The Risk Free Rate, Competitive Markets, and the Capital Asset Pricing Mode

DENNIS E. LOGUE and MICHAEL A. SIMKOWITZ

Abstract

A number of empirical studies have concluded that the predictive content of the traditional capital asset pricing model is seriously hindered by its reliance upon the assumption that investors may lend or borrow all that they like at a single risk free rate. One important revision of the capital asset pricing model which does not use this assumption suggests that investors hold some linear combination of the market portfolio and a minimum variance portfolio which is uncorrelated with the market portfolio. It yields theoretical results which are much more consistent with the empirical findings. Using a single period state preference model, this paper argues that the former model is not consistent with a competitive equilibrium, whereas the latter is. This result derives directly from the fact that the assumption of a risk free rate exogenous to the determination of other market returns implies the existence of a monopolistic security issuer: one who has access to some securities to which other investors do not have direct access.

I. Background

The traditional form of the capital asset pricing model holds that the equilibrium return on any security is a linear function of the return on a riskless asset and the return on a market portfolio. Using a single period state preference model, we argue that the notion of a competitive equilibrium for assets is inconsistent with the concept of a risk free rate which is wholly exogenous to the competitive system which determines the returns on other capital assets. An exogenous risk free rate implies that there is a monopolistic issuer who has access to investment opportunities which are not available to other investors. If there were no such investor, these special opportunities would be a part of the market portfolio and there would be no need to introduce the riskless asset in order to derive an equilibrium situation. Under these conditions, it is not surprising that direct empirical tests of the traditional asset pricing generally have failed, but that the tests of the implications of the theory (in particular, tests of the efficiency of the market in adjusting to new information) have met with outstanding success in demonstrating the competitive efficiency of the capital markets.

Recent direct empirical studies by Black,
Jensen, and Scholes, Friend and Blume, and Blume and Friend have provided evidence that the traditional capital asset pricing model as suggested by Markowitz and independently developed by Sharpe,Lintner, and Moskowitz is deficient in terms of its ability to predict. Friend and Blume, for example, use a cross-sectional regression between risk adjusted performance and risk for the 1960-68 period and find that high risk portfolios have poor performance, but low risk portfolios have good performance. They attribute this result to the empirically incorrect assumption that there is a single riskless rate at which investors lend or borrow all they like. Black, Jensen, and Scholes obtain similar results using much more powerful test procedures. They, too, conclude that the riskless rate assumption is the source of these empirical findings which contradict thereceived theory. They further hypothesize that the true stochastic security return generating model contains two factors: a market portfolio and a minimum variance portfolio which is uncorrelated with the market portfolio. They provide evidence on this hypothesis.

In a separate work, Black develops the underlying theory of the two-factor model. Here the equilibrium return on an asset is shown to be a linear function of the return on the market portfolio and the so-called zero-beta portfolio when there is no riskless borrowing or lending. Conceptually, and very importantly, this model is more consistent with the theory of a competitive market as suggested by Arrow and Debreu; for it does not admit of an exogenous riskless rate.

The question Black addressed is: what will a competitive equilibrium look like if no riskless borrowing or lending is permitted? Whereas our results suggest that Black's model is a considerably more appropriate positive model of economic behavior, use the earlier works of Sharpe and others, we begin by asking a much more fundamental question than Black. That is, we ask whether an exogenous risk free rate is consistent with the spirit of a competitive equilibrium? Our answer is no.

The remainder of this paper is organized as follows. In Section II, the capital asset pricing model and its empirical counterpart, the market model, are reviewed. Section III introduces a simple single period state preference model and presents some preliminary results. Section IV casts the capital asset pricing model in a state preference framework and explores the implications of this model for the riskless rate of interest. In Section V, the Black model is examined in relation to the riskless rate as determined in a state preference context. Section VI presents our conclusions.

II. The Capital Asset Pricing Model

The capital asset pricing model of Sharpe asserts that under appropriate assumptions, the return on the nth security, \( R_n \), is a linear function of the risk-free rate, \( R_f \), and the return on a broad based index of economic activity or a market portfolio, \( I \). Thus, the positive model may be written as:

\[
R_n = R_f + B_n (I - R_f)
\]

(1)

where

\[
R_n = \text{cov}(R_n, I) / \text{var}(I)
\]

The statistical or empirical form of the capital asset pricing model, the stochastic process generating returns or the market model, is written for any time series, \( t \), as:

\[
R_n - R_f = a_n + b_n (I - R_f) + e_{nt}
\]

(2)

where \( a_n, b_n \) are parameters and

\[
\text{E}(e_{nt}) = 0 \text{ for } n = 1 \ldots N
\]

(2a)

\[
\text{E}(e_{nt}) = 0 \text{ for } n = 1 \ldots N, \text{ for all } t
\]

(2b)

\[
\text{E}(e_{nt}) = 0 \text{ for } n \neq n'
\]

(2c)

(2d) defines the second term on the right hand side of (2) to be zero and (2d) defines the final two terms to be zero as well.

In general, both the capital asset pricing model and its empirical counterpart dichotomize the variation of covariation between securities into two components: one a parameter of the variation of the index, \( I \), the so-called systematic risk, and the other a parameter of the variation of the residuals, the unsystematic or diversifiable risk. Thus the variance of \( R_n \) is:

\[
\text{var}(R_n) = \text{var}(I) \times \text{var}(e_{nt})
\]

(4a)

and the covariance between \( R_n \) and \( R_{n'} \) is, in the model,

\[
\text{cov}(R_n, R_{n'}) = \rho_{n n'} \times \text{var}(I)
\]

(4b)

Of critical importance to the analysis at this stage is the nature and composition of the index, \( I \). The vast literature pertaining to the subject has variously described \( I \) as:

i) An index of general economic activity;
ii) An index of industrial stock price averages;
iii) An index of "fit index" of a composite of stocks in different industries; or
iv) An index of specific economic activity, such as investment, manufacturing, or trade.


13 These include:
1. All investors are risk averse, single-period utility maximizers where utility is a quadratic function of returns.
2. All investors have access to identical information.
III. The State Preference Model

To facilitate the exposition of what for our purposes are the most important aspects of the state preference model, a pure exchange economy in equilibrium (with endowments which will increase exogenously at the end of the period) is assumed. Moreover, only the necessary properties of the model will be introduced. Simply put, the state preference model is one in which investors buy state-contingent claims, with the prices investors are willing to pay for those claims determined by their subjective probability assessments of the likelihood of the state's occurrence and the pay-off under the state. In the complete model, there are at least as many real securities (i.e., composite claims paying off in more than one state) as states, as well as a portfolio of securities which will eliminate the possibility of bankruptcy. In other words, when Arrow-Debreu conditions hold, the set of real securities can be reduced to a set of "pure" securities such that at least one pure security will pay off in each possible state. Formally, let there exist \( M \) mutually exclusive and exhaustive states and \( N \) pure securities, where a pure security is defined as one which pays off in one and only one state. 18 We focus on the special case where \( M = N \). By construction, the pay-off of the \( n^{th} \) security is unity in the \( m^{th} \) state (\( m = n \)) and zero otherwise. There also exist \( L (1, \ldots, 1) \) risk averse investors, whose subjective probability assessment of the \( m^{th} \) state's occurrence is \( p_{mn} \), and who all agree on the set of possible states, although not necessarily on the probabilities. 29 Focusing on a single investor or the case where each investor has identical probability assessments, the price of the \( m^{th} \) pure security is \( p_{mn} \), a function of \( p_{mn} \).

The expected pay-off \( (E(P_{mn})) \) on a single unit of the \( n^{th} \) pure security over the \( M \) states (\( m = 1, \ldots, M \)) is defined where \( M = N \) as:

\[
P_t = E(P_{mn}) = \sum_{m=1}^{M} \frac{P_{mn}}{q_{mn}} \tau_m
\]

since

\[
P_{mn} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}
\]

thus

\[
\text{Var}(P_{mn}) = \tau_m (1 - \tau_m)
\]

and

\[
\text{Cov}(R_{ne}, R_{ne}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
R_n = E(P_{mn}) = \sum_{m=1}^{M} \frac{P_{mn}}{q_{mn}} \tau_m
\]

and the variance as:

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]

where

\[
\text{Var}(P_{mn}) = \frac{1}{q_{mn}} \text{Var}(P_{mn})
\]

and

\[
\text{Var}(P_{mn}) = \sum_{m=1}^{M} \left( P_{mn} - \bar{P}_n \right) \tau_m
\]
VAR-COV(R) = QU + U\'PQ\' \quad (12)

Moreover, letting D be a \(1 \times M\) matrix where \(d_m = 1\) for all \(m\), the expected value of returns may be expressed as:

\[
\bar{R} = D(\mu_\bar{R} - \bar{I}) \quad (13)
\]

Turning now from the realm of pure securities to one of real securities, some interesting and useful results can be derived.

Let \(A\) be an \(M \times N^a\) matrix which creates various linear combinations of pure securities and where \(N^a\) is the number of real (composite) securities. The variance-covariance of real security returns (\(\bar{R}\)) may then be expressed as:

\[
\text{VAR-COV}(\bar{R}) = \Sigma = \text{AQPQ}'(\bar{I} - \text{I}')P'Q'A' \quad (14)
\]

Unless one of the real securities is a linear combination of the others, the number of real securities must be \(M\). Since \(A, Q, P,\) and \((\bar{I} - \text{I}')\) are also \(M \times M\), the determinant of the variance-covariance matrix is zero.

Since

\[
\text{det} \Sigma = |Q A Q P Q'| = (\text{det} (\bar{I} - \text{I}'))^{-1} \quad (15)
\]

if the Arrow-Debreu conditions hold and there are \(M\) states and \(N\) pure securities, and \(M = N\) since \(\Sigma = \Sigma_{m=1} = \Sigma_{n=1}\) by definition, then

\[
\text{det} \bar{I} (\bar{I} - \text{I}') = 0
\]

This means, of course, that rank (\(\Sigma\)) \(< M - 1\) for if \(A, Q, P\), and \(\bar{I} - \text{I}'\) are non-singular, then rank (\(\Sigma\)) = \(m - 1\), since rank \((\bar{I} - \text{I}')\) = \(m - 1\). By definition \(Q\) and \(P\) are non-singular; providing that \(A\) is non-singular, rank (\(\Sigma\)) = \(m - 1\). Hence, if the requirements of the Arrow-Debreu state preference model are met, a riskless rate exists. That is, a portfolio of securities can be constructed which has a zero variance.

Letting \(R_p\) represent the real (as opposed to

\[\text{money? In other words, why would all risk-averse investors not hoard money with its presumed zero return and zero variance? The answer lies in the distinction which must be drawn between nominal and real money. In the case of the former, holding nominal money would be similar to holding a nominal risk-free portfolio with the possibility of a real negative return since the investor would not be fully indemnified against inflation—a generally relevant state. Holding real money (having constant purchasing power) would entail its purchasing power being guaranteed by some intermediary or mutual fund. This would imply in the limit that the intermediary has access to at least one or possibly more pure securities than the investor which in turn implies that there are imperfections in the market, which itself suggests an inconsistency with the underlying foundation of competitiveness of the state preference model. Alternatively, real money guaranteed by someone who has no special access but rather can invest only in those states to which the investor has access may still carry a negative return or may have positive variance.}

\[\begin{align*}
\text{VAR-COV}(\bar{R}) &= QU + U\'PQ\' \quad (12) \\
\text{Turning now from the realm of pure securities to one of real securities, some interesting and useful results can be derived.} \\
\text{Let } D \text{ be an } 1 \times M \text{ matrix where } d_m = 1 \text{ for all } m, \text{ the expected value of returns may be expressed as:} \\
\bar{R} &= D(\mu_\bar{R} - \bar{I}) \quad (13) \\
\text{Turning now from the realm of pure securities to one of real securities, some interesting and useful results can be derived.} \\
\text{Let } A \text{ be an } M \times N^a \text{ matrix which creates various linear combinations of pure securities and where } N^a \text{ is the number of real (composite) securities. The variance-covariance of real security returns (}\bar{R} \text{) may then be expressed as:} \\
\text{VAR-COV}(\bar{R}) &= \Sigma = \text{AQPQ}'(\bar{I} - \text{I}')P'Q'A' \quad (14) \\
\text{Unless one of the real securities is a linear combination of the others, the number of real securities must be } M. \text{ Since } A, Q, P, \text{ and } (\bar{I} - \text{I}') \text{ are also } M \times M, \text{ the determinant of the variance-covariance matrix is zero.} \\
\text{Since } \\
\text{det} \Sigma &= |Q A Q P Q'| = (\text{det} (\bar{I} - \text{I}'))^{-1} \\
\text{if the Arrow-Debreu conditions hold and there are } M \text{ states and } N \text{ pure securities, and } M = N \text{ since } \Sigma = \Sigma_{m=1} = \Sigma_{n=1} \text{ by definition, then} \\
\text{det} \bar{I} (\bar{I} - \text{I}') &= 0 \quad (15) \\
\text{This means, of course, that rank (}\Sigma\text{) }< M - 1 \text{ for if } A, Q, P, \text{ and } \bar{I} - \text{I}' \text{ are non-singular, then rank (}\Sigma\text{) } = m - 1, \text{ since rank } (\bar{I} - \text{I}') = m - 1. \text{ By definition } Q \text{ and } P \text{ are non-singular; providing that } A \text{ is non-singular, rank (}\Sigma\text{) } = m - 1. \text{ Hence, if the requirements of the Arrow-Debreu state preference model are met, a riskless rate exists. That is, a portfolio of securities can be constructed which has a zero variance.} \\
\text{Letting } R_p \text{ represent the real (as opposed to } \text{money? In other words, why would all risk-averse investors not hoard money with its presumed zero return and zero variance? The answer lies in the distinction which must be drawn between nominal and real money. In the case of the former, holding nominal money would be similar to holding a nominal risk-free portfolio with the possibility of a real negative return since the investor would not be fully indemnified against inflation—a generally relevant state. Holding real money (having constant purchasing power) would entail its purchasing power being guaranteed by some intermediary or mutual fund. This would imply in the limit that the intermediary has access to at least one or possibly more pure securities than the investor which in turn implies that there are imperfections in the market, which itself suggests an inconsistency with the underlying foundation of competitiveness of the state preference model. Alternatively, real money guaranteed by someone who has no special access but rather can invest only in those states to which the investor has access may still carry a negative return or may have positive variance.} \]
The last term disappears by assumption, and the second right hand term may, since \( EE' = I U' \), be written as:

\[
EE' = I U'
\]  

(21)

where \( I \) as before, an identity matrix and \( U \) is a diagonal matrix. If \( U \) is of full rank, \( M \), then \( IU' \) spans all \( m \) dimensions of the space with orthogonal vectors and the first right hand term of (19b) cannot be orthogonal to the remaining terms, no matter what the construction of the index, \( I \). Thus, the model market may yield biased estimates of systematic risk.

A key feature of the capital asset pricing model is that it facilitates the decomposition of risk into systematic and unsystematic components. Risk forces focus on only the systematic or unavoidable component as the only really relevant source of risk. Equation (19b) separates the variance-covariance matrix of security returns, \( R'K' \), into three kinds, specifically, the three right hand terms of (19b). The first is the systematic risk which is based upon the variation of an index; the second is the unsystematic risk, namely the variation of the residuals, and the third form of risk is the covariance between the index and the residuals which has been assumed away by (20).

More explicitly, letting \( \Gamma \) be a \((1 \times m)\) vector of proportional weights (i.e., the percentage of wealth invested in each security) such that \( X' \Gamma = \gamma_0 \), the risk associated with a portfolio may be written as:

\[
\text{Risk} (\Gamma) = \gamma_0^2 + \sum_{i=1}^{k} \beta_i^2 \gamma_i^2 = 0
\]  

(22)

The case of \( U \) being less than full rank does not help the model. Each \( \beta_i \) corresponding to a non-null diagonal element of \( U \) must be zero if the first and second terms of (19a) are to be orthogonal.

23)The model presented here is a selective treatment of the problem in Eugene Fama, "A Note on the Market Model and the Two-Parameter Model," Journal of Finance XXVIII (December 1973):1181-1190. However, the key result (i.e., that \( \gamma_0 = 0 \)) was simply uncorrelated, but allowed that they may still be not independent. Accordingly, our results and those of Fama's are actually consistent.

When short selling is permitted, then a riskless rate can be achieved even in the absence of any negative \( \beta_i \). This, however, could be accomplished only by selling short a portfolio which reduced the average beta of the whole portfolio to zero. In other words, the investor would be on both sides of the market with beta equivalent portfolio. Of course, this would result in a zero risk-free rate.

When there is a single risk-free asset available, \( \Gamma^* = (0, 0, 0, 0, 0) \). Then (25) may be written as:

\[
\text{Risk} (\Gamma^*) = \beta_0^2 \gamma_0^2 + \sum_{i=1}^{k} \beta_i^2 \gamma_i^2 = 0
\]  

(26)

Only if \( \beta_0 = \beta_i = 0 \) will (26) be satisfied. The requirement to hold portfolios with no systematic risk can be achieved only if it is a risk-free free, i.e., \( \gamma_0 = 0 \), as well. However, if such a risk-free security exists, it will not be part of the relevant system of securities. For it if were part of this system, then buying combinations of real risky securities should produce the same result. For such a security to exist endogenously, it would have to be a composite of securities with negative and positive \( \beta_i \). Without such a risk-free, where the market reaches the real securities rate to be reasonably competitive, such a security could only come about through exogenous forces.

The traditional capital asset pricing model must allow for either negative \( \beta_i \) or an exogenous single risk-free security, whose \( \beta_0 = 0 \) and whose \( \gamma_0 = 0 \), if it is to be consistent with the idea of the existence of a risk-free rate. However, it has been the latter case which has received the far greater portion of attention.

V. The Black Model and the Riskless Rate

The model developed by Fischer Black provides a third alternative. This is the case where there is no riskless asset, but rather a portfolio or security which has zero covariance with the index, \( I \). The formula represents the market portfolio, and has minimum variance of all such portfolios. This is clearly the most consistent with the spirit of a perfectly competitive market, because it allows the return on this so-called zero beta portfolio to be determined within the market, it implicitly assumes that there are perfect substitutes for each of the particular real securities that comprise it.

Figure 1 describes the latter two cases. In one case, there is an asset which is completely risk free, that is \( \beta = 0 \) and \( \gamma = 0 \). This is represented in Risk-Return (E(R), \( \sigma \)) space as point A. All efficient portfolios lie along the line ABC given the model represented in eq. (1). In the case described by Black, the zero beta portfolio would have positive variance. Point Z arbitrarily represents this point, and efficient portfolios lie along the line ZBC. ABC asymptotically approaches ABC at B since as we hold a smaller proportion of the Z portfolio the unsystematic risk of the total portfolio approaches zero. As we move from point B to point C, increasing both risk and return, the new curve would diverge from ABC but its relative amount of divergence would be...
rather small; this is untrue, however, of the divergence of $B$ from $A$. The question, of course, is whether either of these constructs are consistent with state preference model.

Recall that when the Arrow-Debreu conditions are met the variance-covariance matrix of real security returns was written as:

$$ R'K = A^'Q-H(l-I)C'P'Q'C'A' $${28}

If there is a single risk-free real security, then its row vector of $A$, $\tilde{\gamma}$, must be such that

$$ \tilde{\gamma}P = \tilde{\gamma}P(l-I)C'P'Q'C'A' = 0 $$

Since $P = I$, (28) may be rewritten:

$$ \tilde{\gamma}Q = \tilde{\gamma}Q(l-I)C'Q'C'A' = 0 $$

It was previously established that when the Arrow-Debreu conditions are met det $H(l-I)C'Q'C'A' = 0$, thus (29) is solvable if and only if $\tilde{\gamma}Q = (0, \ldots, 0)$ where $\tilde{\gamma}$ is any constant. Among the many solutions to the above, only one satisfies the definition of proportionality:

$$ \sum_{j=1}^{M} \gamma_j = 1 $$

That is,

$$ \tilde{\gamma} = \frac{1}{\text{trace } Q}(q_1, \ldots, q_m) $$

This defines the risk-free security as a linear combination of pure securities such that each pure security is held in equal number or the investment in each is the proportion of its cost to the sum of all costs, trace $Q$. There is, if each producer and investor has access to all states, no such thing as an exogenous risk-free security; it must be endogenous. If not, then some producers and investors are being deprived access to at least one or more states. That is, an exogenous risk-free security will not exist in a perfectly competitive product market.

An exogenous risk-free security will exist if and only if the pure security market is not competitive, that is, if a single monopolist or a collusive oligopoly has access to a pure security, (productive capacity and output in a product market sense) and alone can sell state contingent claims for a particular state. To the extent that potential short sellers of such state contingent claims may fear bankruptcy, may be unable to hedge, or may be institutionally prohibited from selling short, the price charged by the monopolist may be greater than the pay-off; hence this security, even if exogenous to the system, could carry a negative return in a pure exchange economy when the monopolist is not benevolent.

In the traditional version of the capital asset pricing model, however, which is a competitive model of security price determination, an exogenous risk-free rate is assumed to exist.

VI. Conclusion

A. Additional Observations on the Risk-Free Asset

In many empirical tests of the capital asset pricing model, the rate on government Treasury Bills has been used to approximate the riskless rate. The state preference model, however, suggests that this may not be a very good approximation: since subjective probabilities determine the price an investor is willing to pay for a given asset, there would be many investors who would not consider U.S. Treasury Bill rates as representative of what they consider a risk-free return. For those who wholeheartedly approve of all government policies, investments, and projects—presumably determined by who the government considers to be relevant possible future states—the T-Bill rate is a reasonable approximation. For those who disapprove of contemporary government policy, it is not a relevant rate for it is not subjectively risk-free.

As can be seen from (30), government assets should be invested in proportion to their costs when costs are subjectively determined by probabilities and all pay-offs have been, by convention, set to unity. Since probability assessments are likely to differ between individuals, not everyone will be accommodated by a government rate.

From the state preference framework, it is apparent that the rationale for using T-bill rates or any government obligation of proper maturity as an approximation for the exogenously determined riskless rate rests on the idea that the government has a monopoly on at lease one possible futures state. It is the only agent in the economy which can have output and which can provide payoffs in that given state; no linear combination of other real securities will provide an investor with a payoff if this monopolized state should occur. This is inconsistent with the concept of a perfectly competitive market. Moreover, this may explain why such poor results were obtained with these tests.

B. Summary

In this paper the consistency of the capital asset pricing model with the spirit of a perfectly competitive securities market was examined using a simple single period state preference model. In particular, the nature of the so-called “risk-free” rate was explored.

The intended contribution of this paper has been to cast doubt upon the capital asset pricing model of Sharpe and others, and to suggest Black’s model (amended so that the index represents the market portfolio of all available risk assets—not just securities) as more appropriate.