did the Verdoorn Law Hang on Japan

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INTRODUCTION

Many important economic relationships take the form of functions involving two variables. The empirical counterpart of these functions often is expressed conveniently by linear statistical equations. It is well known that the slope coefficient estimated by the least squares method will vary, depending on which of the two variables is chosen as the casually independent variable.

Take, for example, Okun’s Law, which relates the change in the unemployment rate to the percentage change in output. Typically, when output is the independent variable, Okun’s coefficient is significantly lower and the implied output gap is notably higher than when unemployment is the independent variable. Summers, Posner and Schwartz claimed that the “correct estimate” arises when unemployment is the independent variable, but Okun insisted that the correct specification makes output the independent variable. This argument remains unresolved.

A similar argument arose in the case of the Verdoorn law, which relates manufacturing productivity growth to manufacturing output growth. Typically, when output is the independent variable the Verdoorn coefficient is significantly higher than when employment growth (identical to output growth minus productivity growth) is the independent variable. The differences in the values of the estimates typically are large enough to have economic significance, with the former specification implying increasing returns to scale and the latter constant returns to scale.

This paper explores the sort of statistical issues involved in these arguments over specification, using the Verdoorn law as an example.

THE GENERAL STATISTICAL ISSUE

Let us generalize the Verdoorn law problem to the case of any observational identity, where

\[ Y = a + bX + u, \]

Then, the following two equations are logically equivalent:

(i) \[ X = c_i + b_iX + u_i, \]

where \( u_i \) are independently and identically normally distributed with \( \mu = 0 \) and variance \( \sigma_i^2 \), and

(ii) \[ X = c_i + b_iX + u_i, \]

where \( u_i \) are independently and identically normally distributed with \( \mu = 0 \) and variance \( \sigma_i^2 \). It has been pointed out that

(a) \( b_i = b_i(1 - b_i) \),

(b) \( b_i = b_i(1 - b_i) \).

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The sample consists of the exponential growth rates of manufacturing output, productivity and employment in twelve OECD countries for the period 1953/4-1963/4. Kaldor specified the Verdoorn law as an empirical relation between manufacturing productivity growth, X, and manufacturing output growth, Y. The growth of output appeared as the causal, independent variable on the grounds that the supply of output is perfectly elastic in response to effective demand at the going price, which depends on costs alone. The source of manufacturing demand was exogenous to the manufacturing sector, coming as it did from the foreign and the agricultural sectors. Given this economic argument, Kaldor considered that productivity growth was the causally independent variable, which was uncorrelated with the random error.²

In order to state the relevant statistical tests, the authors considered Kaldor's specification of the Verdoorn law by least squares. The estimates, which came very close to Kaldor's, are:

\[ \hat{Y} = 1.048 + 0.480 X, \quad R^2 = 0.823, \]

standard error: (0.462) (0.070), F-stat: 46.53, significance level: (0.047), \( \hat{\phi} = 0.453. \]

The residuals are distributed normally, which validates the use of the statistical test of significance.²

R. E. Rowthorn (1979), using Kaldor's data, estimated manufacturing productivity growth on manufacturing employment growth. Making employment growth the independent variable meant that manufacturing output growth is constrained by the growth of factor supplies to the manufacturing sector. Rowthorn stated that Kaldor followed an inappropriate statistical procedure (3) adopting a two-stage procedure instead of the conventional method of estimating productivity growth \( Y \) directly to employment growth \( X \). The advantages of estimating \( Y \) by OLS the regression of \( Y \) on \( X \) are well known. In particular, if the errors are independent of \( X \), the OLS estimator will be unbiased. Even if the errors are not independent of \( X \), the OLS estimator may provide a sound basis for predicting the effect of a change in \( X \) on \( Y \).

The last sentence of this statement is clearly wrong. The least squares estimates of Kaldor's complete sample using the Rowthorn equation are:

\[ \hat{X} = 2.631 + 0.626 \hat{X}, \quad R^2 = 0.45, \]

standard error: (0.556) (0.220), F-stat = 5.9, significance level: (0.001) (0.018), \( \hat{\phi} = 1.72. \]

Yet, without the Japanese data in the sample, Rowthorn's results by least squares are

\[ \hat{X} = 3.327 + 0.183 \hat{X}, \quad R^2 = 0.05, \]

standard error: (0.541) (0.267), F-stat = 0.5, significance level: (0.000) (0.595), \( \hat{\phi} = 0.014. \]

The estimate of the slope coefficient, \( \hat{\phi} \), is not significantly different from zero, indicating that the appearance of the Verdoorn regularity solely depends on an "outlier."² On this basis Rowthorn stated that

"We may conclude ... that there is no empirical evidence that Kaldor's law has operated during the post-war period in manufacturing. The negative results of Graps and Testing for 1953 to 1963 and Kaldor for 1953-54 to 1963-64 simply cannot be accepted. They are based upon a small sample of countries chosen in such a way that the extreme observations of one special case — Japan — account for the bulk of the observed correlation between productivity growth and employment growth. Moreover, Kaldor used an unconventional and seriously misleading method of estimation which gave results very different from those obtained by the conventional least squares regression of \( X \) on \( Y \)."²

This conclusion, which follows from a preference for one specification over the alternative, entirely lacks a statistical basis.

THE ALTERNATIVE SPECIFICATIONS OF THE VERDOORN LAW

Let us apply this general discussion to the debate over the specification of the Verdoorn law. Equation (5) refers to the accounting identity that manufacturing output growth, \( \hat{X} \), equals the sum of the growth of manufacturing labor productivity, \( \hat{X}_L \), and the growth of manufacturing employment, \( \hat{X}_E \). We shall use the sample of data in Kaldor's 1966 paper, where he debunked the term the Verdoorn law.¹
Using Kaldor's equation to estimate the Verdoorn law by least squares without the Japanese data yields,

\[
\hat{X}_a = 1.359 + 0.416 X_p \quad R^2 = 0.54.
\]

(4)

standard error: 0.711  (0.129)  F-stat = 10.4

significance level: 0.0088  (0.010)  \( \sigma^2 = 0.4946 \)

which confirms the Verdoorn hypothesis. Moreover, assuming that equation (4) gives the maximum likelihood estimates of \( b_1, c, \) and \( \sigma^2 \), then the maximum likelihood estimates of Rowthorn's specification without the Japanese data, according to the definitions (b, c, e, g) are

\[
\hat{X}_a = 2.330 + 0.715 X_p \quad \hat{\sigma}^2 = 1.226.
\]

Because \( \hat{c}_1, \hat{c}_2 \), and \( \hat{c}_3 \) of equation (4) constitute a sufficient statistic for equation (3) and equation (4) is statistically significant, we know that \( b_1 = 0.715 \) is significantly different from zero, which confirms the existence of the Verdoorn regularity.

Taking an alternative approach, let us estimate the relation between \( X_c \) and \( X_p \) by using the two-stage least squares algorithm with \( X_4 \) as the instrumental variable. This method treats output growth, productivity growth and employment growth as occurring simultaneously. The first stage gives the least squares estimates\(^{11}\)

\[
\hat{X}_a = -c + (1 - b_1) X_p + u_a
\]

The second stage gives

\[
\hat{X}_c = \hat{c}_1 + \hat{b}_1 X_p + u_c.
\]

The error terms \( u_a, u_c \) are assumed to be independent of \( X_p \) and \( X_a \), respectively. Using this procedure, the estimates for the complete sample are

\[
\hat{X}_a = 2.07 + 0.925 X_p \quad \hat{\sigma}_2 = 1.72.
\]

The estimates of the constant and the Verdoorn coefficient are consistent mathematically with the estimates of Kaldor's equation (4) according to the definitions in (b, c, e, g). No test statistics are reported because \( \hat{c}_1 \) and \( \hat{b}_1 \) follow an asymmetric Cauchy distribution when \( X_p \) and the random disturbance are statistically independent.\(^{12}\) Without Japan in the sample, the estimates are

\[
\hat{X}_a = 2.33 + 0.725 X_p \quad \hat{\sigma}_2 = 1.46.
\]

which are insignificantly different from those in equation (3). Thus, assuming that output is demanded, it transforms that the Verdoorn law expressed by Rowthorn's equation does not depend on the presence of the Japanese data in Kaldor's sample.

CONCLUSION

Kaldor (1966) presented a statistical study in which he confirmed the existence of a regularity between productivity growth and output growth, which he dubbed "the Verdoorn law." Rowthorn (1972) showed that Kaldor's least squares estimates of the Verdoorn law depended solely on the inclusion of the Japanese data in the sample. Rowthorn's case depended solely on the specification of the Verdoorn law when estimated by the least squares method as a relation between productivity growth and employment growth. This relation can be derived mathematically from the original Verdoorn law given the accounting identity that output growth equals productivity growth plus employment growth. Rowthorn, Kaldor and their supporters acknowledged that the correct specification of the Verdoorn law [when estimated by least squares] depends on whether it is output growth or employment growth which is statistically independent of the random error term.

This paper explained that (A) if employment growth is uncorrelated with the random error term, then the maximum likelihood estimates of Rowthorn's equation arrives at by least squares are sufficient to determine the maximum likelihood estimates and the statistical significance of Kaldor's equation. Conversely, (B) if output growth is uncorrelated with the random error term, then the maximum likelihood estimates of Kaldor's equation arrived at by least squares are sufficient to determine the maximum likelihood estimates and the statistical significance of Rowthorn's equation. Moreover, those estimates of Rowthorn's equation are the same as those arrived at by treating output growth, productivity growth and employment growth as simultaneously determined. We presented the results of the Verdoorn law without the Japanese data in the sample. Nonetheless, the data are powerless to decide which of the competing specifications, (A) or (B) deliver maximum likelihood estimates by the least squares method.

The line of argument stated in this paper applies to other empirical regularities expressed in linear statistical relations between two variables to be estimated by least squares when there is uncertainty as to which variable is causally independent.

NOTES

3. Theil, 1958, discuss the development of the Verdoorn law.
6. Here is a sample estimate of maximum likelihood estimation. The maximum likelihood estimator of the population mean of normal distribution is the sample mean. In general, maximum likelihood estimation occurs when we look at the values of a random sample and choose as our estimate of the unknown population parameter the value for which the probability of obtaining the observed data is a maximum (Freed and Walpole, 1967, pp. 336-338).
7. R.L. Rummage exploited this point to the authors (as in Baumol, 1954, p. 217).
8. The sufficient statistic for one parameter population is discussed in Freed and Walpole, 1967, pp. 349-354. The sufficient statistic for several parameter populations is discussed in Kendall and Stuart, 1961, pp. 27-28. A simple example is as follows: Given a normal distribution with mean \( m \) and variance \( \sigma \), the sample mean and variance constitute the sufficient statistic to describe the population.

<table>
<thead>
<tr>
<th>Country (X)</th>
<th>Output (( X_a ))</th>
<th>Labor Productivity (( X_p ))</th>
<th>Employment (( U_e ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>13.6</td>
<td>7.8</td>
<td>3.8</td>
</tr>
<tr>
<td>Italy</td>
<td>8.3</td>
<td>4.2</td>
<td>3.9</td>
</tr>
<tr>
<td>West Germany</td>
<td>7.4</td>
<td>4.9</td>
<td>2.8</td>
</tr>
<tr>
<td>Austria</td>
<td>6.4</td>
<td>4.2</td>
<td>2.2</td>
</tr>
<tr>
<td>France</td>
<td>5.1</td>
<td>3.8</td>
<td>1.8</td>
</tr>
<tr>
<td>Denmark</td>
<td>5.7</td>
<td>3.2</td>
<td>2.5</td>
</tr>
<tr>
<td>Netherlands</td>
<td>5.5</td>
<td>4.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Belgium</td>
<td>5.1</td>
<td>3.9</td>
<td>1.2</td>
</tr>
<tr>
<td>Norway</td>
<td>4.6</td>
<td>4.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Canada</td>
<td>3.6</td>
<td>4.1</td>
<td>1.3</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>3.2</td>
<td>2.8</td>
<td>0.4</td>
</tr>
<tr>
<td>United States</td>
<td>2.6</td>
<td>2.6</td>
<td>0.0</td>
</tr>
</tbody>
</table>

11. There is strong evidence that the residuals are normal by the Lilliefors test, given 10 degrees of freedom and a test statistic of 0.076 (Plackett and Petterson, p. 1034).
REFERENCES


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DISCRIMINATION IN CONSUMER CREDIT MARKETS

Clifford B. Hawley and Edwin T. Fujii

INTRODUCTION

This paper analyzes consumer credit-seeking experiences with lending institutions. We examine the determinants of (1) the probability that a consumer is rejected for the credit he requests, (2) the probability that the consumer will then continue his search after an initial denial, and (3) the probability that this extended search is unsuccessful. In addition, the data allow us to identify those who are discouraged from seeking credit and we explore the determinants of that discouragement as well.

Our principal interest in this empirical work is with whether the credit allocation process is neutral with respect to the race, sex, and marital status of those who seek to borrow in financial markets. The Equal Credit Opportunity Act (1974) and amended (1976) explicitly prohibits the use of race, sex, and marital status as criteria on which to base lending decisions. We use a national probability sample of households surveyed in 1985 and then the results here apply to an environment in which this federal legislation is in force. Despite this legislation, we find evidence of racial differences in credit experiences that are adverse to nonwhites and that cannot be explained by differences in the distribution of credit-worthiness by race. These results we interpret as evidence of racial discrimination in consumer credit markets.

Most previous empirical studies of discrimination in lending, e.g. Black and Schweitzer (1980), Black, Schweitzer, and Mandell (1978), Warner (1982), and Winton (1980), have generally utilized data on credit applications from banks and other lending institutions to test whether, after controlling statistically for other factors such as income, employment record, and credit history, minorities and women have lower probabilities of success in obtaining credit. The results have been mixed. Black, Schweitzer, and Mandell (1978), for example, provide empirical evidence that "blacks are less likely to be granted loans than nonblacks, ceteris paribus." Lindley, Selby, and Jackson (1984), using survey data, conclude that "there is no significant support for the hypothesis that [racial] discrimination exists in the extension of credit." Similarly, Peterson (1981), finds "no systematic pattern of prejudicial sex discrimination," after finding no significant difference in credit default rates by sex at banks.

With the exception of the contributions by Brandt and Shay (1979) and Lindley et al. (1984), however, the above studies suffer from a potential sample selectivity problem since they ignore the process by which the particular data set is generated. As Bloom et al. (1983) point out, the problem arises because bank data draw their observations from the population of credit applicants and not the entire population of potential applicants. Potential applicants who may be discouraged from applying never appear as actual credit applicants. For example, a lending institution can discourage a potential applicant from applying by a history of discriminatory credit decisions, by suggesting to a potential applicant that the probability of credit denial is high, or by simply being less than attractive to his credit needs and problems. A sample selectivity problem is introduced into the estimation process if such discouragement is more frequent among one sex or one race, other things being equal. The result is that

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21