Real Money Balances in the Production Function: A Comment

Allen Sinai* and Houston H. Stokes**

Jensen-Karnich-Bennet (JKB) (1987) and Jensen-Karnich (JK) (1987) report that the Sinai-Stokes (SS) (1972) empirical results on money balances in production are questionable because a proposed counterexample specified and estimated by them also "confirmed" by the data.

While it is appropriate to propose a counterexample to question the validity of a maintained hypothesis, a difficulty of the JKB approach is that the two counterexamples proposed and estimated are not the same as the original SS result. For a counterexample to work in the manner intended by JKB, it must be logically independent of the example, not a restricted form of the more general functional specification. SS (1972) tested a Cobb-Douglas (CD) production function with three inputs: labor, capital, and money balances. In addition, a time trend was entered in some of the equations estimated. In SS (1972), the elasticities of output with respect to the inputs were not constrained to add to unity. JKB constrained the sums of the output elasticities with respect to the inputs at unity.

Given Q as output, i, t time trend, and K, L, m capital, labor and real balances, respectively, SS estimate

\[ Q = \alpha + \beta_1 L + \beta_2 K + \beta_3 m + \epsilon \]

using annual time series data over 1929-1967. The basic SS results for equations containing a time trend were provided in SS (1972) (Equations 8, 9) and again reported in SS (1989) (Equations 6, 14, Table 7). JKB estimate

\[ \ln Q = \alpha + \beta_1 \ln L + \beta_2 \ln K + \beta_3 \ln m + \epsilon' \]

which is nothing more than a constrained version of (1).

Equation (2) is not a counterexample of (1); it is a special case. For the counterexample methodology used by JKB to be appropriate, it must be logically independent from the example. The other counterexample proposed by JKB is conceptually the same as (2).

There are two ways to estimate a constrained three-input production function and force the coefficients to add to unity. If two of the three inputs in a CD production function are divided by the other input and the result estimated by ordinary least squares (OLS), constant returns to scale are assumed. This was done in (2). The other way is to estimate (1) with a restriction on the coefficients to sum to one. Both approaches will give the results shown in Table 2a of JKB, which indicate support for their counter-example because the equations containing \( m_1 \), \( m_2 \) and \( m_3 \) provide a "GOOD-FTT" (JKB, p. 285), while the "original" SS model shows a "GOOD-FTT" for models containing \( m_1 \) and \( m_2 \), and a "BAD-FTT" for the equation containing \( m_3 \).

* The Boston Company Economic Advisers, Inc. ** University of Illinois at Chicago. The authors thank Dennis Stokes for editorial assistance. Any remaining errors are their responsibility.
What is a "good-fit" and "bad-fit" is not indicated. The results in Table 2 in actually show that a better fit is obtained for all the SS unrelated equations compared with the unrestricted and restricted JKB equations. The only criticism of the SS results reported by JKB might be that the statistic on the coefficient of $M_3$ (1.246). The negative coefficients for $M_1$, $M_2$ and $M_3$ in the JKB constrained case are forced by the form JKB used.

Our interpretation of the JKB results is that the effect of imposing the restriction on (1) is to lower the $R^2$ in all equations, providing evidence that the special constant returns to scale case of the SS interest returns CD specification is an incorrect specification on the data used, rather than a counter-example supported by the data.

For example, using the JKB data, for the M1 regression, the overall fit falls from 0.99229 to 0.95146, for M2 from 0.95005 to 0.89652, and for M3, from 0.95696 to 0.94688. From the sweeping conclusion (JKB, p. 259), "we show that all the evidence produced in support of the Sins-Stokes hypothesis cannot be sustained on theoretical and empirical considerations," what shows is that the constrained three-input CD production function gives an inferior fit compared with the unrestricted form estimated by SS. This JKB conclusion is an incredible overstatement on the results presented.

JKB limit SS (1972) for not checking on multicollinearity when, in fact, such a check was performed by running all prospective variables and checking the $R^2$. These included variables and checking the $R^2$ for each was calculated using the HSAS™ program, documented in Stokes (1991) and has a number of collinearity tests on the regression calculation, including the Faddoese (1959) procedure, which provides a computational check on the estimated coefficients.

While there was some multicollinearity in the data, the problem was not particularly serious. The Faddoese estimates of the computational error in each coefficient for the OLs form of (1) for $M_1$ and $M_2$ are (0.5999-11, 4.6498E-11, 0.1905E-11, 0.1312E-11, 0.3659E-11, 0.1932E-10, 0.1767E-10, 0.7824E-11, and 0.6048E-11, 0.1580E-11) for $M_1, M_2$, $a$, $\beta$, and $\gamma$, respectively. Given that $X$ is the $T$ by $k$ data matrix, the Faddoese procedure calculates the diagonal elements of $[XX'XX']$, which should have an expected value of zero. Substantial deviations from zero provide a computational check on the accuracy of the answers. Since the largest diagonal element found was in the area of 0.46, the accuracy loss due to multicollinearity does not appear to have been a serious problem. Actually, this multicollinearity in the data set, by inflating the diagonal variance-covariance matrix, would work in the direction of reducing the estimated significance of the coefficient for real money balances.

Further evidence on whether money balances belong in the production function awaits implementation of more comprehensive measures of the inflation of the financial system in production and additional experimentation with functional form issues. While we applaud the development of the "counterexample" methodology as a means by which to test and study the effect of the financial system on the potential supply of real output, such counterexamples must be distinct from the original SS unrestricted CD form in order to have validity. All the JKB paper does is to highlight that the constrained form of the CD production function containing real balances is inferior to the specification as the unrestricted form for the 1929-67 data set used in the estimation. It does not controvert or shed doubt on the originally carefully drawn and stated suggestions of the original work.

NOTES

1. Equation 3 comes from taking logs of $Q = A_0 e^{t/2} X_0^{\gamma}$, where $u = \ln v$. The constrained system is $Q = A_0 e^{t/2} X_0^{\gamma}$.

2. The anti-log of equation 2 and multiplying by $y$, we get $Q = A e^{t/2} X_0^{\gamma}$, where $Q$ is the anti-log of $Q$. Hence, equation 2 is just one of two possible ways to estimate a constrained form of equation 1.

3. Bennett (1989) has recently commented on JKB (1987), who again replied in JKB (1989). (Bennett used more recent data that was provided to us. She reports the second-order OLS constrained form of the SS model and finds that the coefficients of $M_1$, $M_2$ and $M_3$ were significant in models without time. With models containing time, she reported significant in the coefficients of money variables, although there was some evidence of multicollinearity.)

BIBLIOGRAPHY


A Counter-counter Critique: A Reply to Sinai-Stokes

S.J. Kamath, K.C. Jensen & R.E. Bennett

In discussing our critique of their 1972 work, Sinai and Stokes (1992) "approve of the use of a valid counterexample to question the validity of a3 examined hypothesis" but then proceed to declare our counterexample as inadmissible because of its being a "special case of their hypothesis and not logically distinct from the (original) example." We are pleased that SS agree with us on the need to remove conventional testing procedures in economics in terms of the counterexample procedure but fear that they do not recognize the methodological implications of their admission and its implications for their hypothesis. We referer some old issues and examine the invalidity of their counter-critique in this section.

As pointed out in our original (1984) paper (most recently published as Kamath, Jensen, & Bennett [1990]), the conventional approach to empirical testing, as outlined by SS in their 1972 paper, involves attempting to confirm predictions derived from models of the theory under examination. There are two major problems with this method. Firstly, rules of evidence, which are used to determine if the prediction fits the data, are generally structured such that a negative result does not mean that the model or theory under consideration is false, Likewise a positive result does not mean that the model or theory is true.2

And secondly, despite the possibility of errors analogous to the type I and II errors of standard hypothesis testing inherent in the usage of all rules of evidence, conventional test procedures do not reflect the need to minimize these errors or reach an optimal trade-off between them.2 These characteristics of the conventional approach to testing reduce the efficacy and efficiency of empirical testing in economics.3 As is well known, a universal theory can be logically contradicted or refuted with the aid of deductive logic only by one singular statement while no amount of confirming statements can ever guarantee its truth.4 Any test procedure for empirical testing needs to exploit this asymmetry in testing theories.