The Role of Factor Intensity Reversals in a Small Country Model of Trade and Growth*

CARLOS ALFREDO RODRIGUEZ

A large number of authors have been concerned in recent years with two-sector models of capital accumulation. None of these authors, however, considered the possibility of factor intensity reversals occurring during the process of capital accumulation. In this note this possibility is analyzed for an open economy. The model describes a small country producing and trading a consumption good (C) and an investment good (I) at a fixed relative price ($p_C = p_I/2$); there are two factors of production, capital (K) and labor (L); the production functions for both goods exhibit the usual neoclassical properties; the domestic demand for the investment good is a constant fraction c of real income, measured in terms of the same good. Within this context, the following issues are discussed:

(a) The shape of the Rybczynski line (Rybczynski, 1955) as capital accumulation proceeds and the factor intensity reversal takes place.
(b) It is shown that there is only one finite long run equilibrium level of the capital-labor ratio ($k = K/L$) irrespective of relative factor intensities, even if they reverse during the growth process.
(c) It is shown that before any factor intensity reversal takes place, the country must become specialized in the production of the
good that was previously capital intensive if the capital-labor ratio is growing.

(d) In a recent article, Ronald Findlay (Findlay, 1970) showed, in the context of a model with no factor intensity reversals, that the comparative advantage of a small country could change once as the capital-labor ratio increases. It will be shown in terms of our model that there could be as many changes in comparative advantage as the number of sets of relative factor intensities; that is, if factor intensities reverse once there could be up to two changes in comparative advantage, if they reverse twice, there could be up to three changes in comparative advantage, etc.

The Rybczynski Line

The Rybczynski line is defined as the locus of production points in the (I, C) plane (both I and C being measured in per-capita terms) which results from varying the relative factor endowments while keeping the relative commodity price constant. It is well known that, in the non-specialization region, the locus described above is a straight line with a negative slope; this line being steeper than the budget lines if $C$ is the labor intensive good or less steep than the budget lines if $C$ is the capital intensive good. Both cases are depicted in Figure 1 where $RR$ is the Rybczynski line when $C$ is capital intensive and $RR'$ is the line when $C$ is labor intensive. Consider point $C_0$ in Figure 1 and the corre-
sponding capital-labor ratio \( k_h \). For any \( k > k_h \) the economy is specialized in the production of \( C \) (the capital intensive good). As \( k \) increases further, the wage-rental ratio (w) increases. If there were no factor intensity reversals this would be the end of the story: for any \( k > k_h \) the Rybczynski line would coincide with the C-axis. If there are factor intensity reversals, however, there will be a sufficiently high \( h \) such that production of both goods will be profitable again. This point is shown with the help of Figure 2. There, \( k_{cw}(w) \) and \( k_{cc}(w) \) represent the capital-labor ratios in both sectors as functions of the wage-rental ratio (*) . The curve \( p = p(w) \) in the lower part of the diagram depicts the relative price of \( C \) in terms of \( I \) as a function of the wage-rental ratio. The elasticity of this curve is given by: \( w[p - sp]/bw - e_{cl} - a_{l} \) where \( e_{cl} \) are the shares of labor costs in production. It is clear that it must be \( 5pQw < 0 \) whenever \( C \) is the capital intensive good and \( 5pQw > 0 \) when \( C \) is the labor intensive good. As depicted— for \( p = p_0 \)—there are two wage-rental ratios \( w_1 \) and \( w_2 > w_1 \) satisfying \( p_0 = p(w) \). Define \( k_1 = k_{cc}(w_1) \) and \( k_2 = k_{cc}(w_2) \)—it must be \( k_1 > k_2 \) since it is \( w_1 > w_2 \) and \( \delta k_{cc} / \delta w > 0 \)..... It is easy to verify in the diagram that for any \( k_0 < k < k_1 \) the consumption of \( I \) will not be profitable. For some \( k > k_h \) both goods can be produced and \( C \) is the capital intensive good. But also for some \( k > k_h \) both goods can be produced although now \( I \) has become the capital intensive good. Since the \( h \) at which the factor intensity reversal takes place must be in between \( k_h \) and \( k_0 \) it is clear that as the capital stock grows, the economy must be specialized in the production of the good which was previously capital intensive at the moment the factor intensity reversal takes place. Define: \( k_{1w} = k_{cc}(w_0) \) and \( k_{2w} = k_{cc}(w_1) \). Then, the pattern of specialization which follows as \( k \) changes—given the terms of trade \( p_0 \)—is as depicted in Table 1: Going back to Figure 1, we

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*Under competition and constant returns to scale production functions of the form \( f_I(k) = f_I(k) = f_I(C) \) it is \( w = f_I(k)/f_I(k) = k_I \) from where it follows that \( k_I = k_I(w) \) and in \( k_I Qw > 0 \) provided \( f_I > 0 \) and \( f_I^2 < 0 \) (this guarantees positive and diminishing marginal product).
TABLE 1

<table>
<thead>
<tr>
<th>Capital-labor Ratio</th>
<th>Goods Produced</th>
<th>Labor Intensive Good</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k &lt; k_0$</td>
<td>$i$</td>
<td>$i$</td>
</tr>
<tr>
<td>$k_0 &lt; k &lt; k_1$</td>
<td>$i,C$</td>
<td>$i$</td>
</tr>
<tr>
<td>$k_1 &lt; k &lt; k_2$</td>
<td>$C,1$</td>
<td>$C$</td>
</tr>
<tr>
<td>$k &gt; k_2$</td>
<td>$i$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

This table shows the different scenarios for capital-labor ratios and their implications on goods produced and labor-intensive goods. The table helps in understanding the relationship between capital-labor ratios and their effects on economic output and productivity.

### The Uniqueness of Long Run Equilibrium

Given the terms of trade, long-run equilibrium is attained at the capital-labor ratio for which savings (the flow demand for investment) is exactly sufficient to provide for the depreciation of the capital-labor ratio. The rate of depreciation equals the sum of the percentage rate of depreciation of capital plus the percent-

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**Figure 3** The dashed lines have a slope $-\gamma$. The path followed as $k$ grows is $0 \rightarrow R_1 \rightarrow R_2 \rightarrow R_3 \rightarrow$.

**Figure 4** All dashed lines have a slope $-\gamma$. 

age rate of population growth. To characterize the steady state of the economy it is convenient to define the following two loci:

I. The set of attainable points in the \((I, C)\) plane where the overall capital-labor ratio would remain stationary—the \(OI\) locus.

II. The set of points in the \((I, C)\) plane which are consistent with the demand side of the economy—the \(OS\) locus.

Long run equilibrium is attained at the intersection of the \(OI\) and \(OS\) loci. These loci were developed by H. G. Johnson (1971) and were further elaborated by T. Bertrand (1973) (both authors assumed that factor intensities do not reverse). Bertrand finds that the \(OI\) locus is a straight line in the non-specialization region and is strictly concave with respect to the \(x\)-axis whenever the economy is specialized in either \(I\) or \(C\). He also shows that this locus starts at \(C = 0\), and that it hits the \(x\)-axis when the economy is specialized in \(I\). Since factor intensity reversals only multiply the number of specialization regions, it follows that the \(OI\) locus will maintain the property of concavity in those regions and that the number of linear segments contained in that locus will equal the number of sets of relative factor intensities. The shape of the \(OI\) locus, following the specialization pattern described in Table 1 is depicted in Figure 4 where it is assumed that the equality between the interest rate and \(n\) occurs for \(k > k^*\). (*

On the demand side, the \(OS\) locus can be obtained from the savings behaviour:

\[ I = s (y_k C + I) \]

or

\[ I = y_k C (1 - s) \]

which represents a straight line through the origin. The fact that the \(OS\) locus is a straight line through the origin in conjunction with the properties of the \(OI\) locus guarantees the existence of a unique intersection between both loci. Thus, the existence of a unique, finite, long run steady state capital-labor ratio is also guaranteed in the presence of factor intensity reversals.

Factor Intensity Reversals and Comparative Advantage

Figure 5 illustrates, on the basis of the specialization pattern set up in Table 1, the possible changes in comparative advantage as the capital-labor ratio grows. At any moment, consumption and investment are attained at the intersection of the prevailing budget line with the \(OS\) locus. Output at any moment is attained at the intersection of the prevailing budget line with the Rybczynski line. The pattern of comparative advantage which emerges as the capital-labor ratio grows is as follows:

<table>
<thead>
<tr>
<th>Production in Region</th>
<th>Produced:</th>
<th>Exported:</th>
<th>Good Labor Intensive Good</th>
</tr>
</thead>
<tbody>
<tr>
<td>OA</td>
<td>I</td>
<td>I</td>
<td>—</td>
</tr>
<tr>
<td>AB</td>
<td>I C</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>BC</td>
<td>I C</td>
<td>C</td>
<td>I</td>
</tr>
<tr>
<td>CD</td>
<td>C</td>
<td>C</td>
<td>—</td>
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<tr>
<td>DE</td>
<td>C I</td>
<td>C</td>
<td>—</td>
</tr>
<tr>
<td>EF</td>
<td>C I</td>
<td>I</td>
<td>C</td>
</tr>
<tr>
<td>F*</td>
<td>I</td>
<td>I</td>
<td>—</td>
</tr>
</tbody>
</table>

It is clear from the above that when factor intensities reverse there can be up to two changes in comparative advantage. It should be clear to the reader that adding one more factor intensity reversal will add the possibility of one more change in comparative advantage. Thus, it is concluded that the maximum possible number of changes in comparative advantage is equal to one plus the number of factor intensity reversals which may take place.

References


