Measuring Expected Inflation; Further Tests in the Frequency Domain of a Proposed New Measure

Houston H. Stokes

1. INTRODUCTION

In an ingenious article, Frankel (1982) proposed extracting a new measure of inflation from the interest rate term structure. His findings for the period 1959-8 to 1979-4 were that the market underestimated inflation in the 1970s and that his measure of expected inflation "does a slightly better job of predicting actual inflation, in terms of mean squared error, than do survey data." (Frankel (1982) page 140) This note provides a further test of the dynamic implications of the proposed new inflation measure by means of a test proposed by Geweke (1982a, 1982b, 1986). It is argued that the Geweke procedure provides additional information on the dynamics of what is being measured in the proposed inflation measure beyond that contained in the mean squared error procedure utilized by Frankel, which just related the new expected inflation measure and the actual price series at the same time period. An underlying assumption of the proposed testing procedure in this paper is that the appropriate way to measure the relationship between two series in the long run and the short run is in the frequency domain where short (long) run is high (low) frequency. The more usual way to relate series is in terms of lag length, with long (short) run being a long (short) lag length. The economic reason for using the frequency approach is that while the market place may be able to detect low frequency cycles, high frequency cycles, by their very nature, are harder to detect. Prior to a discussion of the Geweke procedure in detail and the statistical results of this paper in particular, it is important to relate the models proposed here to prior work on implicit expectations and rational expectations by Mills (1957) and Muth (1961), which have been summarized by Lovell (1986).

2. THE ECONOMETRIC CONSEQUENCES OF "IMPLICIT" AND "RATIONAL" EXPECTATIONS

If we assume that \( F_t \) is an expected (forecasted) inflation measure and \( F_t \) is the actual inflation series, Lovell (1986, equation 4) argues that the Mills (1975) implicit expectation model could be written in terms of an OLS model of the form

\[
F_t = \alpha_0 + \alpha_1 F_t + \epsilon_t
\]

\[(1)\]

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Computer time for this study was provided by the U of Illinois Computer Center. Professor Geweke provided the source code for his program which, although modified slightly by the author, was used for many of the calculations. A preliminary version of this paper was presented to the Business and Economics Statistics Section of the American Statistical Association Meetings in Chicago, 21 August, 1986. Diana A. Stokes provided editorial assistance. Any remaining errors are the sole responsibility of the author.

339
where it is assumed that \( u_t = 0, a_t = 1, E(u_t) = 0 \) and the predicted error \( \epsilon_t \) is uncorrelated with the actual realization \( P_t \). This paper postulates a strong form of the Mills implicit expectations hypothesis (weak implicit expectations), which cannot be tested in practice, but which would require the error process of equation (1) to be uncorrelated with the entire information set available to the researcher at time \( t \). A proposed weak form of the implicit expectations model (weak implicit expectations), which can be tested in practice, would require that the error process of equation (1) be uncorrelated with past forecasts, \( E(\epsilon_t | F_t) = 0 \) for \( j = 1, \ldots, q \), and past realizations \( E(\epsilon_t, F_t) = 0 \) for \( j = 1, q \). If one or more of these added assumptions were violated, it would imply that \( P_t \) cannot be used as a proxy for \( P_t \) in empirical work, as was suggested by Mills (1957), since not all information in the forecasted series is in the actual price series in period 1. To put this point another way, if given an expected price series \( P_t \), that contains more information that is contained in our proxy for the actual price series \( P_t \), it would not be permissible to substitute the actual price series for the expected price series as was suggested by Mills. If \( P_t \) were found to cause \( P_t \) at some frequency, this would cast doubts on the Mills substitution procedure for this problem. By looking at the frequency that violates the Mills procedure, we indirectly obtain information at the frequencies where the Mills substitution is appropriate and thus obtain information on the quality of the expected price series.

Lowell (1980) argues that the Math (1961) rational expectations model requires that the forecast error of equation (2) \( \epsilon_t \) be distributed independently of the anticipated value \( P_t \). This approach reverses the orthogonality condition of Mills. According to Lowell (1986, equation 5A), Math's model in OLS terms is

\[
P_t = \beta^* + \beta F_t + u_t,
\]

where \( \beta^* \), \( F_t \) = 1, \( E(u_t) = 0 \) and \( \epsilon_t \) is uncorrelated with the forecasted or anticipated value \( F_t \). The strong form of the rational expectations hypothesis (strong rational expectations) is difficult to test in practice, since it requires that the errors of the equation (2) \( \epsilon_t \) be uncorrelated with the entire set of information that is available to the forecaster at the time the prediction is made (Lowell 1986 page 113). The weak form of the rational expectations hypothesis (weak rational expectations), which can be tested in practice, requires that the error process \( \epsilon_t \) be uncorrelated with "historical information on prior realizations of the variable being forecasted," (Lowell 1986 page 113) \( E(\epsilon_t, F_t) = 0 \) for \( j = 1, q \) and historical values of the forecasted variable \( F_t \). If the expected price series were found to cause the actual price series, at some frequency, at this point, Math's rational expectations hypothesis which assumed that \( \beta \) was equal to one and the errors of equation (2) were uncorrelated with the entire information set available to the researcher.

Lowell makes the important point that under the Math hypothesis concerning expectations, the variance of \( P_t \) is > than the variance of \( P_t \) while under the Mills hypothesis, the reverse holds. Since weak rational expectations requires that the prediction error \( \epsilon_t \) be uncorrelated with historical information of prior realizations of the variable being forecast, if \( \beta \) in equation (3) is significant, the weak rational expectations hypothesis will be violated.

\[
P_t = \beta \epsilon_t + \beta F_t + \beta P_{t-1},
\]

It is important to note that equation (3) can only be rejected the weak rational expectations hypothesis, since just because \( \beta \) is not significant does not mean that it is not significant. Thus, the correct test reflects a weighted average of "instantaneously short-term interest rate" that is sensitive to the current tightness of monetary policy, and an infinitely long-term interest rate that reflects only the expected inflation rate. The trick is to obtain an estimate of these weights, which were shown by

\[
F_t = E^s(B)P_t + \gamma^s(B)P_t + \gamma \]

\[
P_t = \gamma^r(B)P_t + \gamma^r(B)P_t + \gamma,
\]

where \( E^s(B) \) and \( \gamma^s(B) \) are polynomials in the lag operator \( B \) for lags 1, 2, 3, \( q \) and \( E^s(B) \) and \( \gamma^s(B) \) are polynomials in the lag operator \( B \) for lags 4, 5, 6, \( q \).

Equations (4) and (5) are generalizations of equations (1) and (2). Significant terms in \( C(B) \) for lags 1, 2, 3, \( q \) reject the weak implicit expectations hypothesis and call into question the suggestion of Mills (1957) that \( P_t \) can be used as a proxy for \( P_t \) in empirical work. In equation (5), a finding of significant terms in \( B^r(B) \) would reject the weak rational expectations hypothesis, since it would imply that the error term of a simpler model such as (2) is not orthogonal to prior information (in this case prior expectations values). Later in the paper the above arguments are further refined to look at the relationship between \( P_t \) and \( P_t \) by frequency. The objective will be to test at what frequency the data are consistent with or reject the weak form of the implied and rational expectations models. Frankel (1982) asserted he was developing a long-run measure of inflation. By decomposing equation (4) and (5) into the frequency domain, we can test whether, if it exists, the dynamic relationship between the expected and actual price series is long-run (low-frequency) or short-run (high-frequency). The above section has outlined the theory why equations (4) and (5) provide a means by which to study Frankel's expected price series from the perspective of the implicit (Mills 1957) or rational (Math 1961) expectations hypothesis. In the next section the Geweke (1982a, 1982b, 1984) procedure is outlined and discussed.

The Geweke procedure involves first estimating a vector autoregressive model (VAR) for the series under study and then decomposing the VAR model into the frequency domain to determine the dynamic structure from the frequency perspective. There is much confusion between the concept of the speed of adjustment and the concepts of the long and short run. Although theory suggests that the Frankel expected price series \( F_t \) is an unbiased predictor (up to some constant) of the actual inflation rate \( E(F) - P_t \), there has been no systematic study of how the two series are dynamically related. This paper proposes a test of Frankel's expected price series \( F_t \) which uses a VAR model to determine at what frequency, if any, there is a causal relationship between \( F_t \) and \( P_t \). By decomposing the VAR model in to the frequency domain, we can test whether Frankel's expected price series \( F_t \) is really a long-run (low-frequency) measure of the actual price series \( P_t \).

It will be argued in this note that a time measure of the long-run (long-time delay) and short-run (short-time delay) is not appropriate for the problem at hand. Of more interest is the question "at what frequency (if any) are the series \( F_t \) and \( P_t \) related?". Since the market may be better at figuring out a long-run (low-frequency) component of a cycle than a short-run (high-frequency) component of a cycle, it is important to have a means by which to distinguish between the two components. The question to answer concerns what proportion of the variance of the series is captured by series \( F_t \) or \( P_t \)? In addition, if a relationship between \( F_t \) and \( P_t \) exists, can anything be said about the direction of causality? The information necessary to answer some of these questions is already contained in the VAR coefficients, although it is not apparent from direct inspection. The advantage of the Geweke procedure is that it allows recovery of the frequency information contained in the VAR model and, in addition, via a bootstrap procedure, estimates bounds on the estimated frequency measures.

After first outlining briefly how the Frankel expected price series \( F_t \) is calculated, the Geweke procedure is discussed. Next follows a short discussion of various alternatives for the \( F_t \) series. Finally, the empirical results are presented and discussed.

3. ECONOMETRIC METHODOLOGY

Frankel's expected price series \( F_t \) is developed by assuming that for a given term to maturity, the interest rate reflects a weighted average of "instantaneously short-term interest rate" that is sensitive to the current tightness of monetary policy, and an infinitely long-term interest rate that reflects only the expected inflation rate. The trick is to obtain an estimate of these weights, which were shown by
Franke (1982) to be a function of speed of the adjustment of a macroeconomic system in discrete and continuous time (in Franke's notation B and δ, respectively). As Franke notes, the problem is that the theory underlying the construction of F is not perfect and, given different points on the term structure, various possible combinations of weights are possible. Rather than attempting to develop new estimates of F, this note takes as given Franke's estimated expected price series and measures, using the Geweke procedure, the frequency information contained in this series in comparison with a representative actio pricing series. Before a discussion of the appropriate form of the actual price series to use, a few comments on the Geweke (1982, 1987, 1984) procedure are in order.

A linear time series process can be represented as a VARMA model

\[ (A(B)Z_t = D(B)e_t) \]

where \( Z \) is a column vector of \( k \) random variables and \( A(B) \) and \( D(B) \) are each a \( k \) by \( k \) matrices whose elements are finite polynomials in the lag operator \( B \). It is assumed that the determinantal polynomials \( \det(A(B)) \) and \( \det(D(B)) \) are outside the unit circle so that the VARMA model in equation (6) can be written as a VAR model

\[ \Phi(B)Z_t = \epsilon_t \]

where \( \Phi(B) = \det(D(B))^{-1}A(B) \). If \( k \) is assumed to equal 2, where the first element of \( Z \) is \( F \) (Frankel's expected price series) and the second element is \( P \) (a suitably chosen actual price series), the only assumption needed to estimate the VAR model is the maximum order (\( q \)) of the VAR polynomial \( \Phi(B) \).

In the Geweke procedure, it is not clear how best to determine the maximum order \( q \) of the polynomial in \( \Phi(B) \). In this paper a two-part procedure is employed. First, the SOCR system is used to estimate a VAR model of the form of equation (7), using OLS. Inspection of the autocorrelations and cross correlations of the residuals will indicate if \( q \) is sufficiently large to summarize the information contained in the \( Z \) vector. Once the appropriate \( q \) is found, the Geweke procedure can be applied safely, since all the systematic information in \( Z \) will be captured in \( \Phi(B) \).

Equation (7) can be written as

\[ Z_t = \Phi(B)Z_{t-1} + \epsilon_t, \quad \text{Var}(\epsilon_t) = \Sigma, \]

where \( \Phi(B) \) has no zero-order elements \( (\Phi(B) = \Phi(B) - I) \). Following Geweke, equation (8) can be broken up for the two series in \( Z_t \) as

\[ F_t = E^t(B)F + u_t, \quad \text{Var}(u_t) = \Sigma, \]

\[ P_t = G^t(B)P + v_t, \quad \text{Var}(v_t) = \Gamma, \]

or as transfer functions of the form

\[ \begin{align*}
F_t &= E^t(B)F + C^t(B)P^t + u_t, \quad \text{Var}(u_t) = \Sigma, \\
P_t &= G^t(B)P + H^t(B)F, \quad \text{Var}(v_t) = \Gamma
\end{align*} \]

where if \( i = 2 \), the transfer function models contain only first-order or greater lags, and if \( i = 3 \), the transfer models contain zero-order lage in \( C^t(B) \) and \( H^t(B) \). If \( i = 3 \), equations (11) and (12) repeat equations (4) and (5) above. Since \( \det(C^t) \geq \det(D^t) \geq \det(3) \) and \( \det(T^1) \geq \det(T^2) \geq \det(T^3) \), feedback from \( Y \) to \( X \) (\( F \times Y \rightarrow X \)) can be defined as \( \ln(\det(C^t)/\det(D^t)) \) and feedback from \( X \) to \( Y \) (\( F \rightarrow X \)) can be defined as \( \ln(\det(T^1)/\det(T^2)) \). Instantaneous feedback \( F_{XX} = F_{XX} = \ln(\det(D^t)2\times\det(T^3)) \), while linear independence \( F_{XX} \) becomes

\[ F_2(F \rightarrow X) + F_3(F \rightarrow X) + \ln(\det(C^t)/\det(D^t)) \]

Geweke decomposes the above measures into the frequency domain and (\( F_2(F \rightarrow X(\omega)) \)) measures the feedback from \( Y \) to \( X \) at frequency \( \omega \) and (\( F_2(F \rightarrow X(\omega)) \)) measures the feedback from \( X \) to \( Y \) at frequency \( \omega \).

Although there is one-to-one mapping from the VAR model estimated in equation (7) and the frequency feedback measures, it is important to access the "significance" of an estimated measure of feedback at frequency \( \omega \). Geweke's proposed solution is to perform a bootstrap procedure, where \( \epsilon \) sets of Monte Carlo data are generated with the asymptotic distribution of \( \Phi(B) \) being set equal to the actual estimated value from the data sample. If the frequency decomposition is performed \( v \) times, approximate confidence intervals can be calculated for the estimates, which can be adjusted for small sample bias. The end result is that from this procedure we can determine the frequency relationship between \( X \) and \( Y \) that is implicit in \( \Phi(B) \) in equations (11) and (12). For the purposes of this study, \( v = 250 \) was set at 100.

In the next section, we will discuss the alternatives for the price series \( P \) and proceed to present the findings.

4. FINDINGS

The Franke (1982) expected price series, \( F_t \), runs from 1959(3) to 1979(4) (237 observations). The selection of an actual price series to relate to this series is not unique. It was decided that a reasonable choice to use was the consumer price index-urban, which was obtained from the August 1982 version of the NBER/Chrysler data tape for the period 1954(7) to 1979(4). Three transformations of this series were used in the empirical work. If we assume that \( P \) is the row CPI-U series, then \( \det(D^t) \geq \det(T^3) = \det(T^3) \rightarrow \det(T^3) \). This transformation is comparable to a first difference of the Franke expected long-run inflation rate, \( \det(D^t) \rightarrow \det(T^3) \rightarrow \det(T^3) \). Both series were differenced once to achieve stationarity. An argument can be made that since the Frankel series is an expected long-run inflation rate, the appropriate actual series to use would involve calculation of the percent change over a longer period. To deal with this concern, two additional transformations of \( P \) were tried. An intermediate measure, \( \det(D^t) \rightarrow \det(T^3) \), was calculated on the basis of a year, while a more long-term measure, \( \det(D^t) \rightarrow \det(T^3) \), was selected from inspection of autocorrelations of equation (7) estimated by OLS.

The results from estimating these models are given in Tables 1-3 and will be discussed in turn. In Table 1, the results for the annualized percent change in the CPI-U (DIFPPZU) indicate strong (32.9%) high-frequency (period = 3) feedback from the price series to the expected price series. This indicates that at this frequency (high) the predicted error of equation (1) is not correlated with the entire set of information that was available and thus that at this frequency the weak form of the implicit expectations hypothesis does not hold. In terms of our prior notation, we have significant terms in \( \Phi^t \) in equation (4). The only other feedback found of any note is at somewhat lower frequencies; however, the data for shorter periods equal to 8, 7, and 6, the percent explained was 8.6%, 11.8%, and 13.0%, respectively, for the estimates corrected for small-sample bias.

There is less evidence of lags of the expected price series mapping to the actual price series; the only evidence of any relationship was found at lower frequencies. For example, at periods equal to 60, 40, 22, 16, and 12, the percentages explained are only 6.5%, 8.2%, 9.4%, 11.5%, and 10.0%, respectively. Overall, there was more of a relationship from \( P_B \) to \( F \) (\( F \rightarrow X(\omega) = 8.9\% \)) than from \( F \) to \( P \) (\( F \rightarrow X(\omega) = 3.1\%) \). A similar study to note that the feedback from the actual price series to the expected price series was at a high frequency and much stronger than the much lower frequency mapping from the expected price series to the actual price series. These results are more consistent with the weak form of the rational expectations model than the weak form of the implicit expectations model. The findings support Franke's contention that his expected price measure was a long-run (low-frequency) in our terms measure.
Table 1: Estimated Measures of Linear Feedback: One-Year Data

<table>
<thead>
<tr>
<th>Y vector DIFFPCPZ1</th>
<th>(\text{Estimate} )</th>
<th>(\text{Adjusted Estimate} )</th>
<th>(\text{25%} )</th>
<th>(\text{75%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(Y to X)</td>
<td>(-0.08 ) (4.2%)</td>
<td>(-0.04 ) (6.2%)</td>
<td>(-0.08 )</td>
<td>(-0.07 )</td>
</tr>
<tr>
<td>F(X to Y)</td>
<td>(-0.03 ) (4.3%)</td>
<td>(-0.04 ) (5.4%)</td>
<td>(-0.03 )</td>
<td>(-0.02 )</td>
</tr>
<tr>
<td>F(F to Y)</td>
<td>(0.00 ) (0.0%)</td>
<td>(0.00 ) (0.0%)</td>
<td>(0.00 )</td>
<td>(0.02 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\text{Y vector DIFFPCPZ1} )</th>
<th>(\text{Estimate} )</th>
<th>(\text{Adjusted Estimate} )</th>
<th>(\text{25%} )</th>
<th>(\text{75%} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(Y to X)</td>
<td>(-0.08 ) (4.2%)</td>
<td>(-0.04 ) (6.2%)</td>
<td>(-0.08 )</td>
<td>(-0.07 )</td>
</tr>
<tr>
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<td>(-0.03 ) (4.3%)</td>
<td>(-0.04 ) (5.4%)</td>
<td>(-0.03 )</td>
<td>(-0.02 )</td>
</tr>
<tr>
<td>F(F to Y)</td>
<td>(0.00 ) (0.0%)</td>
<td>(0.00 ) (0.0%)</td>
<td>(0.00 )</td>
<td>(0.02 )</td>
</tr>
</tbody>
</table>

Table 2, where the yearly difference of the CPU-U (DIFFPCPZU) is used, presents somewhat of a different frequency pattern, as might be expected, given the different actual price series used. The overall measures F(Y to X) and F(X to Y) are larger than in Table 1 (6.2\% vs. 5.0\% and 4.4\% vs. 3.1\%, respectively). In contrast to Table 1, where the average percentages for F(Y to X) and F(X to Y) were 5.2\% and 5.3\%, respectively (for adjusted estimates), in Table 2 the corresponding percentages are 13.2\% and 6.0\%, respectively. In comparison to Table 1, we see large low-frequency feedback, which would indicate rejection of the weak form of the implicit expectations hypothesis at low frequencies. For example, at periods equal to 40, 49, and 52, the percent explained \(28.1\%, 67.3\%, \) and 52.2\%, respectively. Giving the other way (F(X to Y)), there is little evidence of a relationship, except for relatively low values at periods equal to 16, 7, and 6, where the percentages are 12.3\%, 12.5\%, and 19.9\%, respectively. It is worth noting that the effect found at 16 for F(X to Y) is relatively similar across the two tables. It is apparent that the low-frequency feedback appear to be sensitive to the construction of the actual price series. The finding that the relationship from P, to F, dominates the relationship from F, to F, is similar to that in Table 1. Table 2, like Table 1, is consistent with the Muth weak rational expectations hypothesis.
TABLE 3
Estimated Measures of Linear Feedback Five-Year Data

<table>
<thead>
<tr>
<th>X vector</th>
<th>Y vector</th>
<th>DIFFPCZS</th>
<th>DIFFPCZS(1)</th>
<th>Estimate</th>
<th>Adjusted Estimate</th>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(Y to X)</td>
<td>.062 (.50%)</td>
<td>.055 (.48%)</td>
<td>.027</td>
<td>.049</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F(X to Y)</td>
<td>.034 (.33%)</td>
<td>.044 (.41%)</td>
<td>.010</td>
<td>.017</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F(X,Y)</td>
<td>.023 (.29%)</td>
<td>.031 (.19%)</td>
<td>.000</td>
<td>.027</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>Estimate</th>
<th>Adj. Est.</th>
<th>25%</th>
<th>75%</th>
<th>Estimate</th>
<th>Adj. Est.</th>
<th>25%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>.105 (12.7%)</td>
<td>.114 (10.7%)</td>
<td>.022</td>
<td>.180</td>
<td>.005 (0.9%)</td>
<td>.001 (0.1%)</td>
<td>.000</td>
<td>.001</td>
</tr>
<tr>
<td>60</td>
<td>.128 (12.0%)</td>
<td>.085 (6.2%)</td>
<td>.025</td>
<td>.125</td>
<td>.017 (1.7%)</td>
<td>.005 (0.5%)</td>
<td>.002</td>
<td>.006</td>
</tr>
<tr>
<td>10</td>
<td>.102 (9.8%)</td>
<td>.064 (4.8%)</td>
<td>.023</td>
<td>.100</td>
<td>.005 (0.5%)</td>
<td>.011 (1.1%)</td>
<td>.006</td>
<td>.019</td>
</tr>
<tr>
<td>32</td>
<td>.088 (6.4%)</td>
<td>.053 (3.4%)</td>
<td>.019</td>
<td>.075</td>
<td>.004 (0.4%)</td>
<td>.027 (2.7%)</td>
<td>.009</td>
<td>.044</td>
</tr>
<tr>
<td>16</td>
<td>.072 (6.8%)</td>
<td>.055 (5.0%)</td>
<td>.024</td>
<td>.107</td>
<td>.121 (11.7%)</td>
<td>.032 (8.0%)</td>
<td>.010</td>
<td>.017</td>
</tr>
<tr>
<td>12</td>
<td>.063 (6.8%)</td>
<td>.072 (7.9%)</td>
<td>.015</td>
<td>.101</td>
<td>.112 (11.8%)</td>
<td>.060 (4.8%)</td>
<td>.027</td>
<td>.123</td>
</tr>
<tr>
<td>10</td>
<td>.135 (15.7%)</td>
<td>.118 (11.1%)</td>
<td>.011</td>
<td>.155</td>
<td>.099 (9.5%)</td>
<td>.065 (6.5%)</td>
<td>.007</td>
<td>.008</td>
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<tr>
<td>8</td>
<td>.215 (19.7%)</td>
<td>.199 (17.9%)</td>
<td>.012</td>
<td>.216</td>
<td>.084 (8.4%)</td>
<td>.027 (2.7%)</td>
<td>.005</td>
<td>.007</td>
</tr>
<tr>
<td>7</td>
<td>.188 (17.1%)</td>
<td>.191 (14.0%)</td>
<td>.016</td>
<td>.222</td>
<td>.115 (12.3%)</td>
<td>.061 (5.1%)</td>
<td>.061</td>
<td>.003</td>
</tr>
<tr>
<td>6</td>
<td>.128 (12.0%)</td>
<td>.090 (6.6%)</td>
<td>.017</td>
<td>.137</td>
<td>.002 (0.2%)</td>
<td>.001 (0.0%)</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>5</td>
<td>.050 (5.1%)</td>
<td>.028 (2.8%)</td>
<td>.019</td>
<td>.050</td>
<td>.006 (0.6%)</td>
<td>.011 (1.1%)</td>
<td>.000</td>
<td>.001</td>
</tr>
<tr>
<td>4</td>
<td>.018 (1.5%)</td>
<td>.004 (0.4%)</td>
<td>.001</td>
<td>.016</td>
<td>.008 (0.8%)</td>
<td>.010 (1.0%)</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>3</td>
<td>.045 (4.4%)</td>
<td>.014 (1.4%)</td>
<td>.006</td>
<td>.027</td>
<td>.005 (0.5%)</td>
<td>.005 (0.5%)</td>
<td>.002</td>
<td>.004</td>
</tr>
<tr>
<td>2</td>
<td>.001 (0.1%)</td>
<td>.000 (0.0%)</td>
<td>.000</td>
<td>.000</td>
<td>.000 (0.0%)</td>
<td>.001 (0.0%)</td>
<td>.001</td>
<td>.005</td>
</tr>
<tr>
<td>Inf</td>
<td>.387 (17.0%)</td>
<td>.124 (11.7%)</td>
<td>.014</td>
<td>.211</td>
<td>.002 (0.2%)</td>
<td>.000 (0.0%)</td>
<td>.000</td>
<td>.000</td>
</tr>
</tbody>
</table>

Mean % 10.03% 7.33% 3.98% 2.52%
SD % 5.70% 2.52% 4.08% 2.09%

See text for further discussion of variable labels and method. B = lag operator, DIFFPCZS = (1 - B), where F = Frankel expected price series.

It must be stressed that the intention of the paper is to investigate the dynamic relationship between the proposed Frankel expected price series and three actual price series, not to test the Muth or Mills expectations hypothesis. To effectively perform the latter tests would require a different econometric setup and would require jointly estimating and testing the Frankel series with appropriate cross equations restrictions. The price model goal of this paper has been to take the published Frankel series as a given, relate this series to three proposed price series, and give some possible economic content to the frequency at which relationships are found.

5. CONCLUSIONS

In an innovative article, Frankel (1982), utilizing relationships implicit in the term structure of interest rates, developed a long-run measure of expected inflation. The Frankel expected price series is causally related, for frequency, to three of many possible transformations of the consumer price index (urban). In most cases it was found that, for the frequencies where there was a relationship, the actual price series caused the expected price series, which is not consistent with the assumptions of the Mills

Notes

1. This paper uses the Granger (1980) definition of causality which states that X causes Y if and only if for a given information set, which includes at least X, Y, and Y, can be predicted better by using past X, than by not using X.

2. Frankel (1980) footnote 16 mentioned that he had used his expected measure of inflation to test the hypothesis of rational expectations. His research, only briefly mentioned in his 1982 paper, reported the null hypotheses.

3. In a later section, there will be a discussion of the appropriate series to use for P, and P. In the empirical work, three alternative specifications are reported. The discussion of the relationship between implicit and rational expectations follows the important survey article by Donald (1980).

4. Lewis (1986) discussed the strong and weak form of the rational expectations hypothesis. This paper proposes the strong and weak form of the implicit expectations hypothesis.


6. In footnote 16, Frankel (1982) outlines a possible cross-section time series procedure to determine the weights and correct for possible liquidity effects, which are assumed away here.

7. Since no constraints are being placed on Q(B), DLS can be used. SCA, which was developed by Liu and Holsan (1980), was used for the VAR estimates in the identification stage.

8. Because of excessive computational cost during the bootstrap procedure, which will be discussed later, the Geweke procedure uses the White's (1983) approach to calculate the estimates of Q(B) rather than DLS. Stokes (1985) discusses some of the costs of this approach. Due to software limitations, in this paper the Geweke procedure has been followed. Calculations have been made using a modified version of Geweke's code. The author is grateful to Professor Geweke for making this code available. The author has made this code under SAS® software as part of the IDA® Data Analysis program.

9. For the purposes of the discussion, we define the X series as the first series in the Z, vector, F, in this case, and the Y series as the second series in Z, P. In this case. The reason for the switch in the notation is to emphasize the fact that, while in the present case there is only one X series and one Y series, in the general case there can be more than one X and Y series. If there are more than one X and Y series, equations (9-12) are VAR models. In this case where there is one X series and one Y series, equations (9-12) are both transfer functions and VAR models.

10. At Geweke has noted, the motivation for these definitions lies in the fact that the natural lag of 1 is zero and the fact that these measures can be interpreted in terms of the one-step-ahead population variances. For added detail see Stokes (1991) which documents the BILS Data Analysis Program used to make the calculations.

11. The degrees of the decomposition will be skipped in this brief note. For further details see Geweke (1980)). Although both the unadjusted and adjusted measures of feedback are reported, only the adjusted measures are discussed in this test because these measures are corrected for small sample bias.

12. A transformation involving a 6-month lag, or some multiple of a 6-month lag, was not attempted because of possible additional complications involving the seasonal in the price series.

13. As the estimates of Q(B) in equation (7), where Z = DIFPCZS and F = DIFPCZS(1), and assuming g = 12 with 24 legs of the residuals autocorrelated and cross correlated, indicated only marginally significant spikes out of (24 - 4) possible cross correlations and autocorrelations calculated. These spikes were found at lag 14 (p14 = 17), lag 17 (p17 = 16), lag 15 (p15 = 16), and lag 19 (p19 = 14). In all subsequent work involving transformations of Z, p was assumed equal to 12. This method of determining the value to assume for p is in keeping with current practice in the VAR time series literature. It is to be noted that the Olsen estimates of Q(B) will not agree with the White's (1983) procedure estimates of Q(B), although they are asymptotically equivalent. For further discussion of the differences between these alternative estimates, see Stokes (1985) footnote 16. In that footnote it was noted that there were differences in the White procedure obtained in the IBM version of Geweke and the program on the DEC-vax. Since Stokes (1983) Geweke, is a private letter, has informed me that there was an error in the DCS results. This leaves only the question of the magnitude of the difference between the White's (1983) estimates of Q(B) and the Olsen estimates. This is a topic for further research.

14. The average percentage for these periods is admittably only a summary measure. Its value is dependent on the periods used for the estimation.
REFERENCES


Eastern Economic Journal, Volume XVI, No. 4, October-December 1990

The Budget Deficit and the Trade Deficit: Insights Into This Relationship

Nazma Latif-Zaman and Maria N. DaCosta

I. INTRODUCTION

During the last decade, the "twins deficits" have been unusually high and have become a major concern for economists and policy makers alike. Since 1989, after a long period of surpluses, the U.S. has had a large trade deficit with every major trading partner. As for the budget deficit, it is true that since World War II, surpluses have been more the exception than the rule, but in the 1980's federal deficits were brought into the spotlight, because of their unusually large size. In the 80's alone, governmental borrowing to finance the deficit exceeded $1 trillion.

This paper addresses the issue of concomitant high U.S. budget and trade deficits. Is it just a coincidence or is there any systematic relationship? If there is a relationship, then what is the nature of such relationship? More specifically, is the relationship between these two variables uni-directional, bidirectional or are they independent?

II. LITERATURE OVERVIEW

Although there seems to be a wide spread political and popular perception that the twin deficits must somehow be interrelated, there is quite a great deal of controversy in the literature as to how and what extent they are related. A rudimentary national accounting identity can be used to relate the trade deficit, the government deficit, and investment and saving:

\[ S + T + M = I + G + X \]

or

\[ T - G = X - M + (I - S) \]

Where:

\[ S \] - Gross private saving
\[ T \] - Government revenues
\[ M \] - Imports
\[ I \] - Gross private domestic investment
\[ G \] - Government spending
\[ X \] - Exports

If savings is kept constant, an increase in the budget deficit will either reduce investment or increase trade deficit, or most likely a mix of both. This simple analysis suggests a positive relationship.

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