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A New Approach to Measuring Utility and Output Levels in Microeconomics

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1. INTRODUCTION

This paper is designed to explore a particular method for operationalizing the general equilibrium and welfare economics analysis of microeconomic theory. In addition, alternative conceptualizations of microeconomic behavior are considered that are not tied so tightly to maximization of utility and output for a given set of factor inputs, as would be the case under the traditional neoclassical microeconomic theory model. Sometimes outliers that don't fit the assumptions of rationality may be more informative than forcing the observations to fit a model that assumes transitivity and the "more is necessarily better" orientation.

Our intent is to spark interest and spawn discussion on the subject of assumptions. Specifically, this paper is designed to discuss both the maximization and constant taste assumptions of traditional economic theory. This paper is not a definitive answer but is rather an exploration of an issue which is often left buried in principles courses. We are not attempting herein to address in any optimum state of either the individual or the economy. We are simply attempting to analyze present and past behavior and forecast the future behavior implied.

The primary means of operationalizing the micro theory general equilibrium model is in finding a method to generate utility indices from observed data directly, rather than through the demand functions or linear expenditure model assumptions. By using this more direct approach we are able to examine alternative sets of assumptions rather than strictly adhering to the assumptions of utility and profit maximization. In addition to inferring ordinal individual utility indices from observed data, this methodology allows for the estimation of a measure of society's overall ordinal level of welfare as determined by an implicit social welfare function.

An important aspect of this approach is that comparisons of interpersonal utilities and aggregation of utilities is not imposed, but rather, is observed through the revealed preference approach used here. Also, instead of assuming a cross sectional approach where tastes and preferences must be considered constant, we have adopted a time series approach in which tastes and preferences are allowed to change without limit. It is this aspect which allows us to estimate a social welfare function without contradiction of Arrow's impossibility theorem. Arrow, in his study of social welfare functions, explicitly states that individuals are assumed to be rational and, by Axiom II, that tastes are consistent (ie, transitivity is maintained).[1] We assert that complete rationality of human beings is a controversial position and may not be applicable in some situations. We are not attempting to assert that humans are completely irrational, for that is obviously untrue. We can only say taste and preference formation is a psychological process which, in all likelihood, will remain a mystery. It is from this position that we advocate the use of revealed preference in order that we may allow any degree of rationality/irrationality to be included in

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analysis. We do not assume tastes are always consistent and we do not require transitivity. We are simply observing past behavior whether it be rational or not.

Additionally, we address a social welfare function as one which conveys information concerning the present state of society in relation to past states which are no longer chosen. We simply look at the actual choices the society is making and has made in the past. We do not pretend to be determining in what state society should be, all we are able to discern is the functional relationship between utility and society's payment for that utility with income. Conversely, Arrow was concerned with the way in which society should choose to allocate resources under conventional optimizing behavior. For these reasons we consider our social welfare function to be significantly different from those discussed by Arrow.

The basic method utilized in this analysis is to consider two goods consumed by two individuals where each good is produced using two factor inputs. For simplicity, the two individuals are referred to as capital and labor and correspond exactly to the two factor inputs being employed in different proportions in the production of the two consumer goods.

For this theoretical exposition we assume a ten week period, however a period of any length would be appropriate. In ten weeks a consumer is faced with ten different sets of prices and spends ten different total amounts of money. Consumers are assumed to be identical in their observed characteristics so that differences in utility or factor market behavior are based strictly on internal preferences. In other words this model ignores such factors as race, religion, age, experience and education. These aspects may be included in future work through examination of specific groups with the desired characteristics.

In ten weeks the consumer chooses among ten different market baskets of goods and services which contain different quantities of two goods, Y1 and Y2. Each week the available market basket may contain different amounts of Y1 and Y2. Prices for Y1 and Y2 may also change.

Deciding if one week's basket is preferred to another requires a comparison. In order to make such a comparison between any two bundles, it is necessary to compare the two purchases at a common price. The cost of the market basket combination that is chosen in any given week (call it week 'i'), is compared with the cost of the market basket chosen in any other week (call it week 'j') at the prices in effect for week 'i'. If, at prices in effect in week 'i', a market basket purchased in week 'j' was not purchased in week 'i' but was available in week 'i' and costs less than the one that was purchased in week 'i' then the presumption is that the chosen market basket was preferred to the less costly alternative. In that case each of the less costly alternative baskets not chosen are given one demerit. If, on the other hand, at prices for week 'i' the basket purchased in week 'j' would be more expensive than the basket purchased in week 'i' (and therefore unavailable to the consumer because all income is assumed spent on actual purchases in week 'i'), then the presumption is that the chosen market basket was not preferred to the more costly alternative. Since no preference has been established, no demerit will be issued. The consumer simply may not have had enough money to afford the more expensive market baskets that were not chosen, or the consumer may have not preferred them even if they were within reach. All of the market baskets are compared each week and demerits are given out in the same manner, always using only that week's prices to make the comparisons. After all ten weeks' outcomes are recorded in this manner, the demerits are added up for each market basket. Finally, this sum is subtracted from the number of market baskets being compared (ten, in this case) to obtain the utility index for this consumer. The subtraction is necessary in order to generate a ranking which has higher numbers corresponding to higher levels of preference. In other words, a market basket with a ranking of ten would be considered the most preferred.

This method is, of course, based on the traditional revealed preference analysis as first suggested by Paul Samuelson. However, the aggregation of demerits is carried out in the spirit of nonparametric estimation as used in constructing the test statistic for a sign test, for example. Note that the aggregated zero-one values for a sign test are then used as a binomial random variable for testing purposes. A similar ordinality property is used here in constructing this quasi ordinal preference ranking function.

The revealed preference approach can be applied to production as well as consumption. Instead of market baskets of consumer goods, there are production baskets of factor input combinations. In place of consumer good prices there are factor input prices. As far as we know the use of this revealed preference method in a production function context is new since it ignores actual production quantities and generates a preference ordering for factor input combinations using only the observed factor inputs and their prices. The production preference ordering might very well not correspond exactly to the production output ordering that results. The advantage of this alternative approach is to drop the assumptions that, given a particular set of factor inputs, a producer will necessarily push the production process to its limits in trying to maximize production. Producers or their agents may have other concerns beyond those that are immediately obvious and measurable. They may be concerned with product quality, for example, that may decline when production is pushed to its limits. Or, producers may be concerned with supplier reliability which may override their inclination to solely focus on cost minimization. By using revealed preference to track how producers actually behave instead of requiring that their behavior fit the profit maximization/cost minimization mold, we may be able to produce better econometric forecasts of producer behavior. Since the ultimate goal of this analysis is to do a better job of forecasting both consumer and producer behavior, the ultimate test should be how this method performs relative to other more traditional approaches in out-of-sample forecasts.

Initially we assume that consumers have constant underlying utility functions and producers have constant technologies and constant output preferences for a given set of factor input values. It should be emphasized that assuming a constant utility function does not imply constant tastes or taste consistency. We acknowledge the possibility of taste changes within the observation period. We will simply fit one utility function to all tastes within the period. Therefore, the utility function will be an average utility function for the given period of observation. The same may be said of our generated production function. The goal of producers may change during the period and we will fit a single average production function to both goal behaviors. These assumptions will be eliminated in later work so as to allow modeling of changes which occur in utility functions, production functions and producer factor-usage choice.

Section 2 of this paper will provide the mathematical notation and basic framework of the analysis. Section 3 will provide the results of the estimation of component parts such as the consumption behavior and production behavior functions. Section 4 will give the graphical and tabulated outcomes results from the simulated consumer/producer model. Finally, section 5 will report conclusions and plans for future research.

2. BASIC MATHEMATICAL FRAMEWORK

Consumption

There are two consumers, K (capital) and L (labor), consuming two commodities, Y1 (wheat) and Y2 (oil), where the earnings from the sale of Y1 and Y2 are used to pay the two factor inputs, K and L. The model simulates from period to period such that the incomes paid the factor inputs are then spent the next period for the consumption of the two goods, Y1 and Y2, thus generating more work and more income for the two factor inputs, and so on.

During the ith period there is a set of commodity prices, Py1(i) and Py2(i), which each consumer faces. The two consumers spend all of their incomes on the market baskets they select for that period. More expensive market baskets are thereby assumed to be out of range of that consumer's budget for that period. The two consumers' incomes may be expressed as:

$$I\_K(i) = Py1(i)\ Y1\_K(i) + Py2(i)\ Y2\_K(i)$$

and

$$I\_L(i) = Py1(i)\ Y1\_L(i) + Py2(i)\ Y2\_L(i)$$

where I_K(i) and I_L(i) being the income of K and L in the ith period. Similarly, Y1_K(i), Y2_K(i) and Y1_L(i), Y2_L(i) correspond to quantities of Y1 and Y2 purchased in the ith period by K and L respectively.

Having defined income in the ith period as the expenditure necessary to purchase a bundle of Y1 and Y2, we are now able to compare it to other bundles to see if one is preferred. Specifically, we need to hold prices constant and then examine the expenditure necessary to purchase two bundles. If one bundle is purchased and the expenditure necessary to purchase the other bundle (at constant prices) is less than that for the chosen bundle, then the chosen bundle is preferred. To introduce our notation, the ith period income is compared with the expenditure necessary to purchase the jth period bundle at the ith period prices. If the necessary expenditure for the jth bundle, under ith periods prices, is less than the ith periods income than the ith period bundle is preferred to the jth period bundle. Consequently the jth period bundle should be issued a demerit. In mathematical notation comparisons are made in the ith time period for consumer K such that

$$\text{If } I\_K(i) > [Py1(i)\, Y1\_K(j) + Py2(i)\, Y2\_K(j)] \quad \text{then} \quad DU\_K(i,j) = 1$$

$$\text{If } I\_K(i) \leq [PY1(i)\, Y1\_K(j) + Py2(i)\, Y2\_K(j)] \quad \text{then} \quad DU\_K(i,j) = 0$$

for the jth alternative market basket where DU_K(i,j) is a binary 0,1 variable representing one demerit when the current market basket (ith) is revealed to be preferred to the jth market basket at the ith set of prices.

In a similar manner for consumer L in the ith period we have

$$\text{If } I\_L(i) > [Py1(i)\, Y1\_L(j) + Py2(i)\, Y2\_L(j)] \quad \text{then} \quad DU\_L(i,j) = 1$$

$$\text{If } I\_L(i) \leq [Py1(i)\, Y1\_L(j) + Py2(i)\, Y2\_L(j)] \quad \text{then} \quad DU\_L(i,j) = 0$$

for the jth alternative market basket where DU_L(i,j) is a binary 0,1 variable representing one demerit when the current market basket (ith) is revealed to be preferred to the jth market basket at the ith set of prices.

The utility indices for the jth market basket for consumer K and consumer L, respectively, are found as:

$$U\_K(j) = n - \sum_{i=1}^{n} DU\_K(i,j) \quad \text{and} \quad U\_L(j) = n - \sum_{i=1}^{n} DU\_L(i,j)$$

where n is the number of periods, and therefore the number of market baskets being compared. This summation technique will yield a ranking in which higher utility states will be represented by higher numbers. In this case the most preferred state will be represented by a ranking of ten, and the least preferred state would be represented by a ranking of one.

Special note should be taken of the possibility for a seeming contradiction to arise from the above ranking procedure. It is possible for a particular basket 'x' to be preferred to a particular basket 'y' when directly compared (ie. basket 'y' would receive a demerit when compared to basket 'x' and yet the sum of demerits for basket 'y' could be less than the sum of demerits for basket 'x'). Thus the utility index ranking of basket 'y' would be higher than that of basket 'x', revealing basket 'y' as preferred to basket 'x'. The explanation for this seeming difficulty lies in the fact that transitivity is not imposed. While basket 'x' may have been preferred to basket 'y' when directly compared, it is possible that some other basket or baskets were preferred to basket 'x' and not to basket 'y' (accounting for the high number of demerits accrued to basket 'x'). As has been held throughout this paper, tastes are allowed to change. Thus, if such a seeming contradiction were to appear, the model is capable of explaining it through changed tastes. The task then would be to gather further data and decide what the new taste is and then incorporate the new taste in prediction.

The social welfare index, W, is constructed in a similar manner except that the "quantities" being considered are the values of the utility indices for K and L derived above. The corresponding "prices" are calculated by dividing the individual's income level by his/her utility index to get the average income

(i.e. "price" or social value) per observed occurrence of preference as follows:

$$P\_UK(i) = I\_K(i)/U\_K(i) \quad \text{and} \quad P\_UL(i) = I\_L(i)/U\_L(i)$$

where U_K(i) (utility for consumer K in the ith period) can be shown to possess an equal amount of utility as revealed by U_K(j)[2] And U_L(i) can be shown to possess an equal amount of utility as revealed by U_L(j).[3]

The P_UK(i) and P_UL(i) equations may not seem appropriate upon first glance. The reason for this is that apparently U_K(i) and U_L(i) are ordinal rankings and therefore cannot be devisers. However, the utility rankings have at least some aspects of cardinality. That is, the numbers represent the exact amount of times a particular basket was preferred. Suppose U_K(1) = 5. The number five represents five instances in which the bundle purchased in week one was preferred over other bundles. These specific instances give the utility index a quasi cardinality, even though they are based on ordinal preferences. Given there are at least some aspects of cardinality present in the utility indices, we are able to use the rankings as devisers to get a "price" for utility which is also at least a quasi cardinal number.

Thus, the social welfare index, W, reveals the world's true preferences (or, at least the ones that are currently in effect) for choosing among the utility combinations of consumer/earner, K, and consumer/worker, L. The total "value" of the world's utilities are expressed as:

$$T(i) = P\_UK(i)\, U\_K(i) + P\_UL(i)\, U\_L(i)$$

The total "value" of utilities is, of course directly analogous to income in traditional revealed preference analysis. It may be considered the total utility income of society in the ith period.

Comparisons are then made in the ith time period for the utility combinations such that

$$\text{If } T(i) > [P\_UK(i)\, U\_K(j) + P\_UL(i)\, U\_L(j)] \quad \text{then} \quad DW(i,j) = 1$$

$$\text{If } T(i) \leq [P\_UK(i)\, U\_K(j) + P\_UL(i)\, U\_L(j)] \quad \text{then} \quad DW(i,j) = 0$$

For the jth alternative combinations of utilities DW(i,j) is a binary 0,1 variable representing one demerit when the current combination of utilities (ith) is revealed to be preferred to the jth combination at the ith set of prices.

The social welfare index, W, for the combination of utilities generated in the ith period is given by:

$$W(j) = n - \sum_{i=1}^{n} DW(i,j)$$

where n is the number of periods, and therefore, the number of utility combinations being compared.

At this point, Arrow's impossibility theorem deserves comment. As previously mentioned we are not imposing any condition of rationality or transitivity in this analysis. Also, as is apparent, there may not be conventional optimizing behavior. Therefore, we believe we are within the bounds of relevant theory when we offer this social welfare index.

Production

So far the analysis has dealt with unobserved but implicit utility and welfare index values. That is, only prices and quantities were initially observed. On the production side, however, we observe input prices and factor input quantities as well as output quantities. The observed output quantities provide a "competitor" to the corresponding production preference index that will be generated. There is no reason that the production preference index would necessarily have the same rank ordering of input combinations as implied by the observed output levels. This is particularly true when input substitution rather than input scale changes are observed.

In this exposition model there exist two firms corresponding to the two goods being produced. Firm FY1 produces good Y1 and firm FY2 produces good Y2. During the ith period there is a set of factor input prices, P_K(i) and P_L(i), which each firm faces. The firms spend all of their revenues on the factor input combinations they select for that period. More expensive factor input combinations are thereby assumed to be out of range of that firm's budget for that period.

The total budgets of the firms are expressed as:

$$FY1(i) = P\_K(i)\ KY1(i) + P\_L(i)\ LY1(i)$$

and

$$FY2(i) = P\_K(i)\ KY2(i) + P\_L(i)\ LY2(i)$$

Comparisons are then made in the ith time period for firm FY1 such that

$$\text{If } FY1(i) > [P\_K(i)\ KY1(j) + P\_L(i)\ LY1(j)] \quad \text{then} \quad DV1(i,j) = 1$$

$$\text{If } FY1(i) \leq [P\_K(i)\ KY1(j) + P\_L(i)\ LY1(j)] \quad \text{then} \quad DV1(i,j) = 0$$

for the jth alternative market basket where DV(i,j) is a binary 0,1 variable representing one demerit when the current factor input combination (ith) is revealed to be preferred to the jth factor input combination at the ith set of prices.

In a similar manner for firm FY2 in the ith period we have

$$\text{If } FY2(i) > [P\_K(i)\ KY2(j) + P\_L(i)\ LY2(j)] \quad \text{then} \quad DV2(i,j) = 1$$

$$\text{If } FY2(i) \leq [P\_K(i)\ KY2(j) + P\_L(i)\ LY2(j)] \quad \text{then} \quad DV2(i,j) = 0$$

The production preference index, V, for the combination of factor inputs utilized in the ith period is given by:

$$V1(j) = n - \sum_{i=1}^{n} DV1(i,j) \quad \text{and} \quad V2(j) = n - \sum_{i=1}^{n} DV2(i,j)$$

where n is the number of periods, and, therefore, the number of utility combinations being compared. Thus, the production preference index, V, reveals the firm's true preferences (or, at least the ones that are currently in effect) for choosing among the factor input combinations of capital, K, and labor, L.

It is clear that revealed preference analysis can provide a framework for generating consumer utility rankings of specific commodity bundles. Also clear is that this framework is equally suitable for analyzing producer behavior. As will be made clear below however, the production preference index does not necessarily mimic preference rankings generated under the assumption that output is the sole determining characteristic.

3. ESTIMATION OF COMPONENT PARTS

Preference rankings from revealed preference analysis provide all information necessary to estimate utility and production functions. In this section the estimation technique utilized will consist primarily of ordinary least squares regression (OLS). The utility indices from above will allow two consumer utility functions, for individual K and individual L, and for an observed "social welfare function" to be estimated. The production preference index will allow an alternative production function to be generated and compared to a traditional output oriented production function. It is assumed herein, that the utility functions are of the Cobb-Douglas variety, however the procedure would be the same for any functional form.

Throughout this section, indices which have been created for the jth commodity bundle (U_K(j), U_L(j), W(j), V1(j), V2(j)) will be assumed equal to their counterparts for the ith time period (U_K(i), U_L(i), W(i), V1(i), V2(i)). This is a valid assumption, as footnotes 2 and 3 demonstrate.

Estimation of the utility function for each consumer is a relatively simple process. Given log linear form

$$\ln U\_K(i) = \beta 0 + \beta 1 \ln K\_Y1(i) + \beta 2 \ln K\_Y2(i)$$

and

$$\ln U\_L(i) = \beta 0 + \beta 1 \ln L\_Y1(i) + \beta 2 \ln L\_Y2(i)$$

with U_K(i) and U_L(i) corresponding to the utility indices generated above. K_Y1(i) and K_Y2(i) correspond to the amount Y1 and Y2 in the ith period purchased by individual K. Similarly, individual L purchased L_Y1(i) and L_Y2(i). OLS can be used to obtain values for all coefficients.

Utilizing the same principle, an observed "social welfare function" may be estimated from the social welfare index. The function is, again, assumed to be the log linear Cobb-Douglas form as follows:

$$\ln W(i) = \beta 0 + \beta 1 \ln U\_K(i) + \beta 2 \ln U\_L(i)$$

where W(i) is the social welfare index and U_K(i) and U_L(i) are the utility levels of individual K and individual L respectively. All coefficients are easily obtained using OLS.

Using the same procedure, it is possible to extract from the production preference index a preference-oriented production function which does not simply assume higher levels of output are preferred with given input factors. Here, we don't know why certain input factor combinations are preferred, we simply know they are preferred. If the factor input combinations that are revealed as preferred through the production preference index coincide with high output levels, then it may be inferred the producer is attempting to maximize output. However, if the preferred input factor combinations do not coincide with high levels of output it may be inferred that the producer is, at the very least, considering other aspects to production aside from output levels.

The preference-oriented production function is generated from capital and labor inputs of a given product when regressed onto the production preference index for that product. The two production functions for good Y1 and Y2 respectively are

$$\ln V1(i) = \beta 0 + \beta 1 \ln KY2(i) + \beta 2 \ln LY1(i)$$

and

$$\ln V2(i) = \beta 0 + \beta 1 \ln KY2(i) + \beta 2 \ln LY1(i)$$

This alone may be of interest but to truly appreciate its relevance it should be combined and compared to the traditional output oriented production function. The above preference-oriented production function reveals only the relative happiness of a firm (if such a phrase has any meaning), based on factor combinations. It does not reveal the level of output which would correspond to those preferred factor input combinations. In order to see output, two things must be allowed. First, the preference-oriented production function must be maximized. That is to say, whatever the producer's goal is, it is assumed the producer is trying to get as much of that as possible.

Secondly, an output-oriented production function must be generated. This may be generated from the observed data by regressing factor inputs on output levels rather than, as previously, on the production preference index. Then it is possible to see the output levels of the preferred production function. By imputing the values which maximize the preference oriented production function into the output oriented production function, the levels of output which correspond to goal seeking may be seen.

It is important to note that the goal being sought need not be known. Any goal oriented behavior will exhibit some producer preference index and hence a preference-oriented production function which can be combined with an output oriented production function to see the corresponding output levels.

As evidence that this preference-oriented production function is significantly different from the traditional output-oriented production function, we offer the rankings of good Y2 for our hypothetical

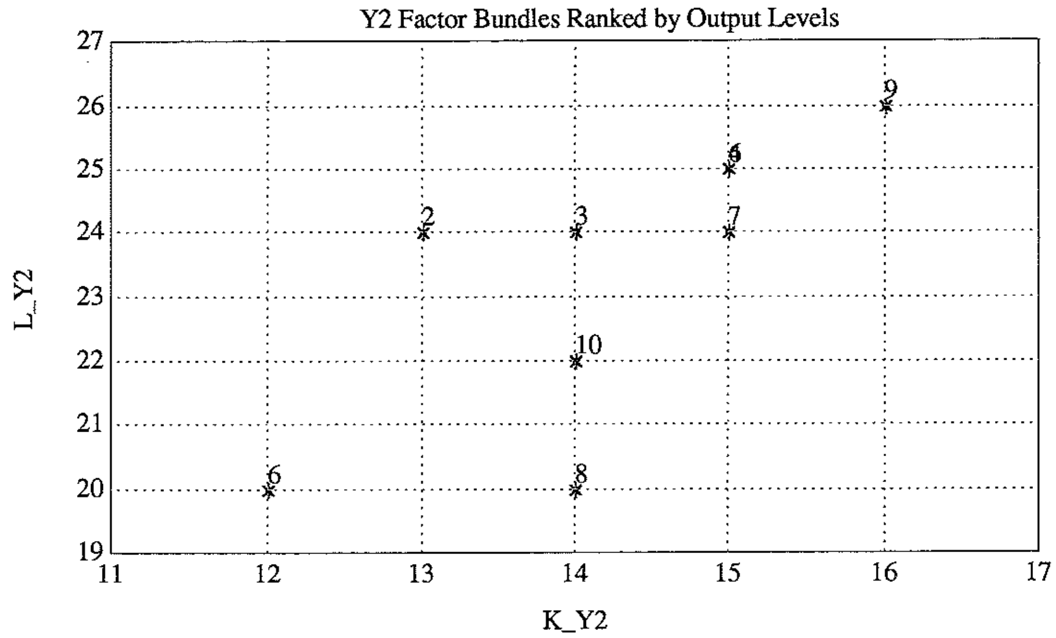

Figure 1. Y2 Factor Bundles Ranked by Output Levels

producers behavior. In Figure 1 the levels of factor inputs are ranked strictly in order of output.[4] That is the higher the output, the higher the ranking, with ten being best.

Physical production function engineering efficiency would require that more of both inputs would lead to more output. However, as seen in Figure 1, this need not always be the case. These data demonstrate a case where the combination ranked 10 represents a higher output level than that marked 9, although the latter observation used more of both inputs. We could, of course, impose the "larger than" principle that requires that more of one or both inputs must not lead to a reduction in output. This, in fact, is what standard microeconomic theory does when defining a production function to be an upper bound of efficient points.

Clearly the potential exists for rankings according to some other criteria (such as that given through revealed preference), to differ from those in Figure 1. That situation is present in Figure 2. For example, notice the rankings in accordance with output levels in Figure 1 have the most desirable input factor combination at a level of 22 labor units and 14 capital units employed. On the other hand, the rankings according to revealed preference analysis in Figure 2 show that the most desirable input factor combination is that of 26 labor units and 16 capital units employed. We do not attempt to explain why some input combinations are preferred over others. Many considerations such as adequate product quality, supplier reliability, and a longer term effort to retain skilled workers might all play a role in making a particular input combination desirable even when output levels fall.

The goal of this section has been to describe the process by which revealed preference indices may be used to generate functions of utility and preference oriented production functions. Additionally, differences between preference oriented production functions and output oriented production functions were discussed. Finally, it was shown that output levels corresponding to any goal oriented behavior may be found.

4. THE SIMULATED CONSUMER/PRODUCER MODELS

The output and input data used to estimate the empirical production and preference rankings for the first good, Y_1, and the second good, Y_2, along with the factor prices, are given in Table 1. Using the

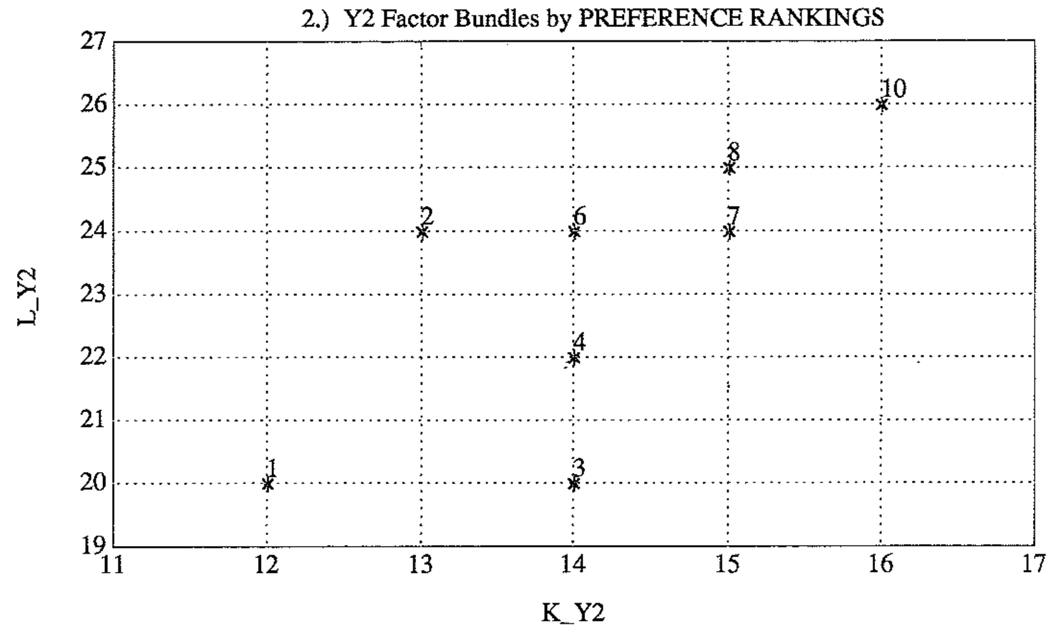

Figure 2. Y2 Factor Bundles by PREFERENCE RANKINGS

Cobb-Douglas production function with ordinary least squares applied to the log-linear form, yields the estimated output-oriented production functions:

$$Y_1 = 43.71319\, K_{Y1}^{.25815}\, L_{Y1}^{.04615}$$

and

$$Y_2 = 7.764639\, K_{Y2}^{.30996}\, L_{Y2}^{.40355}$$

while the corresponding preference-oriented production functions are:

$$V_1 = 4.476694037^{-52}\, K_{Y1}^{16.35161}\, L_{Y1}^{14.71554}$$

and

$$V_2 = 1.644589699^{-10}\, K_{Y2}^{7.33314}\, L_{Y2}^{1.45519}$$

TABLE 1

K and L Used in the Production of Y_1 and Y_2 and Prices of K and L

Y_1	K_{Y1}	L_{Y1}	Y_2	L_{Y2}	K_{Y2}	P_K	P_L
150	40	60	60	14	20	100	15
150	32	68	55	12	20	130	15
152	34	69	60	14	22	120	14
154	34	69	62	14	22	110	14
159	33	70	63	13	24	90	15
155	31	70	64	14	24	100	14
153	32	70	66	15	25	90	13
152	33	69	65	15	24	90	14
153	35	69	66	15	25	80	14
152	35	69	67	16	26	90	13

Both the preference and output oriented production functions reveal the standard positive coefficients. This result is consistent with the assertion that at least some of the producers general happiness is determined by the amount of output produced.

The two consumers in our model are called capital, K, and labor, L. Table 2 shows the amounts of goods Y_1 and Y_2 consumed by K and L as well as their prices. With this information it is possible to derive a preference ranking for capital's choices of bundles of Y_1 and Y_2, and similarly for labor's choices. We can thus obtain an ordinal preference ranking among commodity bundles for each of these two individuals. Using the Cobb-Douglas utility function with ordinary least squares applied to the log-linear form, yields the estimated functions:

$$U\_K = 0.00101235\ Y1\_K^{.35384}\ Y2\_K^{1.64921}$$

and

$$U\_L = 4.527284981^{-26}\ Y1\_L^{2.64088}\ Y2\_L^{13.00191}$$

Using these derived ordinal preference rankings for these two individuals and noting the amount of income allocated by society to each of these individuals in each time period, we can derive an overall, grand ordinal preference ranking or Social Welfare Index:

$$W = .991080021\ U\_K^{.88795}\ U\_L^{.20251}$$

4. CONCLUSIONS

The intended goal of this paper has been to discuss a method of using revealed preference to expand and enhance economic analysis in not only the area of consumption but in production as well. From a hypothetical raw data set, we have been able to generate indices which convey utility rankings for consumption and production activities. Given these indices, it was a simple matter to generate utility and preference-oriented production functions. Additionally, we were able to repeat the process of revealed preference and determine an observed "social welfare function."

Concerning the generation of a "social welfare function," we are aware this may engender some criticism. Arrow's impossibility theorem would seem to suggest our attempts have been futile. However, in his theorem, Arrow was discussing a social welfare function generated under the premise of maximization. We discuss a "social welfare function" in a slightly different climate. We consider society's actual valuation of personal utility (by normalizing it to income) and then assert this is how society *is* choosing. From the utility functions under which society is operating we are able to discuss the implications of the points on the grand utility possibility frontier society has revealed it prefers. Arrow was considering the question of how society *should* choose under optimizing conditions. The primary addressed issue was, given a grand utility possibility frontier, what utility functions could be considered consistent. We consider our social welfare function to be outside the realm of the impossibility thereom.

We cannot stress enough the fact that this model is designed with the sole purpose of predicting future behavior as implied by past observed behavior. Our primary objective has been to eliminate some assumptions which are commonly used but may, at times, be inappropriate. The regression equations we have generated are wholly unable to answer the question of why certain economic relations exist. All they are capable of is illuminating relations that seem to exist and utilizing those relations in prediction.

This paper has limited itself to surface analysis of the implications of nonmaximization, goal oriented behavior. In future we intend to consider the issue in much more theoretical and empirical depth. There are several areas of particular interest, among them: consideration of field transformation impacts on an observed "social welfare function" generated herein, application of the model to actual macro and possibly micro data, and further considerations on the impact of the preference-oriented production function on a dynamic version of this model. The most immediate refinement we are considering is one in which transitivity is maintained without requiring that tastes remain constant. We will be able to detect when tastes have changed when we observe a transitivity violation. At that point we will disregard all observations prior to the taste change and allow only data from after the taste change to enter into our forecasting. This will maintain transitivity and allow tastes to change.

TABLE 2
Amounts of Y_1 and Y_2 Consumed by K and L and Prices of Y_1 and Y_2

Period	Y_{1K}	Y_{1L}	Y_{2K}	Y_{2L}	P_{Y1}	P_{Y2}
1	17	9	110	67	30	13
2	9	7	96	70	37	15
3	12	4	42	67	36	17
4	46	5	137	65	21	15
5	21	7	65	67	19	14
6	4	6	72	68	21	13
7	33	7	96	69	21	13
8	41	9	104	67	17	15
9	26	10	83	65	25	17
10	13	9	120	63	29	15

NOTES

1. Arrow; pp. 8 and 13.
2. $U\_K(i) = \Sigma_{j=1}^{n} DU\_K(i,j) = n - \Sigma_{i=1}^{n} DU\_K(i,j) = U\_K(j)$. This is simply summing merits across a row, rather than summing demerits down a column and subtracting it from n.
3. $U\_L(i) = \Sigma_{j=1}^{n} DU\_L(i,j) = n - \Sigma_{i=1}^{n} DU\_L(i,j) = U\_L(j)$. This is simply summing merits across a row, rather than summing demerits down a column and subtracting it from n.
4. The graphs of figure 1 and figure 2 seem to have missing data and numbers which are "garbled." This is simply the result of multiple observations having the same factor inputs.

REFERENCE

Arrow, K.J., *Social Choice and Individual Values* New Haven, Conn.: Yale University Press, 1951.