Joint Natural Resources and Government Policy: Helium and Natural Gas

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INTRODUCTION

Most exhaustible resource theory is developed for single resources. However, there exist resources which are found and mined together, creating interdependencies which have been ignored. Examples of jointly mined resources include natural gas and helium, natural gas and oil, air extraction of oxygen and noble gases (including helium), as well as ores such as gold and uranium. To determine the effects of government policies on such resource markets, it is important to account explicitly for that interdependence. This paper uses the case of helium and natural gas, which raises interesting policy questions, as an illustration.

Government concern about natural resource extraction is based on the assumption that private markets are not allocating resources in a socially optimal manner, most likely because of differences between private and social discount rates. If resource owners do indeed use the same discount rate as society, given the same information, the optimum for private (perfectly competitive) producers will also be the social welfare maximizing solution. On the other hand, if private owners discount the future more heavily than society, profit-maximization will lead to too rapid depletion of the resource from society’s point of view. Under these circumstances, conservation of the resource by government may be justified.

Helium has several properties which reduce its potential for substitution by other substances, among them its use for cooling materials until they become superconducting. The latter characteristic figures prominently in future technologies such as fusion energy generation, superconducting underground power lines, and fast transport based on superconducting magnets as well as the proposed superconducting super collider to be built in Texas.

When natural gas containing helium is extracted from the ground, the helium can be removed before the natural gas is burned. If non-purified natural gas is burned, the co-extracted helium will be vented into the atmosphere, where its concentration is approximately 0.001%. The costs of removing this helium from the atmosphere are estimated to be more than 160 times greater than separation costs from natural gas [2].

The demand for helium is at present relatively small, but is expected to increase drastically as new technologies, especially fusion energy generation, become increasingly viable; separated helium can be stored until that time. On the other hand, most natural gas storage serves to smooth out seasonal fluctuations and the storage issue is of less concern for this reason. Therefore, the model developed in the following sections ignores natural gas storage.

Because helium demand is expected to increase dramatically in the future, and because of the high cost of the secondary source, there is government interest in helium separation and sale. This paper analyzes the effects of some government policies in the context of a simple model of joint resource extraction. The model will take into account the residual nature of one of the resources, as well as its demand structure, which is accommodated through the postulation of the appearance of a (positive) helium demand curve at an exogenously given time in the future. Although the discussion is couched in terms of helium and natural gas, this analysis provides, more generally, an extension of resource taxation to jointly extracted resources.

In the following section, the joint resource model is introduced and equilibrium paths are derived. The third section analyzes the effects of these government policies. This is followed by some concluding remarks.

THE MODEL

The joint resource owner’s problem is to maximize the present discounted value of the total profits from the sale of the two resources, given a known resource stock. The resource industry is assumed to face negatively sloped demand functions for each of the two resources and to be perfectly competitive in both markets. Therefore, individual extractors are price-takers.

Each unit of pure natural gas is extracted with a unit of pure helium, where \( x = 0, 1 \) is the upper limit to helium separation. Any helium which is not separated is vented into the atmosphere when the natural gas containing it is burned.

Helium demand is assumed to be zero at every price level until an exogenous time \( T_1 \) when it becomes positive. It is also assumed that both the natural gas and helium demand have choke-off prices; prices above which there is no demand for the product (both can be interpreted as prices of backup technologies in the case of helium the backup is atmospheric extraction).

The problem can be conveniently split into three time periods: during period \( [0, T_1] \) the joint resource is mined, the natural gas is sold, and the helium can be separated and stored or left unseparated to be vented into the atmosphere when the natural gas is burned. This interval will be separated into two sub-intervals through solution of the maximization problem; we will find that in the initial period, \( [0, T_1] \) no helium is separated from the natural gas. In the second sub-interval, \( (T_1, T_2] \), helium separation takes place. During \( [T_1, T_2] \) the joint resource is still mined and both resources are sold; the separated helium is either sold or stored (for later sale). At time \( T_2 \), the joint resource in the ground has been exhausted, so that in the interval \( [T_2, T] \) only the remaining helium in storage is sold. \( T_1 \) and \( T \) are the solutions to \( q(T_1) = q_{min} \) and \( q(T) = q_{min} \) respectively, where \( q(T_1) \) and \( q(T) \) are the market prices of natural gas and helium respectively, and \( q_{min} \) and \( q_{max} \) are the respective choke-off prices. Since the resource owners are maximizing profits, they will exhaust each resource just as the backup technologies become viable.

The profit-maximizing producer’s problem is:

\[
\max \int_0^{T_1} \left[ q_1(n(t)) - E(n) - PH(t) - b(t)w(t)^{\alpha} \right] dt + \int_{T_1}^T \left[ q_2(n(t)) - PH(t) - b(t)w(t)^{\alpha} \right] dt
\]

subject to:

\[
\begin{align*}
\dot{S}_1 &= -N(t) \\
\dot{S}_2 &= \alpha - E(t) + n(t) \\
\dot{H} &= \beta - n(t) \\
\dot{S}_1(t) &= S_{min} \text{ for all } t \in [0, T_2] \\
S_2(t) &= S_{max} \text{ for all } t \geq T_2,
\end{align*}
\]
The solution of (II.1)–(II.7), using the methods of optimal control theory, yields equilibrium price paths for the two resources. For natural gas:

\[ q(t) = \begin{cases} \lambda(t) + \delta(t) \gamma \sigma(t) + E & \text{if } t < T^* \\ \lambda(T_1) + \delta(T_1) \gamma \sigma(T_1) + E & \text{if } t > T^* \end{cases} \]

The price of helium, during the time it is sold, follows:

\[ q(t) = \frac{\gamma(t) - \lambda(t)}{\sigma(t)} \]

where \( \lambda(t) = \lambda(T_1) \) for all \( t \) and \( \gamma(t) = \gamma(T_1) \), the time at which helium separation begins, is when the discounted benefits of helium separation are just equal to the discounted costs. The relationship between the separation costs and the choke-off price of helium will determine whether or not any helium will be separated. If \( P > \lambda(T_1) \), there will be no helium separation because costs cannot be covered at any time. If \( P < \lambda(T_1) \), there will be helium separated since the costs can be covered at prices less than the choke-off price.

The equilibrium price paths, (II.8) and (II.9) are the only paths along which resource owners are indifferent between selling in the present and the future; therefore, the only price paths along which sales will take place at all times in the appropriate intervals. The consumption and extraction paths for the two resources are demand determined, so that the markets are in equilibrium at all points in time. Helium separation is directly proportional to natural gas extraction.

**THREE SUBSIDIES**

Public policy provides three residual resource subsidies: a subsidy to storage costs, and a subsidy on separation costs. With respect to helium the policy goal is to increase its future availability from natural gas.\(^4\) The equilibrium paths in the presence of each of the subsidies will be analyzed, in order to determine whether the goal is fulfilled. In each case, the post-subsidy price paths will be compared to the paths identified by means of (II.8) and (II.9).

**HELIUM SALES SUBSIDY**

Consider a subsidy on the sales price of helium at time \( T_1 \). This subsidy is a proportion \( s \) of (II.1) of the consumer price, so that the price received by the producer for each unit of helium at time \( t \) becomes \( (1 + s)q(t) \). In the single resource case this policy would serve initially to decrease the market price and increase consumption rates, resulting in a shorter consumption horizon for the resource.

The maximization problem after the imposition of the subsidy is found by replacing \( q(t) \) with \( (1 + s)q(t) \) in equation (II.1). Solving, using the same procedure as in Section II, yields equations for the price paths for \( T_1 \geq T_0 \), analogous to (II.8), and (II.9).

\[ q(t) = \left( \lambda(t) - \delta(t) \gamma(t) + \theta(t) \right) \sigma(t) + E + \omega(P + \theta(t)) t \leq T_1, T_2 \]

The natural gas price path after the imposition of the subsidy, where \( \lambda(t) \) and \( \gamma(t) \) are the new values of \( \lambda_1 \) and \( \gamma_1 \).

The post-subsidy helium price is:

\[ q(t) = \left( \lambda(T_1) + \delta(T_1) \gamma(T_1) + (1 + s) \theta(T_1) \right) \sigma(T_1) + E + \omega(P + \theta(T_1)) t \leq T_1, T_2 \]

Under the assumption of perfect foresight, the equilibrium paths for both resources will be such that each resource is exhausted just as its choke-off price is reached. In particular, this means that if the initial stock of a resource is unchanged, its post-policy price path, and its price path in the absence of the policy, must either cross or coincide. Otherwise, the resource will not be exhausted optimally. Only if more of the resource is available to be sold can the post-policy price path be everywhere below the original price path. This is because a lower market price corresponds to a higher quantity demanded. Similarly, only if less of the resource is available can the post-policy price path be everywhere above the original path, since a higher price corresponds to a lower quantity demanded.\(^1\) Hence, the following proposition:

**Proposition:** In the presence of a helium sales subsidy, the helium price will be higher initially \( q(t) < q(T_1) \) with a lower slope \( q(t) < q(T_1) \) and the natural gas price path will be identical to its no-tax levels \( q(t) = q(T_1) \) for all \( t \geq T_2 \) (where \( q(T_1) \) is the post-subsidy value of \( q(T_1) \)).

If imposed after helium separation has begun, the sales subsidy results in a later exhaustion of the helium from storage, an increase in its value \( q(T_1) \), and an increase in the value of a unit of resource in the ground \( \lambda(T_1) \). The natural gas price path and consumption rates, though are unaffected. The helium results are the same whether the subsidy is imposed before or after time \( T_2 \).

If the sales subsidy is announced before the onset of helium separation \( T_2 \), helium separation will begin at an earlier date because of the higher benefits to separation and sales, and a greater amount of helium will be separated from the natural gas (with a longer time horizon for helium consumption). The natural gas will also be exhausted at a later date as a result of this policy.

This effect is opposite to the single non-storable resource case; in this model a helium sales subsidy encourages a shift from current to future helium consumption, i.e., it is conservative. This redistribution occurs because the helium is storable and, therefore, its price, \( q(T_1) \), grows at a percentage rate greater than the discount rate, \( r \). In present value terms, the sales subsidy is worth more in the future than it is now. Hence, the jointness of the resources is crucial to the analysis of the policy.

**HELIUM STORAGE SUBSIDY**

Consider a storage subsidy imposed at time \( T_1 \), equal to a proportion \( s \) (0, 1) of the storage cost per unit b, on helium. One would expect this type of policy to be used if it is desired to encourage increased storage of helium directly.

The firm's maximization problem after \( T_1 \) is derived by replacing \( bl \) in equation (II.1) with \( (1 + s)bl \).

Solving this problem for \( T_1 \geq T_0 \), yields the post-subsidy values:

\[ q(t) = \left( \lambda(T_1) + \delta(T_1) \gamma(T_1) + (1 - s) \theta(T_1) \right) \sigma(T_1) + E + \omega(P + \theta(T_1)) t \leq T_1, T_2 \]

where \( \lambda(T_1) \) and \( \gamma(T_1) \) are, respectively, the new values of \( \lambda_1 \) and \( \gamma_1 \).

As illustrated in Figures (I.a) and (I.b), as a result of the subsidy, the natural gas (resource in the ground) is exhausted more quickly, while the helium is exhausted at a later date. This is to be expected.
since the costs of holding the helium until a date further in the future have decreased. Since natural gas extraction is slowed down in the presence of the helium, when helium storage becomes cheaper the natural gas extraction program more closely approximates what it would have been in the absence of the jointly extracted helium.

The total amount of helium separated is unchanged if the subsidy is imposed after $t^*$. It increases in value (as measured by $\gamma(t)$) initially, but its value rises at a slower rate than without the subsidy. The change in the value of the resource in the ground ($\lambda$) is indeterminate, and depends on the magnitude of the movement in $\gamma$.

If the subsidy is announced before the separation of helium begins, we find that the direction in the time at which helium separation begins, and therefore, also in the amount of helium separated, is indeterminate. Whether the amount separated has increased or decreased, the helium will be exhausted at a later date, and the natural gas at an earlier date.

Therefore, the storage subsidy has the unambiguous effect of increasing the length of time over which consumption of helium occurs. At the same time, depending upon the parameters of the problem, it may actually result in a decrease in the amount of helium separated from the natural gas.

**Figure la.** Natural Gas Price Path under Helium Storage Subsidy Imposed after $t^*$

**Figure lb.** Helium Price Path under Helium Storage Subsidy Imposed after $t^*$

**HELIUM SEPARATION SUBSIDY**

At time $T^*$, which can occur at any time between $0$ and $T$, a separation subsidy is imposed on helium. The subsidy is equal to $sP$, where $s \in (0, 1)$, so that the costs of separation to the producer become $(1 - s)P$. The maximization problem after imposition of the subsidy is found by replacing the term $(1 - s)P$ in (II.1).

Solving for the equilibrium paths using the same methods as in Section II, we find the analogous paths to (II.8) and (II.9), under the subsidy imposed after $t^*$:

(III.5) \[ q_1(t) = \lambda^*_1 - d/\gamma^*_1 + b
\]

(III.6) \[ q_2(t) = \gamma^*_1 + b/\lambda^*_1 - b
\]

where $\lambda^*_1$ and $\gamma^*_1$ are the post-subsidy values of $\lambda_1$ and $\gamma_1$.

The helium price and consumption paths are unaffected by the subsidy if it is imposed after the onset of helium separation. The new natural gas price path leads to an earlier exhaustion of the resource in the ground. The value of the resource in the ground ($\lambda^*_2$) increases as a result of this subsidy, while
there is no change in the helium shadow price ($y(t)$). This subsidy operates in the same manner as a subsidy on natural gas extraction costs if imposed after $t^*$, because, given no change in the total amount of helium separated, the helium price and consumption paths do not depend on the costs of separation.

The effect of the subsidy on the natural gas paths can easily be understood by noting that the present discounted value of this subsidy decreases over time. Therefore, the subsidy is worth more now, leading the resource owner to re-benefit extraction from the future to the present.

The result is different, though, if the subsidy is implemented before helium separation has began (at $t^*$). In this case, helium separation begins at an earlier date, and, as a result, there is more helium separated from the natural gas. The helium will be exhausted at a later date under the subsidy, and the natural gas at an earlier date. The associated price paths are pictured in Figures 2a(2) and 2b.

**CONCLUDING REMARKS**

The relatively simple model used in this study has shown that, of the three subsidies discussed, the helium sales subsidy is the one which will have the most substantial conservationist effects: specifically, social benefits in the helium market, without adverse effects on natural gas. It is also clear that the separation subsidy when imposed after time $t^*$ is the least desirable alternative. If announced before separation of the residual resource begins (less applicable to current helium policy decisions), the sales subsidy has the added advantage of delaying the exhaustion of the resource in the ground.

This analysis has also substantiated the hypothesis that when natural resources exist jointly, the effects of government policies are likely to differ from what would be predicted by a single resource model.

**NOTES**

1. [14] discusses some of the reasons why this might be the case.
2. Some substitution for helium has been made in certain uses, for instance argon is used for welding, but it is believed that other substitution possibilities are limited. See [6] and [16] for further discussion of the characteristics of the problem.
3. At this point, although helium separation is increasing, it has not reached its maximum rate.
4. Various aspects of the helium industry have been studied in [7], [10], [11], [12].
5. Alternatively, we can consider there to be some completely inelastic minimal initial level of demand. In addition, demand uncertainty can be analyzed. In this paper we consider the deterministic case in order to focus on government policies. One type of helium demand uncertainty is analyzed in [6].
6. The two gases can be measured by volume (cubic feet) at constant pressure in order for all measurements to be taken in the same units.
Foreign Lobbying: A Theoretical Analysis

Steven Husted

INTRODUCTION

Recently, the theory of commercial policy has moved rapidly in the direction of incorporating into general equilibrium analysis several phenomena which have been characterized as directly-unproductive profit-seeking (DUP) activities (Bhagwati, 1982). These include lobbying to seek policy changes (e.g., tariff seeking) or to profit from existing policies (e.g., rent seeking for prenica on quotas).

Very little effort has been made in this vast literature to analyze the phenomenon of foreign lobbying (lobbying on behalf of foreign interests to change domestic policies). On the other hand reports in the popular press suggest that such lobbying activity is by no means negligible in the United States. Congress, for its part, seems concerned over the influence foreign lobbying might have on U.S. policy. Legislation is being considered that would sharply limit the ability of foreign governments to lobby for foreign interests. Doubtless foreign lobbying is not limited to the United States. It needs to be analyzed.

Section II provides a brief review of the DUP theoretic literature on lobbying. Section III then analyzes foreign lobbying, utilizing the general equilibrium theoretic framework that has become popular in the literature. Section IV discusses some underlying issues and presents some data on foreign rent seeking in the United States. Section V offers some concluding remarks.

Existing Analysis: A Review

The introduction of domestic revenue (rent) or tariff (quota) seeking into international trade models has taken the following general form. The (domestic) economy is usually assumed to be small so that world prices are given. There are two factors of production (e.g., labor and capital) and three goods (two tradable and one non-tradable, lobbying services). In the rent-seeking model seeking, protection is assumed to be in place and then factors are diverted into the production of lobbying services in pursuit of the revenues rents generated by protection. In the tariff/quota seeking literature, it is assumed that lobbying is required in order to produce any given level of protection. Again resources are drawn from otherwise productive uses into DUP activities.

A number of theorems exist regarding the effects of DUP activities on an economy. For instance, Bhagwati and Stiglitz (1980) have shown that tariff revenue seeking in a small economy is necessarily welfare-worsening compared with the tariff distorted equilibrium. Consider, for instance, Figure 1. With AB as the production possibility frontier, PC = P_C, the given foreign price ratio, and P_C, the tariff inclusive price ratio, free trade yields consumption at C, welfare at U. The imposition of a tariff shifts production to P, from P, consumption to C, and welfare to U. When competitive revenue seeking is introduced into the model, equilibrium production shifts to P which lies on the generalized Rybczynski line, P_C. Consumption then is at C, and welfare at U, on.

REFERENCES