

REFERENCES


Throughout this paper, the references are cited in parentheses following the quotations.

"The Effect of Children on the Housewife's Value of Time," Journal of Political Economy,

(March/April, 1977), Part 2, 168-90.

The general theoretical framework of the model is discussed in detail in the following pages.


Throughout this paper, the references are cited in parentheses following the quotations.

The general theoretical framework of the model is discussed in detail in the following pages.

The Demand for Labor with Heterogeneous Hours

Bryan L. Boulier, Vince Foy, and Robert Goldfarb

The usual microeconomic model of the firm's demand for labor gives no insight into the choice between number of workers and hours per day, since all man-hours are assumed to be homogeneous. Models that allow for variation in worker productivity over the day permit analysis of this choice (Barzel, 1973; Elbergren, 1971(a), 1971(b); and Rosen, 1968, 1969, 1978). Our previous work on heterogeneous hours (Fosler, 1985) develops this approach for one grade of labor in the presence of fixed costs of hiring and legal requirements for time-and-one-half overtime pay. This paper generalizes our earlier work by treating how hours for several grades of labor are determined at the firm level in a framework that takes explicit account of the imperfect substitutability between workers and hours within each labor category.

This extension generates a number of results which testify to the fruitfulness of the model. First, the model allows the derivation of plausible conditions under which a firm would choose to employ one class of labor full-time and another type of labor part-time or overtime. Second, the analytical framework provides a more fundamental theoretical explanation for ambiguous empirical results (Grasmich, 1976; McKee and West, 1984) about the effects of raising the minimum wage on relative levels of part-time versus full-time employment. Third, the general comparative static properties of the model produce a striking implication about the demand for labor. While the firm's demand for total labor hours is negatively related to the wage, the demand for the number of employees need not be. That is, the demand for the number of employees can be upward sloping. Besides its obvious theoretical interest, this finding has useful implications for interpreting empirical studies of the demand for labor.

THE MODEL

In this paper, we assume that the firm produces output using two grades of labor as inputs. For simplicity, we exclude capital and other inputs used in production. The production function is:

$$ Q = (L_1 g(H_1))(L_2 g(H_2)) $$

where $L_1$ is the number of homogeneous workers of type $1$, $H_1$ is the total number of hours per day per worker of type $1$, and $g(H)$ is a labor effectiveness function for type of labor. The labor effectiveness function is assumed to be $S$-shaped and in its derivative $g'(H)$, the "marginal effectiveness per extra hour" function, is an inverted U. The $S$-shape results from low marginal productivity at the start of the day due to start-up costs and low marginal productivity at the end of the day because of boredom and fatigue (Rosen, 1976, p. 150). The product of $g(H)$ and $L_1$ is labor input of type 1. The way equation (1) is written assumes that the production function is weakly separable in $L_1$ and $H_1$ and that the shape of $g(H)$ is independent of capital and the number of employees of either labor type. We indicate below how some of our comparative statics results are altered when the assumption of separability is relaxed.

Suppose the firm is faced with a legal requirement to pay time-and-one-half after 40 hours and must pay fixed costs of hiring workers, where $F$ is the anticipated daily fixed cost of hiring a worker of type 1 ($\Omega, 1962)$. Given our assumption that the worker's effectiveness function $g(H_1)$ is identical for each day of


We would like to thank Michael Bradley, Daniel Hamenoh, Mark Montgomery, Susan Vroman, Anhomy Yoran, and participants in seminars at George Washington University and Georgetown University for comments on an earlier version of this paper.

239
Demand for Labor with Heterogeneous Hours

Next, consider a case where one (or both) of the grades of labor works overtime. Equilibrium for the grades working overtime requires:

\[
\frac{g(s + H)}{1.5w_s} = \frac{g(s + H) + (\alpha + H)}{(1.5w_s + 1.5w_H + F_s)/(8 + H)}
\]

which has an interpretation analogous to equation (5) above.

Finally, consider the case where one or both of the grades of labor works exactly eight hours. Then equilibrium requires that the following inequalities be satisfied:

\[
\frac{g(0)}{1.5w_s} = \frac{g(0)}{1.5w_H} = w_s = w_H
\]

The LHS of the inequality represents the marginal output per dollar cost of lengthening the worker's day beyond eight hours; the 1.5 in the denominator indicates that hours in excess of eight must be paid at time-and-one-half. The RHS must be less than or equal to the middle term, which gives the extra hourly output per dollar of cost (wage plus hour's worth of fixed costs) from hiring another worker. The middle term must in turn be less than or equal to the RHS, which represents the output per dollar of cost for the eighth hour. The LH inequality ensures that it does not pay to raise hours above eight. The RH inequality ensures that it does not pay to cut back hours below eight.

THE FIRMS DEMAND FOR PART-TIME VS. FULL-TIME VS. OVERTIME HOURS OF WORK

The equilibrium conditions embodied in equations (5), (6), and (7) can be used to investigate the conditions under which a firm would choose to employ one class of labor part-time while simultaneously employing another class full-time and another overtime. Rearrangement of (5) yields the following result, where the subscript s is used to indicate labor of type s:

\[
\frac{F_s}{w_s} = \frac{g(s)(1)}{g(s)(H)}
\]

This condition must hold if labor of type s is to be employed part-time. Rearrangement of (7) for full-time workers produces

\[
\frac{F_s}{w_s} = \frac{g(s)(1)}{g(s)(H)}
\]

where the h indicates labor of type h. Finally, manipulation of equation (6) for workers hired for more than eight hours produces:

\[
\frac{F_s}{w_s} = \frac{g(s)(1)}{g(s)(H)}
\]

where the subscript c is used to indicate labor of type c. Using (9), (10), and (11) and assuming identical g's for each grade of labor, then

\[
\frac{F_s}{w_s} = \frac{F_c}{w_c}
\]

That is, firm equilibrium requires that the ratio of fixed cost per day to the wage must be lower for part-time workers than for full-time workers, and lower for full-time than for overtime workers. Thus, the lower the ratio of fixed cost per day to the wage, the fewer the hours per worker the firm is likely to desire. In addition, for a firm to hire one class of labor part-time and another full-time or overtime, the part-time labor must have a lower ratio of fixed cost per day to its wage than does the full-time or overtime labor.
The result in (11) is derived as follows. Using equation (8) and the L.H inequality of (9), \( F_0/w = F_0/w < F_0/w \), if
\[
g\left( H_0 \right) > g\left( H - 8 \right)
\]
where \( H_0 < 8 \). But this inequality must hold because the function \( g\left( H \right)/g\left( H - 8 \right) \) can be shown to have a positive first derivative with respect to \( H \). A similar rearrangement of (9) and (10) can be used to show that \( F_0/w < F_0/w \).

**COMPARATIVE STATIC ANALYSIS**

Further interesting results can be obtained from comparative static analysis. In particular, this analysis leads to surprising results about the shape of the "head count" demand for labor, and to results about increasing the minimum wage on hours of work relevant to the conflicting findings of Gramlich and McKee-West. We present comparative statics for three cases:

- **Case 1:** \( L_0 > 0 \) and \( H_0 < 8 \).
- **Case 2:** \( L_0 > 0 \) and \( H_0 = 8 \).
- **Case 3:** \( L_0 > 0 \) and \( H_0 > 8 \).

For each case, we can show that the elasticity of demand for labor with respect to the wage \( w \) equals
\[
\eta_{w} = \frac{w H_{0}}{\Delta \eta_{w}}
\]
where the elasticity of demand for labor with respect to fixed costs \( \omega \) and the elasticity of demand for hours per week with respect to the wage \( \eta_{w} \) are both less than zero. If the demand for hours is sufficiently elastic that \( \eta_{w} > -1 \), then \( \eta_{w} \) increases with the increase of wages. In the absolute value of \( \eta_{w} \), is positively related to the ratio of variable costs per worker \( \omega/w \) to fixed costs per worker \( \omega/w \). Finally, the wage elasticity of demand for total hours \( \eta_{w} \) equals the sum of the wage elasticities of demand for workers and for hours:
\[
\eta_{w} = \eta_{w} + \eta_{w}.
\]

While \( \eta_{w} \) can be positive, \( \eta_{w} \) must be negative. Consequently, even if \( \eta_{w} > 0 \), there is a demand for total hours is more elastic with respect to the change in the wage than the demand for workers, since \( \eta_{w} > 0 \).

As for cross-wage effects, Table 1 indicates that a rise in \( w \) has no effect on \( H_0 \). Note, however, that this result is generated by the assumption of separability. Because \( H_0 \) is unchanged, the effect of \( w \) on \( L_0 \) must be in the same direction as the effect on \( H_0 \). The table indicates that the effect of \( w \) depends on whether effective hours \( L_0 \) of type 1 and type 2 labor are complements \( (\delta H_0 > 0) \) or substitutes \( (\delta H_0 < 0) \), an intuitively appealing result.

The effects of a rise in \( F \) or \( L_0 \) and \( H_0 \) are also plausible; as \( F \) rises, hours rise to economize on fixed costs per employee, and \( L_0 \) falls because employees are more expensive. These conflicting effects on \( L_0 \), versus \( H_0 \), produce the ambiguity of the effect of an increase in \( F \) on \( L_0 \). The cross effects of \( F \) on \( H_0 \) and \( L_0 \) are precisely analogous to the cross-wage effects.

The price effects are also of some interest. Price increases do not affect hours per worker, because they do not affect the relative cost of hiring one more worker versus raising hours. (This can also be seen by examining equation (5) and the associated discussion.) That demand for \( L_0 \) and \( H_0 \) will rise with a price increase if the two grades of labor are complements agrees with intuition; the ambiguity if the two are substitutes reflects the fact that an output increase may only cause one of the two inputs to rise.

The reader may wonder how dependent these effects—especially the ambiguous sign of the "head count" demand for labor—arise on the assumption of weak separability between number of workers and hours per worker. If a more general production function without separability is assumed, the comparative static results become less determinate. In fact, the only remaining determinate comparative static effects are the negative effect of \( w \) on \( L_0 \) and the negative effect of \( F \) on \( L_0 \). Thus, the ambiguous sign on the "head count" demand for labor is not a result of the separability assumption.

In Case 2, type 1 labor works overtime \( H_0 > 0 \) and type 2 labor works exactly 8 hours \( H_0 = 8 \). Comparative static results are given in Table 2. Note that the format of Table 2 differs from that of Table 1. In particular, the \( dH_0 \) column and value of \( H_0 \) is missing and a new row labelled \( "w" \) or \( "w" \) is added. The \( dH_0 \) column is omitted because all \( dH_0 \) effects are zero by assumption.

The price effects of an increase in \( w \) or \( F \) on \( L_0 \) are the "mirror images" of the cross-effects given by the upper two entries in the fourth column, so that it is not surprising that the same signs are displayed. The "zero" in row three, column two results from our separability assumption.
TABLE 2
Comparative Statics When Type 1 Labor Works Overtime and Type 2 Labor Works Full-Time

\[
\begin{array}{cccc}
\delta_{L_i} & \delta_{L_0} & \delta(L_i + H_0) & \delta(L_i H_0) \\
\text{\text{complement}} & + & + & + \\
\text{\text{substitute}} & - & - & - \\
\end{array}
\]

explained in our comments on Table 1. The "complement/substitute" result in row three, column three follows directly from the first two entries in row three. All else being equal, column three must be identical to row three, column one, since both cells embody only the identical effect on \( L_1 \).

The effect in row three, column four is quite striking in that the head count demand for labor of grade 2 is downward sloping, in contrast to our ambiguous result for part-time labor (Table 1) and overtime labor (Table 2, upper left-hand cell). The reason is straightforward. For labor "bunched" at eight hours, we assume (as explained in footnote 8) that exogenous comparative static changes do not change hours. Thus, the ambiguous effect of a wage increase on \( L_1 \), which can arise when hours fall is ruled out when hours are constant.

In Case 3, type 1 labor works part-time \( (0 < L_1 < 8, H_0 = 0) \) and type 2 labor works exactly eight hours. We have not provided a separate table for these results, because they are identical to Table 2 with column two relabeled \( d\delta \), and column three relabeled \( d(L_i H_0) \).

TWO APPLICATIONS OF THE COMPARATIVE STATICS

We believe that the ambiguous sign of the head count demand for labor in several of the cases is an interesting finding. Such an ambiguity does not, to the best of our knowledge, arise in other standard labor demand models. Thus, adding an intuitively plausible assumption about how labor effectiveness varies with hours produces a notable departure from standard results.

the elasticity of the number of workers, \( -7.2 \), Brown, Gilroy and Cohen (1983) try to deal with the hours issue by converting part-time teenage workers into full-time equivalents. They do this by assuming the average part-time worker halves the hours of the average full-time teenage employee. They then estimate the effect of the minimum wage on full-time equivalent (FTE) employment. They argue that if raising the minimum wage results in a rise in the fraction of part-time workers, then the minimum wage in FTE equations should be larger than in simple head count equations. This result is precisely what they find, leading them to conclude that the minimum wage has a larger effect on hours corrected employment than on simple counts of the number of employed.

A second application of the comparative statics concerns the Gramlich-McKenzie-West results about how a rise in the minimum wage would affect the relative quantities of full-time versus part-time work. Gramlich (1957) produced data indicating that a rise in the U.S. minimum wage increased the proportion of part-time work. McKee and West (1984) claim that contrasting outcomes are possible and use Canadian data to show a rise in the proportion of full-time employment. Our framework analyzes the question at the firm level. Clearly, several conflicting effects are possible. Consider Case 3 first, and suppose that the part-time labor \( L_1 \) is working at the minimum wage \( \omega = \omega_p \) while full-time labor \( L_0 \) is working at \( \omega = \omega_e > \omega_p \). Clearly, if \( H_0 \) is constant, we would expect \( L_0 \) to contract. McKee and West rely on differential coverage of the minimum wage across firms to explain ambiguous effects of minimum wage increases on the proportion of part-time to full-time employment. Our analysis reveals that this indeterminacy has an even more fundamental cause and would exist even without differential coverage.

QUALIFICATIONS AND POTENTIAL EXTENSIONS

While we believe the analysis given above is useful and illuminating, there are a number of relevant aspects of the firm's hours choice that it does not incorporate. One important aspect is that it may be necessary for the firm to coordinate the hours of both types of labor because they must work in physical proximity. This kind of constraint is not incorporated in our analysis. Another example concerns a firm's decision to hire some workers in a single grade of labor part-time and others in the same grade full-time. This behavior is undoubtedly a reaction to daily peak load demand (e.g., extra waiters at lunch).

Peak-load demand is not included in the current model. Such practices may also arise when there is an upward sloping supply curve of hours among workers and there are differentiated fixed costs of hiring. (Montgomery, 1988b). An extension of the current model to incorporate upward sloping supply curves of hours worked would be especially desirable.

There are other possible applications of extensions of the model. Equation (11) indicates that hours per worker depend on the ratio of \( \delta \) to \( \omega \). This finding implies that hiring subsidies have different effects on the daily hours worked by subsidized workers than do wage subsidies. Moreover, the analysis of employment subsidies is complicated by the fact that the response of total hours \( L(H) \) to a decrease in fixed costs is of ambiguous sign because of the substitution of employees for hours per worker. This result suggests that hiring subsidies based on number of employees, job banks, and other programs that reduce fixed costs of employment have more complex effects than has been supposed.

A quite different application involves analyzing the biases in head count based measures of elasticities of substitution. The labor input in our production function model \( g[H] \) is considerably more complex than simple head counts or even simple hours counts. Given that hours per worker will vary with the wage in our model, what biases would head count measures of substitution elasticities reflect?

than the elasticity of the number of workers, \( -7.2 \). Brown, Gilroy and Cohen (1983) try to deal with the hours issue by converting part-time teenage workers into full-time equivalents. They do this by assuming the average part-time worker halves the hours of the average full-time teenage employee. They then estimate the effect of the minimum wage on full-time equivalent (FTE) employment. They argue that if raising the minimum wage results in a rise in the fraction of part-time workers, then the minimum wage in FTE equations should be larger than in simple head count equations. This result is precisely what they find, leading them to conclude that the minimum wage has a larger effect on hours corrected employment than on simple counts of the number of employed.

A second application of the comparative statics concerns the Gramlich-McKenzie-West results about how a rise in the minimum wage would affect the relative quantities of full-time versus part-time work. Gramlich (1957) produced data indicating that a rise in the U.S. minimum wage increased the proportion of part-time work. McKee and West (1984) claim that contrasting outcomes are possible and use Canadian data to show a rise in the proportion of full-time employment. Our framework analyzes the question at the firm level. Clearly, several conflicting effects are possible. Consider Case 3 first, and suppose that the part-time labor \( L_1 \) is working at the minimum wage \( \omega = \omega_p \) while full-time labor \( L_0 \) is working at \( \omega = \omega_e > \omega_p \). Clearly, if \( H_0 \) is constant, we would expect \( L_0 \) to contract. McKee and West rely on differential coverage of the minimum wage across firms to explain ambiguous effects of minimum wage increases on the proportion of part-time to full-time employment. Our analysis reveals that this indeterminacy has an even more fundamental cause and would exist even without differential coverage.

QUALIFICATIONS AND POTENTIAL EXTENSIONS

While we believe the analysis given above is useful and illuminating, there are a number of relevant aspects of the firm's hours choice that it does not incorporate. One important aspect is that it may be necessary for the firm to coordinate the hours of both types of labor because they must work in physical proximity. This kind of constraint is not incorporated in our analysis. Another example concerns a firm's decision to hire some workers in a single grade of labor part-time and others in the same grade full-time. This behavior is undoubtedly a reaction to daily peak load demand (e.g., extra waiters at lunch).

Peak-load demand is not included in the current model. Such practices may also arise when there is an upward sloping supply curve of hours among workers and there are differentiated fixed costs of hiring. (Montgomery, 1988b). An extension of the current model to incorporate upward sloping supply curves of hours worked would be especially desirable.

There are other possible applications or extensions of the model. Equation (11) indicates that hours per worker depend on the ratio of \( \delta \) to \( \omega \). This finding implies that hiring subsidies have different effects on the daily hours worked by subsidized workers than do wage subsidies. Moreover, the analysis of employment subsidies is complicated by the fact that the response of total hours \( L(H) \) to a decrease in fixed costs is of ambiguous sign because of the substitution of employees for hours per worker. This result suggests that hiring subsidies based on number of employees, job banks, and other programs that reduce fixed costs of employment have more complex effects than has been supposed.

A quite different application involves analyzing the biases in head count based measures of elasticities of substitution. The labor input in our production function model \( g[H] \) is considerably more complex than simple head counts or even simple hours counts. Given that hours per worker will vary with the wage in our model, what biases would head count measures of substitution elasticities reflect?
NOTES

1. For example, if it costs $Z$ dollars to hire a worker and his expected length of stay is $Y$ days, $F$ is obtained as the solution to the equation $Z = \int_{0}^{Y} (1 - e^{-t}) dt$, where $e$ is the daily interest rate relevant to the firm in question. Depending upon institutional arrangements, contributions to social welfare schemes and unemployment and disability insurance may vary with the number of workers employed and hours of work. The daily annuitized value of these non-wage costs are also included in $F$. For a detailed treatment of non-wage labor costs, see Hart (1984).

2. Barrett (1973) considers the case where the market supply curve of labor is not perfectly elastic. Montgomery (1983a, 1988b) considers the firm's demand for full-time versus part-time employees when it faces an upward sloping wage-hours function. In Montgomery's model, hours of part-time and full-time labor are perfectly substitutes.

3. If $F = 0$ hours per worker do not vary with the wage. When $F > 0$ in equation (5), the firm maximizes average effectiveness per hour of each worker, setting the marginal product of another hour equal to the average product. As $w$ changes, labor services are adjusted by altering $L_{1}$; the firm continues using each worker for the number of hours that maximizes his average "input effectiveness" per hour.

4. Obviously, different $g(h)$ functions can be assumed which will "explain" virtually any differences between full-time and part-time work across groups. We want to see how far we can get in explaining hours differences without assuming "convenient" variations in $g(h)$ functions.

5. Our earlier paper (1985) incorrectly implied that $dL/dw$ was unambiguously negative.

6. In a model similar to the case used here, Hart (1984, pp. 76-78) derives the substitution effect of a change in the hourly wage on the number of workers employed, but does not consider output effects of wage changes.

7. Derivations of the elasticity formulae and the output and substitution effects of wage changes are in an appendix available from the authors on request.

8. The comparative static analysis assumes that $H_{0} = 8$ and that $H_{1}$ does not vary with changes in exogenous parameters. It is, of course, possible that changes in exogenous parameters could be of sufficient magnitude that this corner solution would not be optimal.

REFERENCES


