Summary and Conclusions

The problem addressed here is that of determining the appropriate basis for including a monetary asset balance in the money stock. It has been shown that the traditional composition of the nonbank public is based in part upon an accounting convention which is inapplicable to the question of money stock construction.

The suggested alternate rule proposed here limits the money supply to those balances whose size and disposition are functions of market forces. The quantitative importance of the changes suggested above is difficult to establish. Legally required cash balances of thrift institutions generally cannot be directly estimated from available data, and currency holdings of government agencies are unavailable.

The only readily available figures are those for excess reserves of commercial banks. Using FDMC call reports and a Treasury compilation of state reserve requirements, this component was estimated to be $5.12 billion in 1973 or about two percent of money narrowly defined. A thorough evaluation of the quantitative importance of this change requires a time series on effective cash reserve requirements imposed on state member banks and nonbank financial intermediaries, a task beyond the scope of the present paper.

This, however, neither diminishes the logic of the argument nor obviates the need for altering existing definitions. In this regard it is worth repeating that there is no final answer to the empirical measure of money; that the measure must be altered periodically to account for changes in the institutional setting; that the only way to insure the appropriateness of any measure of money is to see to it that it is based on a conceptual definition consistent with general economic theory.

Bibliography


On Stability With A Foreign Exchange Reserve Target For Monetary Policy

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Introduction

In the literature of the policy mix for an open economy under a fixed exchange rate, the target variable for monetary policy, when it is directed to the external goal, has been some desired magnitude of the balance of payments, usually a "zero" balance of payments. Mundell (22), Fleming (7), Whitman (31). In the literature of the reaction function of the monetary authority or of endogenous monetary policy, some desired value of the balance of payments is considered to be one of the target variables to which monetary policy instruments are applied. Dewald and H. G. Johnson (4), Harrity (14), Phreyen (11).

It has been pointed out repeatedly, however, that the target variable of policy directed to the external goal should be the "stock" level of foreign exchange reserves rather than a particular value of the rate of change of it in a non-growing, non-inflationary, open economy with a fixed exchange rate. This argument is based on the following two conceptually distinct but related reasons. First, balance of payments deficit or surplus per se, does not cause trouble or comfort. It is the level of foreign reserves from which services come, and it is the low level of such reserve from which discomfort arises. Mundell (22), Neihans (25). Second, any balance of payments deficit or surplus automatically tends to correct itself. In a non-growing, non-inflationary economy, the balance of payments goes automatically to zero value as long as the economic system has a stable stationary state. Aghiveil and Borts (1), Schweda (29), Whitman (31). Unless the authority is not indifferent about the level of reserves when the balance of payments has the equilibrium value of zero, or unless it is not indifferent about the time path of the balance of payments which goes to zero value in equilibrium, there is no clear reason why it should utilize monetary policy for stabilization of the external balance.

The model in this paper adapts the portfolio balance approach and emphasizes the stock adjustment nature of the balance of payments adjustment process. McKinnon (20), Jones (17), Frenkel and Rodrigo (10), Dombusch (6), Allen (2). We will introduce the external "stock" balance and the internal "stock" balance into the model. Monetary policy is assumed to be directed to achieving external stock balance, i.e. equality between the actual level of foreign reserves to the hands of the monetary authority and the long-run desired level of such reserves. External balance is defined in terms of the stock but not in terms of the current flow. A zero balance of payments is simply one particular point of flow equilibrium in the money market, such equilibrium occurring when the domestic flow supply of money is just absorbed by the private sector. The internal stock balance, which involves equality between the long-run desired level of wealth and the actual level of wealth, is assumed to be attained by the market response in the private sector. The internal stock balance implies equality between income and expenditure, not just flow equilibrium in the
good market. The specifications of, and interaction between the dynamic behavior of the monetary authority and the private sector determine the nature of the adjustment process to the long-run stationary state.

A dynamic model of an open economy which contains the interaction between monetary policy reaction and market response was developed by R. Mundell (22) in terms of flow definitions of the external and internal balances. The results of the model developed here will be compared with the results of his model to highlight the difference in the two approaches.

Framework of the Model

(1) There is one homogeneous good which is tradable but not storable. The economy is assumed to be fully employed and non-growing. The rest of the world is neither growing nor inflationary. For a small open economy under a fixed exchange rate, the domestic price level is given from the rest of the world and is equal to the product of the fixed exchange rate and the world price level. The units of the traded good and foreign exchange are so chosen as to set the domestic price level equal to unity.

(2) There are three paper assets: money, foreign bonds $BF$, and domestic bonds $Bd$. These three assets are assumed to be gross substitutes in the private portfolio.

(3) The foreign bond is assumed to be internationally traded and is in infinitely elastic supply at the world rate of interest, due to the small country assumption.

(4) Money is issued by the monetary authority in exchange for the domestic bond or foreign exchange. It is assumed that open market operations are conducted only through the domestic bond market. Hence the money supply will change only through surplus or deficit in the balance of payments and through open market operations. The money multiplier is assumed to be unity for simplicity. The total money supply can be written as,

\[ M = R + Ba \]

where $R$ is foreign exchange reserves and $Ba$ is domestic bonds in the hands of the monetary authority. We can alternatively interpret $R$ as the part of the total money supply based on foreign reserves, and $Ba$ as the part of the total money supply based on domestic bonds. Although the private sector is indifferent between backing of $M$ by $R$ or $Ba$, the monetary authority is not, since only $R$ is the money acceptable to the rest of the world.

(5) The domestic bond, $Bd$, which exists as the result of past deficits in the government budget, is assumed to be non-traded. This assumption means that the government cannot continually finance a deficit in the current account by printing and selling bonds. The domestic bond is assumed to be a fixed-price variable-coupon type, i.e., a short-term bond or bill. Hence its price is not affected by a change in interest rate on it. This assumption will be modified in Appendix (1). It is also assumed that the private sector does not discount or capitalize taxes and transfers. Hence the domestic bond is part of the private sector's net wealth.

(6) The government is assumed to keep its budget in balance during the period in question.

\[ T - G - (Bd) = 0 \]

Since an open market operation exchanges interest-bearing government debt for non-interest-bearing government debt, an open market purchase (sale) will cause a budget surplus (deficit) if the budget is initially balanced. We assume that taxes are reduced (raised) so as to keep the budget in balance and also to keep disposable income constant. Since tax changes are assumed to be not capitalized, this operation keeps the total rate of net private wealth intact.

(7) Total net private wealth, $W$, can be written as,

\[ W = R + Ba + (Bd - Bd') + Bf' = M + Bd + Bf' \]

where $Bd$ is total domestic bonds outstanding in the economy and $Bf'$ is total domestic bonds in the private sector.

Since the government does not issue new bond and since open market operations do not change total wealth, total wealth can be changed only when $(R + Bf')$ is changed through surplus or deficit in the current account. It will prove convenient to describe $(R + Bf')$ as "foreign assets," $Wf$. Then, we can write $W$ as, $W = Wf - Bd'$. Finally, it is assumed that expectations about the price of bonds are static in the sense that people expect whatever price of bonds occurs to continue forever.

Short-Run Stock Equilibrium

(1) The short-run is defined as a given $W$ and $Bd'$. We assume that reallocation of the composition of $W$ is costless and is done instantaneously. Hence there is always short-run stock equilibrium for each asset at each point in time, given the total size of $W$. Since at each point in time $W$ and $Bd$ are given from past asset accumulation and deficits in the government budget, $Wf$ is also given. What is not given at each point in time is the composition of $Wf$, that is, $R$ and $Bf'$ separately are not given from the past.

The short-run demand for assets, given $Wf$ and the world interest rate $i^*$ can be postulated as,

\[ M^* = m(i^*)Wf \]

\[ l^* = \frac{m^*}{M^*} < 0 \]

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(1) For the analysis of balance of payments based on this framework, see Mundell (22) and Johnson (16).

\[ M^* = m(i^*)Wf \]

\[ l^* = \frac{m^*}{M^*} < 0 \]

\[ l^* = \frac{m^*}{M^*} < 0 \]

(2) For the specification of short-run asset demand, see Tobin (20). Feeny and Sbokrovsky (19).
(3) The short-run effects of open market purchases and sales can be studied by means of Figure (1). Open market sales raising private holdings of the domestic bonds to $B_M$ raise $i$ up to $i_1$ in such a way that the increased stock of privately available bonds is willingly held by the private sector. Hence, the $B_M$-$B_h$ locus shifts up. The $M$-$M$ locus also shifts up since with given $R$ the reduction of $B_h$ causes an excess demand for money which must be matched by a higher $i$ so as to clear the money market. The $B_h$-$B_h$ locus stays the same since neither $B_h$ nor $B_M$ has been changed by this open market sale. Hence the new equilibrium point, $E_1$, is on the $B_h$-$B_h$ locus. At $E_1$, $i$ and also $K$ are higher than at $i_1$ and $R$ of the original equilibrium. Open market sales raise the level of reserves, $R$, although supply of and demand for money have been reduced. This unambiguous result comes from the fact that at this point in time $W'$ is constant; $B_h$ can be changed only through surplus or deficit of the current account over a period of time. Hence, with given $W'$, $R$ must be raised, since $B_h$ has been unambiguously reduced due to the higher $i$.

Algebraically, these effects can be written as:

$$\frac{dW'}{dB_h} \bigg|_{OMO} = \frac{1}{W_1} > 0$$  \hspace{1cm} (8)

$$\frac{dR}{dB_h} \bigg|_{OMO} = q_1 < 0$$  \hspace{1cm} (9)

$$\frac{dR}{W'} \bigg|_{OMO} = 1 + \frac{1}{\sigma_1} > 0$$  \hspace{1cm} (10)

A positive sign of $\frac{dW'}{dB_h} \bigg|_{OMO}$ implies that $X$ rises as the domestic bonds in the private sector, $B_h$, rises keeping $(M + B_h)$ and $W'$ constant; i.e., an open market sale of $B_h$ raises $X$. A negative sign of $\frac{dR}{dB_h} \bigg|_{OMO}$ implies that an open market sale of $B_h$ reduces $X$. It can be seen that equation (9) and (10) add up to zero. The effect of open market sales is the same as if the private sector switched $B_h$ with $R$ instantaneously.

If capital is perfectly internationally mobile in the sense that the domestic and the foreign bonds are perfect substitutes in the private portfolio, the equilibrium loci of the $B_h$ market and $B_f$ market, the $B_h$-$B_h$ locus and the $B_f$-$B_f$ locus, coincide and they become a horizontal line at the world rate of interest, $R_w$. In this case, $R$ will flow in by the same amount of open market sales and there will be no change in $i$. Open market sales are "as if" done with the rest of the world.

(4) The short-run effects of a change in $W'$ and $B_h$ keeping $B_f$ constant, are shown in Figure (2). Since $W'$ can change only through a change in $W_f$, the change in $W'$ must be equal to the change in $W_f$.

An increase in $W'$ reduces $i$ since the demand for $B_h$ is raised and the supply is given. Hence the $B_h$-$B_f$ locus shifts down to $B_f$-$B_f$. The $M$-$M$ locus shifts to the right since with given $R$ and $B_h$ a rise in $W'$ causes an excess demand for $M$ which must be offset by a higher $i$. The new equilibrium point is at $E_2$, through which $B_f$-$B_f$ must pass. Clearly $R$ and $B_f$ are increased since both a reduction of $i$ and an increase in $W'$ raise the demand for money and foreign bonds.

Algebraically, these results can be written as:

$$\frac{dW'}{dB_h} \bigg|_{OMO} = \frac{n}{W_1} < 0$$  \hspace{1cm} (11)

$$\frac{dB_f}{dB_h} \bigg|_{OMO} = \frac{q - n}{\sigma_2} > 0$$  \hspace{1cm} (12)

$$\frac{dR}{dB_h} \bigg|_{OMO} = -1 + \frac{1}{\sigma_1} > 0$$  \hspace{1cm} (13)

Equation (12) and (13) must add up to unity since a change in $B_f$ must come from changes in $B_h$ and/or $R$.

If capital is perfectly internationally mobile, an increase in $W'$ only raises $R$ keeping $i$ constant. The rise in $R$ in this case would smaller than that in the case of gross-substitute bonds since there is no substitution effect caused by a lower $i$. 
Short-Run Flow Equilibrium

(1) The saving function. Along with decisions about the allocation of assets and determination of \( i, R \) and \( \bar{B}_f \), the private sector and the monetary authority are assumed to make flow decisions at each point in time. The private sector decides the rate of accumulation of \( W \) through the saving decisions. Saving is assumed to involve the adjustment of actual wealth to long-run desired wealth, \( W^* \), over time. The non-continuous nature of adjustment is explained by the existence of the cost of adjustment; foregone consumption in this case. The increasing marginal cost of foregone consumption together with either a constant or decreasing marginal return of filling the gap between actual and the desired level of wealth determine a desired rate of saving at each point in time. The particular functional form postulated here is that of partial adjustment.\(^4\)

\[
S = \bar{W} = \phi(W^* - W) \quad 0 < \phi < \infty \tag{14}
\]

where \( \phi \) is the speed of adjustment with dimension of 1/time. \( W^* \) is the long-run desired level of wealth planned at a point in time and is assumed to be a positive function of the domestic interest rate, \( i \), the world interest rate, \( i^* \) and disposable income, \( Y_d \).

\[
W^* = W^*(i, i^*, Y_d) \quad W^i > 0, W^y > 0, W^i > 0 \tag{15}
\]

Saving is clearly increasing function of \( i, i^* \) and \( Y_d \), giving the total size of initial wealth, \( W^* \).

Short-run equilibrium saving is determined by substituting the short-run equilibrium domestic interest rate, \( \bar{i} \), into (14).

\[
\bar{S} = \phi(W^* - \bar{W}) = \phi \bar{i} \tag{16}
\]

In a reduced form, we can also write,

\[
\bar{S} = \phi \bar{B}_f, \bar{W} \quad \bar{S}_i > 0, \bar{S}_W < 0 \tag{16}'
\]

Since an increase in \( \bar{B}_f \) through open market sales, keeping \( \bar{W} \) constant, will raise \( \bar{S} \), it will raise \( \bar{i} \). An increase in \( \bar{W} \), keeping \( \bar{B}_f \) constant, will reduce \( i \) and raise \( \bar{W} \); hence it will reduce \( \bar{i} \). Algebraically, the effects of an open market operation and of a change in \( \bar{W} \) on the short-run equilibrium rate of saving can be seen as,

\[
\frac{d\bar{S}}{d\bar{W}_f} \bigg|_{\bar{W}_f = \text{const.}} = \phi \frac{\bar{W}_f}{\bar{W}_f} > 0 \tag{17}
\]

\[
\frac{d\bar{S}}{d\bar{W}_f} \bigg|_{\bar{W}_f = \text{const.}} = \phi \frac{\bar{W}_f}{\bar{W}_f} + 1 < 0 \tag{18}
\]

(2) The monetary authority’s reaction function. The monetary authority decides the rate of open market purchases or sales so as to adjust the actual level of foreign reserves to the long-run desired level, \( R^* \), over time. The particular functional form postulated is,

\[
\bar{B}_f = \alpha(R^* - R) \quad 0 < \alpha < \infty \tag{19}
\]

where \( \alpha \) is the speed of adjustment with dimension of 1/time. \( R^* \) is the long-run desired level of foreign reserves planned at each point in time by the authority. \( R^* \) might be proportional to the domestic source money \( B_s \) as in the case of the rule of the gold standard system. It might be a rising function of variability of balance of payments, or a function of degree of openness of the economy.\(^5\) It might depend on the world rate of interest rate \( i^* \) similarly to the holding of real cash balances in a private portfolio. Whatever the reason, these factors can be assumed to result in a constant \( R^* \) in our simple model.

This partial adjustment type policy reaction function can be explained by the existence of the cost of adjustment of changing policy instruments. The increasing marginal cost of quicker adjustment of policy instruments together with either a constant or a decreasing marginal return of filling the gap between \( R^* \) and

\(^4\) For the arguments along this line, see McFadden (21) and Jones (17).

\(^5\) For the arguments along this line, see Heise (15), Kelly (18) and Frankel (9).
R determine a desired rate of open market purchases or sales at each point in time. The short-run equilibrium rate of open market operation is determined by substituting the short-run equilibrium level of foreign reserves, \( \tilde{R} \), into (19),

\[
\tilde{R} = a(R^* - R) = \tilde{b}h(B, \tilde{W}) \tilde{R} \leq 0, \quad \tilde{b}h < 0 \tag{20}
\]

An open market sale, keeping \( \tilde{W} \) constant, raises \( \tilde{R} \), hence it reduces \( \tilde{b}h \). An increase in \( \tilde{W} \), keeping \( \tilde{b}h \) constant, raises \( \tilde{R} \), hence it also reduces \( \tilde{b}h \).

Algebraically,

\[
\frac{d\tilde{b}h}{dbh} \bigg|_{\text{cost}} = -a \left( \frac{\frac{1}{n_1}}{\frac{1}{n_1}} \right) < 0 \tag{21}
\]

\[
\frac{d\tilde{b}h}{dW} \bigg|_{\text{in cont.}} = -a \left( -\frac{\frac{1}{n_1}}{\frac{1}{n_1}} \right) < 0 \tag{22}
\]

(3) The short-run flow equilibrium condition in the goods market can be written,

\[
Y = C + X - I + M + G \tag{23}
\]

where \( Y \) is total output, \( C \) is consumption, \((X - I)\) is the trade balance and \( G \) is government consumption. From the budget constraint of the private sector, we have

\[
Y + i^*f + \tilde{b}h - \tilde{R} = C + S \tag{24}
\]

Substituting (24) into (23) and using the assumption of a balanced government budget, we have

\[
S = X - I - \tilde{b}h + i^*f \tag{25}
\]

It can be seen that the flow equilibrium condition in the goods market is identical with the flow equilibrium condition for total wealth. Since total wealth can only be changed by surpluses or deficits in current account, we can rewrite Equation (25) as

\[
\tilde{W} = \tilde{R} = X - I + X + \tilde{b}h = \tilde{R} = \tilde{W} \tag{26}
\]

Short-run equilibrium saving is equal to the trade account surplus plus service account surplus, which is by definition the surplus on the current account. The surplus on current account must be equal to the increase in foreign assets. Hence Equation (26) determines the short-run equilibrium current account, which is identical with the short-run equilibrium rate of increase in foreign assets.

The short-run flow equilibrium condition for the domestic bond market can be written as,

\[
\tilde{b}h = a \left( \frac{\frac{1}{n_2}}{\frac{1}{n_2}} \right) \tilde{W} = \tilde{b}h \tilde{W} \tag{27}
\]

Since the domestic bond, \( \tilde{b}h \), is non-traded, the flow market for \( \tilde{b}h \) has to be cleared domestically. Subtracting the short-run equilibrium saving \( \tilde{S} = \tilde{W} \) into (27), we have the equilibrium rate of change of \( \tilde{W} \),

\[
\frac{d\tilde{W}}{dt} = \tilde{b}h - \tilde{h}w \times \tilde{R} \tag{28}
\]

We have specified, so far, the private flow demand for total wealth and the authority’s flow supply of domestic bonds. But we have not specified the private flow demand for each asset separately. This is because there is only the private flow demand for total wealth but there is no flow demand for each asset separately. This independency of individual flow demands is due to our assumption of costless reallocation to achieve portfolio balance. Since \( \tilde{b}h \) is costless for each individual to change the composition of his total wealth at each point in time, he will be indifferent about which asset he wants to accumulate at a point in time. He can get the desired portfolio composition in the next instant no matter which asset he accumulates now. This independency of flow demand for each asset separately, however, does not mean independency of the time path of accumulation of each asset. Since we assumed that portfolios are always in equilibrium at each point in time, the time path of wealth \( \tilde{W} \), i.e., saving, and the time path of the short-run equilibrium interest rate \( i^* \) will completely determine the time paths of \( \tilde{b}h \) and \( \tilde{W} \).

\[
\tilde{b}h = \tilde{b}h \tilde{W} \tag{29}
\]

The equilibrium rate of \( \tilde{b}h \) is the equilibrium capital account. \( \tilde{R} \) is the equilibrium time path of \( \tilde{R} \), which is the equilibrium balance of payments. The equilibrium balance of payments is not necessarily a zero balance of payments. The balance of payments is equal to zero only when the desired speed of open market purchase \( \tilde{b}h \) happens to be equal to the rate of accumulation of real each balance in the private sector. "Equilibrium" implies only that these magnitudes are determined along the short-run equilibrium time path of the economy.

Short-run flow equilibrium, given \( \tilde{W} \), is depicted in Figure (3), together with the determination of the short-run equilibrium interest rate, \( \tilde{i} \). It should be noted that in Figure (3) saving is positive and open market sales are being conducted. Hence total wealth is accumulating and the balance of payments is in surplus. The capital account may be either in surplus or deficit since open market sales and wealth accumulation affect it in opposite directions.

**Long-Run Equilibrium and Policy Dynamics**

(1) The long-run equilibrium is reached when both internal and the external stock balance are attained. In the private sector, the actual level of wealth is equal to the desired level of \( I_t \) and saving is zero. For the monetary authority, the actual level of foreign reserves is equal to the desired level and open market operations will cease. In this long-run equilibrium, all the stocks are constant and all of the constant income is spent. The balance of payments, the capital account and current account are all equal to zero. The trade account deficit is offset by a surplus in the service account due to the holding of foreign bonds.

(2) The schedule of internal stock balance, \( XX \), that is, \( X^* = 0 \) locus, can be shown in \( BB-WF \) space. If its slope is positive, since an increase in \( BB \) will raise \( R \) and hence raise \( S \), to reduce \( S \) to zero again, \( BB \) has to be reduced or \( W \) has to be raised. This requires higher \( WF \). Algebraically, the slope can be written as,

\[
\frac{dBB}{dWF} \bigg|_{E=0} = \frac{\tilde{b}h \tilde{R} + \tilde{h} \tilde{W} - \tilde{R}}{\tilde{W} \tilde{W}} > 0 \tag{31}
\]

If \( BB \) and \( WF \) are perfect substitutes, \( WF \) has to be equal to \( P \) and there will be no change in \( P \) or \( S \). Hence \( XX \) must be vertical in the case of perfect substitution between domestic and foreign bonds. This can be easily seen from Equation (31). When \( n_1 = \bar{n}_1 = \bar{m} \) becomes infinite, the solution to Equation (31) becomes infinite. In this case open market purchases or sales do not affect the saving decision of the private sector.

The schedule of the external stock balance, \( FF \), that is, \( BB = 0 \) locus, also can be shown in \( BB-WF \) space. Its slope is negative. Since an increase in \( BB \) raises \( \tilde{R} \) and hence makes \( BB \) negative, to keep \( BB = 0 \), \( WF \) and hence \( W \) have to be reduced in such a way that \( BB \) is unchanged. Algebraically, its slope is,

\[
\frac{dWF}{dBh} \bigg|_{Bh=0} = \frac{\bar{n}_1 \bar{m}_1}{\bar{m}_1 + \bar{m} + n_1} \geq 0 \tag{32}
\]

If \( BB \) and \( WF \) are perfect substitutes, i.e., \( n_1 = \bar{n}_1 \), the slope becomes \( -1 \).
(3) The relevant phase diagram is shown in Figure 4. To the right of the XX locus, $R^*$ and hence $W$ are too high, and this implies decumulation of wealth. To the left of it, the story is the opposite. Above the FF locus, $B_r$ is too high and hence $B_r$ is too low. This causes $R > R^*$, given $W$, and hence implies declining $B_r$. Below FF, the story is the opposite. Figure (4a) is for the case when $B_r$ and $B_f$ are gross substitutes; Figure (4b) is for the case when $B_r$ and $B_f$ are perfect substitutes. ($\downarrow$) denotes long-run equilibrium value.

In both cases the system is stable for "any" speed of adjustment. (Appendix II) Contrary to the case of the flow model of Mundell (22), a monetary policy directed to the stock of reserves does not make the system unstable for any speed of adjustment.

The instability of Mundell’s model is due to his insufficient distinction between stock and flow in his treatment of capital flows. Although he explicitly treated the accumulated effect of past surpluses or deficits on the balance of payments on the current stock of reserves, he neglected the accumulated effect of past capital inflows or outflows on the current interest rate. In other words, for one component of external transactions he assumed that past flows affect the current level of the stock, but for the other component of it he ignored this effect.

If capital is highly internationally mobile, a small rise in the domestic interest rate will cause a large “vicious for all” capital inflow and the domestic interest rate will bounce back to the original level. We may not even observe any significant interest rate rise in this process of capital flow. It is not a “high” level of $T$ relative to $T^*$ that causes continuous capital inflow, but a continuously “rising” level of $T$.

A once for all change in the interest rate causes only a once for all “stock shift,” i.e., reshuffling of the composition of the portfolio, but it does not cause continuous capital flow unless that interest rate change affects the saving decision. A continuous capital flow will occur only if a continuously changing interest rate causes continuous reshuffling of the portfolio, or the total size of the portfolio itself is changing due to saving.

Since Mundell assumed that high $T$ relative to $T^*$ causes continuous capital inflow, there must be some value of $(T^* - T^*)$ which is large enough to push $R$ up to $R^*$. When $R = R^*$ is attained, capital will still flow in, since $(T^* - T^*)$ is assumed to be unchanged. In the process of reducing this differential, capital is assumed to flow in, and this makes $R$ overshoot $R^*$. This mechanism causes undamped cyclicity in his system.

In our model, capital flow is treated as a stock adjustment phenomenon and also as a flow phenomenon. When $T$ is changed by open market operations, this will cause a stock shift effect of a capital flow through a reshuffling of the composition of $W$, and this will also cause a flow effect, a capital flow through a change in the total size of $W$. In the long-run equilibrium, our model has $R = R^*$ with zero open market operations, and $W = W^*$ with zero saving. Therefore there will be no stock shift or flow effect of capital flow. This mechanism gives the stability to our model.

(4) When bonds are gross substitutes, the time path may be cyclical, but when bonds are perfect substitutes the time path becomes direct. Perfect substitutability of bonds implies that as long as the stock of foreign bonds in the private sector is positive, capital is deemed to be perfectly mobile. This result is similar to that of Mundell. In his article, he says, “Under fixed exchange rates and virtually perfect capital mobility, the approach to equilibrium is direct rather than cyclical.” Our model provides the same result in the context of a portfolio balance approach. When an open market sale is conducted, the private sector will shift out $B_f$ by the same amount as the increase in $B_r$, and obtain $R$ by the same amount. This increased $R$  

See, on similar arguments in the context of growth, Brunner (9) and Ford (8).
will be bought by the authority in exchange for domestic money. There will be no change in W or \( V \) and \( BF \) is simply replaced by \( R \) from the point of view of the economy as a whole.

(5) In order to see how the model works over time and how the assumption of substitutability of bonds plays a crucial role, suppose that the desired level of reserves \( R^* \) is raised. The XX locus stays the same and the FF locus shifts to the right. Given \( B_B \) and \( B_A \), a higher \( R \) requires a lower \( i \) and/or a higher \( W \). Since a rise in \( BF \), keeping \( B_B \) constant, reduces \( i \) and raises \( W \), the FF locus shifts to the right. When the bonds are gross substitutes, \( B_B \), \( BF \) and \( W \) will be higher at the new equilibrium, \( R^* \). The effect on \( BF \) is ambiguous since \( W \) is higher but \( i \) is also higher. (Figure 5) Initially the authority undertakes an open market sale. This will raise \( i \) and hence \( W^* \). This rise in \( W^* \), in turn, induces saving, and hence \( BF \) and \( W^* \) start to accumulate. The actual \( R \) hits \( R^* \) at \( A \) and subsequently overshoots it. It will not stay in such a position since \( W^* \) is still accumulating at \( A \). The direction of open market operations will be switched from sale to purchase. Then, \( W \) eventually hits \( W^* \) at \( B \) and subsequently overshoots it. It will not stay in such a position since \( W^* \) is still changing due to changing \( i \). People will switch from saving to dissaving. This cyclical path converges towards the long-run equilibrium. The cyclicity is not, of course, necessary; it depends on the speeds of adjustment and other parameters.

When the bonds are perfect substitutes, it is obvious that there is neither cyclicity nor change in wealth. The increase in \( BB \) reduces \( BF \), i.e., causes capital inflow, and raises \( R \) by the same magnitude while \( W \) and \( BF \) remain constant. Monetary policy is completely offset by capital flow which keeps the private sector unchanged. The open market sale is "as if" conducted with the rest of the world. (Figure 5b).

An Alternative Assumption Concerning Domestic Bonds; Inside Bonds

The assumption of zero discount or capitalization of taxes or transfers is dropped in this section. It is assumed that the private sector discounts part of its tax liability. Let \( 100r \) be the percentage of this tax discount. Hence the definition of net private wealth is,

\[
W = R + B_B + (1 - r)(B_B - B_B) + BF
\]

\[
= M + (1 - r)B_B + BF
\]

\[
= BF + Bd - dB_R
\]

(34)

The total value of the domestic bonds in the private sector, \((1 - r)B_B\), and the value of net private wealth, \(W\), are not independent with respect to an open market operation. However, it can be shown that the short-run effects of open market purchases or sales on \( i \), \( BF \) and \( R \) do not qualitatively differ from those in the previous case. An open market sale raises \( i \) and reduces \( BF \) by \( dB_R \). This implies \( BF \) is reduced. The reduction of \( BF \) implies \( R \) is unambiguously increased. Although a higher \( i \) and a lower \( W \) reduce a demand for cash balance, the constancy of \( BF \) at each point in time again gives us this unambiguously result. Algebraically,

\[
\frac{dB}{dB_R} = \frac{1 - r(1 - n)}{W_{R1}} > 0
\]

(8)

\[
\frac{dB}{dB_R} = -iq \ast \frac{1 - r(1 - n)}{R_1} < 0
\]

(9)

\[
\frac{dR}{dB_R} = -(1 - r) + \frac{1 - r(1 - n)}{R_1} > 0
\]

(10)

The short-run effects of a change in \( BF \), keeping \( BF \) constant, on \( i \), \( BF \) and \( R \) are the same as those in the previous case. Since the short-run effect of open market purchases or sales do not differ qualitatively from the previous case and the short-run

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\(^{28}\)For the argument along this line, see Peltokas (26) and Meyer (19).
effect of a change in W are the same as those in the previous case, we have qualitatively the same reduced forms of the equilibrium rate of savings and of the equilibrium rate of open market operations as those in the previous case. We also have qualitatively the same phase diagram as in the previous case. All the results in the previous outside bond case hold in this inside bond case except the directness of the time path in the case of perfect-substitute bonds. If the domestic and the foreign bonds are perfect substitutes in private portfolio choice, an open market operation does not leave the private sector unchanged. The saving decision is affected by the operation, not because of the change in i but because of the change in W. Hence, the directness of the time path disappears even if the domestic and foreign bonds are perfect substitutes. The slope of the XX locus is not vertical but 1/r since the change in W due to an open market operation, dBB, has to be matched by dW so as to keep R constant. The slope of the FF locus is 1/(r - 1), which is still negative. Hence, the qualitative nature of the time path to the long-run equilibrium in the perfect-substitute bonds case is the same as that in the case of gross-substitute bonds in spite of the constancy of i.

In general, the directness of time path to the long-run equilibrium of the fixed exchange rate system in a world of perfect capital mobility disappears wherever open market operations change saving decisions, through changes in i, W, or any other variables. This directness, which is considered an advantage of the fixed exchange rate system in a world of high capital mobility, crucially depends on the independency of the saving decision in the private sector with respect to open market operations.

Concluding Remarks

This paper has presented a portfolio balance model of an open economy whose monetary authority conducts active stabilization policy respecting the level of foreign reserves. The model was so constructed as to shed light on the exact channels through which monetary policy affects a short-run reshuffling of the wealth portfolio; on the flow decision respecting saving; and on the long-run equilibrium of the stationary state. It heftily utilized the distinction between short-run stock reshuffling given the total size of wealth, and the long-run stock adjustment process over time toward the stationary state. It also relied on the assumption of gross-substitute paper assets and non-tradability of domestic bonds.

Some specific outcomes of the model which are worth noting are as follows. First, although the domestic bond is non-traded, an open market sale of domestic bonds will instantly raise the level of reserves. The private sector will shift foreign bond out and get foreign reserves instantaneously so as to have an optimal composition of its portfolio. This result holds even if the open market sale causes capital loss and the reduction of net private wealth. Second, an open market sale raises the domestic interest rate and hence saving. This rise in saving causes a flow demand for total wealth and hence increases the inflow of reserves. This additional flow effect due to the change in saving tends to produce an overshoot of the level of reserves beyond R*.

Third, even if the authority is concerned only with the level of the reserves, the system is not usable for any speed of adjustment. Fourth, the advantage of a fixed exchange rate system in a world of perfect capital mobility in terms of the directness of the path to the long-run equilibrium is shown to depend on the independence of the saving decision with respect to monetary policy. If the monetary policy affects saving either directly through the size of wealth or indirectly through the interest rate, this directness disappears.

Since in some countries, such as Japan, monetary policies have been used systematically to ease problems of external balance, it is clear that we need a model which contains a policy
reaction function as a crucial part of the model rather than one which assumes that policy is parameterically given, in order to study the exact mechanism of the adjustment process applying to the various accounts of the balance of payments over time.

(Appendix I)

We argue in this Appendix that all of the results obtained in the text will hold qualitatively if the assumption of a fixed-price variable-coupon bond, i.e. a one-period bond, is replaced with the extreme alternative assumption of a fixed-coupon variable-price bond, i.e. a consol. Contrary to the previous case in the text, the total value of domestic bonds in the private sector, \( B_d \), and the value of net private wealth, \( W \), are not given by net accumulation at each point in time. The short-run equilibrium values, \( i, B_f, R, B_h \) and \( W \) are all endogenously determined. Although open market sales (purchases) raise (reduce) the price of the domestic bonds and hence raise (reduce) the value of net private wealth, it can be shown that the short-run effects of open market sales (purchases) on \( i, B_f \) and \( R \) do not qualitatively differ from those of the previous case in the text. An open market sale raises \( i \) and reduces \( W \). This implies \( B_f \) is reduced. The reduction of \( B_f \) implies \( R \) is unambiguously increased. Although a higher \( i \) and a lower \( W \) reduce a demand for cash balance, an open market sale causes excess demand for cash balance and hence induces an inflow of money. This is because \( B_f \) cannot change instantaneously even though \( W \) can. Since \( B_f \) is unambiguously reduced by an open market sale, \( R \) must be increased. Algebraically,

\[
\frac{dR}{dh} = \frac{\epsilon}{W_1 - (1-n)(\epsilon/2)} > 0
\]

we can write the fixed coupon payment on a unit of the domestic bonds and \( dh \) is a number of the domestic bonds in the private sector. The short-run effects of a change in \( W_f \), keeping \( B_f \) constant, on \( i, B_f \) and \( R \) do not qualitatively differ from those in the text. Algebraically, these effects can be written as,

\[
\frac{dW_f}{dW} = \frac{n}{W_1 + (1-n)(\epsilon/2)} < 0
\]

where \( \epsilon \) is the fixed coupon payment on a unit of the domestic bonds and \( dh \) is a number of the domestic bonds in the private sector. The short-run effects of a change in \( W_f \), keeping \( B_f \) constant, on \( i, B_f \) and \( R \) do not qualitatively differ from those in the text. The characteristic roots are,

\[
\lambda_1, \lambda_2 = \frac{1}{2} \left( \frac{1}{2} 2\sigma \right) \pm \frac{1}{2} \sqrt{\left( \frac{1}{2} 2\sigma \right)^2 - 4np}\] 

The roots are complex, they have negative real parts; and if they are real, they must be negative for any \( \alpha, \sigma > 0 \). When \( B_d \) becomes perfect substitutable with \( B_f \) we have,

\[
f_1 = -1, f_2 = -1, g_1 = 0, g_2 = -1
\]

and the discriminant becomes,

\[
(\alpha + \beta) > 0 + 4\sigma > 0
\]

(Appendix II)

Our dynamic system can be summarized as,

\[
\begin{align*}
\frac{dR}{dh} &= \frac{\epsilon}{W_1 - (1-n)(\epsilon/2)} > 0 \\
\frac{dW_f}{dW} &= \frac{n}{W_1 + (1-n)(\epsilon/2)} < 0 \\
\frac{dW}{dh} &= \frac{\epsilon}{W_1 + (1-n)(\epsilon/2)} < 0
\end{align*}
\]

Expanding these into Taylor series around the long-run equilibrium position and collecting linear terms,

\[
\frac{dR}{dh} + \frac{dW_f}{dW} + \frac{dW}{dh} = \frac{\epsilon}{W_1 + (1-n)(\epsilon/2)} > 0
\]

where \( \epsilon \) is the fixed coupon payment on a unit of the domestic bonds and \( dh \) is a number of the domestic bonds in the private sector. The short-run effects of a change in \( W_f \), keeping \( B_f \) constant, on \( i, B_f \) and \( R \) do not qualitatively differ from those in the text. Algebraically, these effects can be written as,

\[
\frac{dW_f}{dW} = \frac{n}{W_1 + (1-n)(\epsilon/2)} < 0
\]

References

Small-Disturbance and Large-Sample Approximations in Mixed Regression Estimation

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Introduction

Recently Kakwani [2] considered a regression model containing only one explanatory variable and made a numerical comparison of the variance of the exact distribution, derived by Swamy and Mehta [6], of the mixed regression estimator of the slope coefficient with the large-sample approximation obtained by Nagar and Kakwani [3]. Employing the small-disturbance approach, introduced by Kadane [1], for analyzing the properties of estimators in simultaneous equation models, we obtain the expression for the variance-covariance matrix of the small-disturbance distribution of mixed regression estimator under a general set-up and examine the appropriateness of small-disturbance approximation with the exact large-sample approximation for the special case considered by Kakwani [2].

Small-Disturbance Approximation

Let us consider the regression model

\[ y = X\beta + u \]  

where \( y \) is a column vector of \( T \) observations on the variable to be explained, \( X \) is a \( T \times \Lambda \) matrix, assumed to be of full column rank, of \( T \) observations on \( \Lambda \) explanatory variables, \( \beta \) is a column vector of \( \Lambda \) unknown coefficients, and \( u \) is a column vector of \( T \) disturbances assumed to be temporally independent and distributed with mean zero and variance \( \sigma^2 \) (unknown).

The stochastic prior information is expressible in the form

\[ r = \beta \Psi + \upsilon \]  

where \( r \) is a column vector of \( G \) known elements, \( \Psi \) is a \( G \times \Lambda \) matrix, assumed to be of full row rank, with known elements and \( \upsilon \) is a column vector of \( G \) disturbances with mean zero and variance-covariance matrix \( \Psi \) assumed to be known and nonsingular.

It is further assumed that the elements of \( \upsilon \) and \( u \) are independently distributed.

For estimating \( \beta \), Theil [7] proposed the \( f \)-class of estimators:

\[ \hat{\beta} = (X'X + R'\Psi^{-1}R)^{-1} (X'y + R'\Psi^{-1}\upsilon) \]  

where \( f \) is the characterizing scalar.

Nagar and Kakwani [3] considered the estimator (2.3) with \( f = \frac{1}{\Lambda} \) so that the resulting estimator is

\[ \hat{\beta} = \left( \frac{1}{\Lambda} X'X + R'\Psi^{-1}R \right)^{-1} \left( \frac{1}{\Lambda} X'y + R'\Psi^{-1}\upsilon \right) \]  

(2.4)

where

\[ \hat{\sigma}^2 = \frac{1}{(T - \Lambda)} y'My \left[ M = I - X(X'X)^+X' \right] \]  

(2.5)