INTRODUCTION
The virtual nonexistence of price-level adjusted (indexed) financial instruments has puzzled economists for a long time. Because the benefits of indexed instruments seem obvious, their proponents have blamed their paucity on market imperfections or regulatory obstacles. (See, for example, McCulloch (1985).) Their nonexistence, however, follows from an asymmetry in the economy. While individual agents prefer constant real consumption and thus real income, the economy does not naturally supply it, as the real rate of return on physical assets varies over time. If real income and profits are uncertain, then financial instruments, whose cash flow is derived from the underlying real asset, cannot provide a constant real cash flow. This paper represents an attempt to show that the near absence of indexed financial instruments is a response to this uncertainty.

The next section contains a review of the literature that describes the supply and demand for indexed financial instruments. The reason why monetary theory favors indexed instruments while others are satisfied with nominal financial instruments is given in section 3. A basic model is used to show why households or firms do not readily supply indexed instruments. The model is extended to demonstrate that indexed instruments in general provide no clear improvement over nominal instruments. The relationship between real rates of interest on nominal and indexed instruments, which bears on the issue of supply and demand, is examined in the fourth section. The last section contains a summary and conclusion.

LITERATURE REVIEW
The focus of this paper is on the use of indexed financial instruments by ultimate borrowers and lenders. Ultimate borrowers are usually firms and households who supply indexed instruments such as mortgages and bonds. Ultimate lenders demanding indexed instruments are typically households saving for retirement. The government supply of indexed instruments as well as the supply and demand for indexed instruments by financial intermediaries is omitted. (Leeds (1989) describes potential problems of duration-mismatched financial intermediaries when they use indexed instruments.)

Economists usually assume that households desire steady real consumption and income and thus demand instruments that enable them to reach that goal. Because the presumption in favor of indexed financial instruments is so strong, only two papers build a model to study it more closely. Fischer (1975) examines the market for indexed bonds most thoroughly. He assumes that the rate of inflation, the real rate on equity and nominal bonds, and the nominal rate on indexed bonds and equity follow a stationary stochastic process. Fischer derives the demand for indexed bonds from consumer's maximization of expected utility of infinite consumption constrained by stochastic income from the above instruments.

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In a market equilibrium, assuming no outside bonds and homogeneous expectations, consumers hold equity, but no nominal bonds. The demand for real bonds depends on the attitude toward risk, as "more risk adverse individuals hold positive amounts of indexed bonds and less risk adverse individuals borrow, leasing indexed bonds to buy equity" (p. 120). Nominal bonds in this special case are dominated by real bonds. Fisher is thus able to reproduce equilibrium by using indexed instruments.

The introduction of stochastic wage income alters the demand for real bonds. If no outside bonds exist and expectations are homogeneous, both nominal and real bonds exist in market equilibrium. Fisher cannot find any reasons within his model for the complete absence of indexed bonds in the U.S. economy.

Livian and Levhari (1977) examine the demand and supply of indexed bonds by households in a two-period model with uncertain inflation in the second period. They show that if price-linked and nominal transactions are mutually exclusive, then linked transactions dominate nominal transactions if no money is held in the second period because the variance of the linked portfolio is less than the variance of the nominal portfolio.

If money balances are held, however, the dominance of the linked portfolio disappears for all but very large loans. The lender still prefers the linked transaction because the payments he receives are not variable. But the borrower prefers to give nominal payments, because they are a hedge against the loss of purchasing power of his nominal balances. Thus, linked transactions do not arise because they do not benefit borrowers.

Similar results obtain in a mixed portfolio where borrowers and lenders issue and hold both nominal and real bonds. With no money balances, price-linked bonds dominate nominal bonds, which disappear in equilibrium. This is analogous to Fisher's derivation showing that with no outside bonds consumers use only indexed instruments. If a part of the portfolio is adjusted, however, indexed bonds should appear.

Livian and Levhari thus admit that they cannot explain the complete nonexistence of a market for price-linked bonds. In contrast, (Fisher 1979), Livian and Levhari use comparative statics to show that an increase in inflation uncertainty leads to a decrease in supply and an increase in demand of indexed bonds, thus producing an unclear effect on the size of the market.

THE MARKET FOR Indexed FINANCIAL INSTRUMENTS

This section reviews some of Livian and Levhari's results and links them to the advocacy of indexed instruments. Extending Livian and Levhari, it is shown that uncertainty about adjustment of future income to inflation leaves both borrowers and lenders virtually indifferent between linked and nominal bonds.

Let M represent nominal income in period 1 when the price level is 1. M is thus also real income. Let the inflation rate, , in period 2 be a discrete random variable taking on values p and with equal probabilities. Thus, purchasing power is distributed as U(1' + p) and U(1' + p').

Lenders purchase nominal or real bonds issued by borrowers. With nominal bonds, borrowers repay and lenders receive B(1' + p)'. With real bonds, borrowers repay and lenders receive B(1' + p)' denoted as R, where r is the real rate of interest. Assume that the expected value of both payments, expressed in real terms, is the same:

\[ R = E(U(1' + r)) \]

Then,

\[ 1' + r = E(U(1' + r)) \]

It is assumed that real and nominal bonds are mutually exclusive and that there are no other items in the portfolio.

**Adjustment for Inflation**

Assume that both borrowers and lenders know that their incomes are adjusted for inflation in period 2. Their nominal income is then M(1' + p) and their real income M. With nominal bonds requiring repayment, the borrower's expected real value of the second-period portfolio, P, is:

\[ E(P) = M - U(1' - p) \]

and its variance is

\[ var(P) = p^2U(1' - p) \]

With real payment, R, the expected real value is

\[ E(P) = M - R \]

and the variance is zero.

The general trend of monetarists is that the economy is more prone to nominal than to real shocks. Thus monetarists view real income and real rates of interest as steady. If income is fixed in real terms, as above, then both borrowers and lenders (who receive 1' or R) are better off with indexed financial instruments, associated with real payment, R, because the variance of their portfolio is reduced, while the expected values are the same by assumption. This explains why Milton Friedman advocates indexing in general and price-level-adjusted mortgages in particular (1963).}

**No Adjustment for Inflation**

Assume now that borrowers and lenders anticipate no adjustment for inflation in their incomes, thus expecting to receive a nominal amount M in period two. Still, however, they expect a change in the price level. The borrower's expected real value of the portfolio with nominal payment I is:

\[ E(P) = M - U(1' - p) \]

and its variance is

\[ var(P) = p^2U(1' - p) \]

With real payment, R, the expected value is

\[ E(P) = M - U(1' - p) + R \]

and its variance is

\[ var(P) = p^2U(1' - p) \]

In this scenario, inflation brings real changes to borrowers' and lenders' incomes, and the results from the previous section are now reversed. With real shocks, the expected values of the portfolio are the same for nominal and real payments, but the variance of the portfolio is lower with nominal payments which hedge the loss of value on nominal income. To the extent to which real shocks characterize economic history, borrowers prefer nominal instruments. Since borrowers issue them, they tailor them to their circumstances. Repeated attempts to introduce price-level-adjusted mortgages have therefore failed because of insufficient supply by house buyers. (Ch. Winter 1965) and Woodward and Crowe (1963). Lenders, however, prefer to receive real payments. For them, (3) exceeds (7), as it replaces 1p in expression (5).

**UNCERTAIN ADJUSTMENT**

The preceding examples show that agents can correctly anticipate the future as follows. If borrowers know that real (nominal) income is stable, they use indexed (nominal) instruments. Generally, however, agents cannot predict the structure of the future as there is a fundamental uncertainty about future regimes. For example, the markets expected the 1980s to be inflationary as reflected in high nominal rates at the beginning of the period, but the decade experienced very low inflation. This uncertainty is modeled by allowing for both real and nominal shocks. The assumption of homogeneity of expectations is preserved by assigning the same probability of each regime to all agents.

Most realistically, borrowers and lenders have mixed forecasts. They do not know whether future income is adjusted for inflation. If the probability of adjustment is one half and inflation is distributed
as a and p with equal probability, then nominal incomes in period two are:

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<th>No adjustment</th>
<th>Adjustment</th>
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\[-p(1/2) M \quad M(1-p)\]

\[+p(1/2) M \quad M(1+p)\]

Translation into real incomes produces:

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<tr>
<th>No adjustment</th>
<th>Adjustment</th>
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\[-p(1/2) M(1-p) \quad M\]

\[+p(1/2) M(1+p) \quad M\]

With real payment, R, the expected real value of the portfolio is

\[E(P) = M2 - R + (M2)(1-p)^2.\] (10)

With nominal payment, R, the expected value is the same by assumption.

\[E(P) = M2 - (M2)(1-p)^2.\] (11)

The variances are harder to evaluate, as they do not reduce to a simple expression. The long-term variances of both portfolios are presented and compared term by term in an Appendix, available upon request.

The difference is small, as there is some expected mis científicas between adjusted and non-adjusted returns and payments regardless of the choice of instruments.

Thus, if agents are uncertain about the nature of their future incomes, neither payment dominates the other. If indexed bonds do not offer unambiguous benefits, they will not displace nominal bonds, especially when rates of financial innovation are taken into account. Indeed, switching into indexed bonds would result

**ESTIMATION OF REAL RATES OF INTEREST ON NOMINAL AND INDEXED BONDS**

The existence of a market for indexed financial instruments is naturally closely tied to their prices. Their attractiveness depends on the relationship between real rates on real and nominal bonds. (See expression (3.)) This relationship can be examined empirically and theoretically.

At empirical examination is difficult for two reasons. First, indexed instruments are scarce. Two such securities, however, are described in the literature: Wilcox (1985) analyzes interest rates on a 15-year indexed bond issued in the United Kingdom in 1981, and he compares them to nominal interest rates of similar maturity. Between 1981:1 and 1981:4 (from private correspondences) the average nominal interest rate was 11.71 percent and the real rate was 3.81 percent. Since Wilcox uses these rates to impound inflationary expectations, he must assume that real rates on nominal and real bonds are the same. Thus, no comparison between the expected real rate on these two instruments can be made from his work.

Bouchen and Newman (1988) present interest rates on a 10-year bond issued in Argentina in 1974. They use data between 1976:7 (because the central bank fixed interest rates when these bonds were first issued) and 1982:6, when the bonds were repurchased by the government. The average real rate of interest for the period was 9.4 percent. This rate cannot be compared with expected real rates on nominal instruments either because the authors did not find nominal bonds of similar maturity.

Second, long-term expected real rates on nominal instruments are also difficult to determine because long-term expectations are inherently unreliable. Thus, we do not have any measure of the expected rate of inflation for 10-30 years necessary to calculate the expected real rate for contractual nominal rates. One could assume unbiased expectations and calculate realized rates on bonds that have matured. Such measures, however, frequently reveal negative real interest rates, thus invalidating the assumptions. Empirically, then, the relationship between real rates of interest on real and nominal instruments is difficult to ascertain.

It is thus important to have a theoretical group of the issue. The easiest point of departure is the model of Livian and Levich as used in this paper. If the shocks to the economy are nominal, incomes are linked to inflation and indexed bonds dominate nominal bonds with the same real rate of interest. This rate is represented in this paper by the same expected real payment on the loan.

If, however, the shocks to the economy are real as opposed to a fixed nominal income, borrowers prefer nominal and lenders real payments and prices have to adjust to bring borrowers and lenders together. Thus, there is a premium on nominal bonds, as the real rate on indexed bonds is less than the real rate on nominal bonds (Livian and Levich, p. 369).

The same insight is provided in a more sophisticated model by LeRoy (1984a, 194b). He shows that the relationship between expected real rates on nominal and indexed instruments depends on the degree of shocks to the economy. In a representative individual or two-state rational-expectations model with a stochastic steady state equilibrium, the Fisher relationship holds if monetary shocks are the source of variation in nominal GDP. In this scenario, expected real rates on the two classes of instruments are the same (p. 198). Real endowment shocks, however, disturb this equality, as the real rate falls with the price level. Thus, the expected real rate on real instruments is less than the rate on nominal instruments; there is a positive premium on nominal instruments.

Fisher (1932) allows for other possibilities that depend on the covariance of the nominal rate of return on equity and nominal rate of return on real bonds (or inflation) and on the covariance between the real rate of return on equity and the real rate of return on nominal bonds (or inflation). If the second covariance is positive, then indexed bonds command a premium over nominal bonds in terms of the real rate of interest (p. 519). In other words, if equity is a hedge against inflation, which seems to be the short-run empirical observation, indexed bonds offer a higher real rate. Conversely, if equity is a hedge against inflation, when holds empirically in the long run, then nominal bonds offer a higher real rate of interest than indexed bonds.
Pochter concludes by suggesting that factors left out from his model would influence the demand for indexed bonds and their premium. In particular, uncertainty about future rates of return could increase the demand for indexed bonds and reduce their required rate of return.

In the presentation of fundamental uncertainty in this paper, however, the portfolios of borrowers and lenders are very similar in terms of expected values and variations with real and nominal financial instruments. Uncertainty regarding income adjustment therefore produces similar real rates on nominal and indexed instruments.

CONCLUSION

Economists assume that consumers smooth out real consumption over time. Thus, they are puzzled by the absence of indexed financial instruments that would help consumers achieve that goal. In an economy with stable real income, the benefits of indexed instruments are easy to demonstrate, as the variance of a portfolio with indexed bonds is less than the variance of nominal bonds. Thus, investors, who believe that the shocks to the economy are mostly nominal, advocate indexation.

With stable nominal income, the results are very different. A market for indexed bonds exists in an economy characterized by real shocks only if the real rate on indexed instruments is less than the real rate on nominal instruments.

If agents are uncertain about the nature of their incomes, however, these conclusions must be modified. Neither instrument dominates the other when both command the same real rates. Thus, given the costs of innovation and the lack of familiarity of consumers with price-indexing, there is no incentive to abandon nominal instruments in favor of indexed instruments.

REFERENCES


The Role of Empirical Analysis in the Investigation of Situations Involving Ignorance and Historical Time

Donald W. Katzner*

Empirical analysis is the employment of observed facts to shed light on a particular problem. It involves, in general, the gathering of appropriate data, the organizing and summarizing of that data, and the interpretation of the results of the organizing and summarizing process in reference to the issues raised by the original problem. The data may be numerical in character or consist of collections of verbal descriptions. In either case, the techniques for organizing and summarizing are similar.

Although the problems to which empirical analyses are addressed can spring from a variety of sources, many arise in connection with theoretical models. Theoretical models, after all, are built to explain what is seen or to predict future events. Such models are metaphorically taken to approximate the generating mechanisms that determine, subject to error, the observations. From this perspective, then, observations can be interpreted either as the solution points of static models consisting of simultaneous equations, or as lying along equilibrium or non-equilibrium paths produced by dynamic models containing differential or periodic (including difference) equations.

Most empirical analyses today are set in a context in which the relevant data are numerical in character and are analyzed, as described above, by a theoretical model and error. In addition, it is frequently assumed that the error has its own generating mechanism, and that this latter mechanism is based on some law of probability. The purposes of these empirical analyses often include one or more of the following overlapping goals: describing the observed phenomenon, estimating or fitting particular functional forms to the data, predicting future observations, testing specific theoretical hypotheses in particular, and empirical confirmation or falsification of theoretical conclusions in general.

Difficulties arise, however, when examining situations in which either an actor under study, say a decision-maker, or an investigator pursuing an inquiry (or both), views time as historical and confines ignorance in a significant way. Historical time has the quality that each moment of it is attached to a distinctive history and is, therefore, unique. This uniqueness precludes the possibility of replication of decision situations and contemplations of future economic outcomes. Ignorance is a fact of humanity which comes about because man's limited ability to know certain things. Recognizing both historical time and human ignorance in a situation (and such situations may, in fact, be the more important ones in Economics) produces a true uncertainty that renders the assumption of a probabilistic error-generating mechanism untenable. Non-empirical models make the same explanatory meaning in the presence of this uncertainty. The formulation of probability functions and the standard interpretation of theoretical models simply presume more information than is available. It follows, then, that empirical analysis under conditions of ignorance and historical time must take a rather different flavor from that which it has when these elements are absent.

This paper is concerned with the role and extent of empirical analysis in the simultaneous presence of both ignorance and historical time. It begins with a description of the historical-time environments in which ignorance appears, and then proceeds to a discussion of the nature of empirical analysis in such situations.

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