THE TO-BE-EXPECTED ANGST CREATED FOR ECONOMISTS BY MATHEMATICS

Paul A. Samuelson

Massachusetts Institute of Technology

Professors Quddus and Rashid [1994] have provided useful samples that confirm what I predicted more than four decades ago — that the virus of mathematics would spread in economics and cause grave psychological discomfort in those scholars who lag behind the external-margin frontier of its extreme cultivation.

The authors present a selection of quotations from writers of eminence who by and large have themselves pioneered in the use of mathematics in economics. Yet *even they* warn against others who carry mathematics *too far*. Admissions against self-interest, jurists know, carry special weight in court.

In response to the editor's request for any reactions by me, I jot down a few thoughts.

BOTTOM LINE

Whatever the *ad hominem* interest of this discussion, it neither adds to nor subtracts from an effort to judge what is the optimal degree of use of mathematics by end-of-century economists.

PREDICTABLE REACTIONS

I do not find the number and eminence of the quoted savants at all surprising. One could double or quadruple these samplings. And oral denunciations of pseudo-mathematicians far exceed in expletives the cautious wordings culled from journals and printed interviews.[1]

EXPLANATORY PARADIGM

I have a fancy [quasi-mathematical] model to rationalize the quotations presented. It employs the *exact* [truncated] *Pareto Distribution*. As is well known the empirical distribution of large incomes obeys pretty well the P.D. property: "However high is your income, there are people richer than you. Calculate the mean income of such people and, no matter what yours is, their mean is always two to three times your income. There is never reason to cease complaining about others' 'excesses'. "

So it is with mathematical pretension as judged by degree of complexity that each scholar averages. Hicks and Allais used much math. (My teachers Jacob Viner, Frank Knight, and Edward Chamberlin would swear to [and at!] that.) But there are others who use more. And this is even more true about each of us as we age. "This far and no further," is our natural motto. (That is tautology, for if we believed in the usefulness of greater complexity, we would engage in it ourselves and recalibrate the frontier from which we complain.)

THE ONCE-AND-FOREVER BATTLE

Historians know that ever there is a *Methodenstreit* between shirt-sleeve hard-fact economists and fancy-dan theoretical economists. Thus, if you want quotes, turn to the 1880s writings of John Kells Ingram, the enemy of theory and "the leading advocate" in Britain of "the historical method." He was no great shakes, but the burden of his complaint against "vicious abstraction" and the deductive methods had to do with his revulsion against the *geometric diagrams* of his age! Who but antiquarians these days reads Cliff Leslie, the best of Jevons' opponents?

MATHEMATICS THE HANDMAIDEN

It was Nicky Kaldor — no mathematician he — who said: "Think of the power of the mathematical method: it enabled a mediocre high-school teacher like Erich Schneider to become a respected scholar." That was malicious and snobbish.

But Charles Darwin said something of the same in a nicer way: "I have always envied those who had the gift of mathematics because it was as if they possessed an extra sense that I lacked." His cousin, Francis Galton, one of the most creative persons who ever published, independently expressed a similar sentiment when he thanked a Cambridge friend who wrote out the Bravais mathematics of bivariate gaussian distributions that unified and clarified Galton's own ingenious reinvention of the wheel of *correlation* theory.

HOW TO SNOW THE COMPETITION

One could compile a compendium of quotations on how careerists can use the complexity of mathematics to snow their superiors, rivals, and pupils. One way of being vaccinated against being snowable is to acquire some competence in mathematics, but there will have to be better reasons than that to justify diverting energies from the study of rhetoric and the Statistical Abstract.

Science, in my observation, is somewhat self-purifying. The Native American adage tends to apply: "You fool me once, shame on you. You fool me twice, shame on me." I do not wish to over-glorify the self-monitoring process.

On the other hand, the burgeoning movements of anti-science, which quite transcend economics and profess to see in mainstream physics, biology, and medicine only a self-serving coterie of reciprocal back-scratchers, have thus far not been

observed to display convergence toward some different version of verifiable and useful science. Dissident movements have special characteristic dynamics, which display weak correlations with any accumulation of reproducible findings.

BITS OF WHAT, FOR WHAT?

The authors correctly say, "To us quotes are what data [are] to a statistical paper" [1994, 253]. As will be developed further, their article illustrates the strengths and weaknesses of quotations as evidential data.

When Jane Doe discovers and justifies a novel fact that is true, in the end this is just as important as when Albert Einstein discovers that fact. When Einstein, or Goethe, says,

> I do not like you, Dr. Fell
> The Reason why I cannot tell.
> But this I know and know full well,
> I do not like you, Dr. Fell.

this testifies something to the great person's mood and digestion of the moment. What else it signifies is problematic.

There can be no unqualified criticism of using quotations as evidence. For the description and documenting of *attitudes* they are indispensable. Still they are truncations of the full texts, and with some authors there can be found quotations that contradict or qualify each other; where this is so there is a duty to present more than one quotation. That is why a perceptive paraphrase can sometimes be more representative than an exact quotation. When confronted with a paraphrase, the reader cannot always tell the difference between a perceptive paraphrase and a misleading summary. By reputation some paraphrasers earn our tentative credence, just as by reputation we learn to distrust some who select for verbatim quotation what rereading the text shows not to be faithfully representative. Although such misleading selection can be artful and deliberate, as often it is semi-unconscious among temperaments who, in Samuel Johnson's phrase, "argue for victory" and who are prone to read into others agreement with and confirmation of their own views.

The study of attitudes is respectable in its own right. But the truth of a proposition cannot be well tested by a count of the number of noses of those who believe in that truth and of those who doubt it. Weighting the number of yea-sayers by their quality cannot do much to redeem the quotation technique as a tester of logical cogency or of empirical relevance.

SAD CALLS RECALLED

Let me adduce two examples of great scholars who prophesized independently while at the top of their powers that mathematical economics was a passing fad that more or less deserved its impending decline. I refer for one to Lionel Robbins at the end of World War II.

Even sadder to me was the essay written by my Chicago teacher Jacob Viner to introduce the 1936 *Festschrift* for his Harvard teacher Frank Taussig, then the dean of American economists. Viner was justly called the most erudite economist of his time: he was then but a year away from publishing his classic doctrinal *Studies in International Economics*. He had in the years just before returned from helping establish in the New Deal Treasury Department a corps of professional economists. In a great Chicago department that was a bit stronger on ideology than on wisdom, Viner stood out as an economist of eclectic judgment and subtle knowledge. And I will remind readers of what they know by hindsight but which Viner could of course not yet know in 1936 — of the impending explosive renaissance in trade and finance to be associated with the names of Lerner, Leontief, Meade, Tinbergen, McKenzie, Stolper, Samuelson, Metzler, Pearce, Kemp, Jones, Johnson, Mundell, and innumerable others, *all of whom by Viner's standards had to be regarded as mathematical economists*.

One doesn't know whether to cry or laugh at Viner's conclusion that math had been carried about as far as it could usefully go in post-Taussig international trade. I will not quote selectively from Viner's text; however, as Casey Stengel used to say, "You could look it up in the book."

QUOTING THE UNIVERSE

My title here refers to the corpus of folklore, fables, aphorisms, and *bons mots* that have survived the Darwinian competition of time. These also our authors could count as units of data. But note the pitfalls in trying to distill from them, say, a guide to ethics or to tactical habit formation.

You know the names that appear most frequently in *Bartlett* or the Oxford Volumes of quotations: Mark Twain, Oscar Wilde, Bernard Shaw (*not* Will Rogers), Ambrose Bierce, La Rochefoucauld, Karl Kraus (less than he should), and all the other usual suspects. Perhaps the greatest name of all is that of Anonymous. Her/his pearls cast before us, by definition, must stand on their own merits without the boost that comes from the name of genius such as Smith or Keynes or Einstein. The same can be said for the Bible, Homer, and Shakespeare (whomever he/she may have been). Moreover, I have noticed that many of the best of the Twain or Wilde sayings are those of doubtful attribution.

To get into the quotation anthologies, don't work to formulate *a new truth*. Repackaging an old untruth will prove as rewarding. Indeed many of our happiest old chestnuts consist of phrasing in an interesting way what catches our attention for its patent falsehood.

As with any industry based on selecting quotations, ransacking folklore to create a code of conduct and belief is peculiarly sterile. Folklore's point is to assert propositions and their exact opposite. Do animal spirits today caution you against explicit risk taking? Then you utter solemnly, "Don't put all your eggs in one basket." Do you wake up hellbent to take a flyer? Then you quote Mark Twain: "Put all your eggs in one basket and — WATCH THAT BASKET!" [as if watching something averts

the harm it can do you.] Did Will Rogers really say: "Buy land, they ain't making any more of it"? Efficient-market economists will laugh, not *with* the cowboy, but *at* him. They like better, and like it for its absurdity not truth, his alleged saying: "Only buy stocks that are going to go up. After they have gone up sell them. If they fail to go up, don't have bought them." If Will Rogers "never met a man he didn't like," that ought to disqualify him from any jury. "An open mind is [too often] an empty mind."

In societies professing to believe in astrology, astrology serves a purpose unrelated to its ability (inability?) to predict future events. Like tossable coins, astrology breaks ties. If I like (dislike) a prospective son-in-law, astrology will come to my rescue. Exactly so with folklore.

I am free to choose between: "Out of sight, out of mind" and "Absence makes the heart grow fonder." To choose between: "Never look back, someone may be gaining on you" and "Always learn from your mistakes" Between: "An eye for an eye" and "Turn the other cheek." From "Penny wise and pound foolish" and "Take care of the pence and the dollars will take care of themselves," or "Many a mickle makes a muckle."

From my own writing a sharp-eyed reader can line up clauses pro more math in economics and clauses pro more common sense. (Years ago my colleague Charles Kindleberger was vastly amused when his former student, Jaroslav Vanek called him up to say: "I've changed my mind about flexible exchange rates." "Why did you do that?" Charlie asked. "Because I found a mistake in sign in one of my determinants." CPK thought that deliciously funny. My reaction to him was "How do *I* ascertain where (uncommon!) good sense lies except, so to speak, by the signs in my determinants?" (Unto which of the two PAS pro-and-con columns would this last sentence go?)

AN EXPLANATORY EPICYCLE

All the esteemed people quoted I happen to know quite well. Earlier I gave a Pareto Distribution paradigm to rationalize the quotations data. Now I venture a factor-analysis addendum *ad hominem*.

People in any group differ in the degree to which they are "Napoleonic." In advance of observing the facts, one hypothesizes that "The more Napoleonic the scholar, the more content that person is with his/her own choice of mathematical complexity; and the more critical such persons will be of the level of mathematics employed by those who use more of it than they and those who use less of it."

By applying this regression-variable to the authors' data, it seemed that as much as one-third of the observed variance could be explained by my calculated $R^2 = .33\ 1/3$. (Correction for degrees-of-freedom have not been made because that would be a just-unnecessary refinement.)

HOW REPRESENTATIVE OF THE UNIVERSE IS A SPECIFIED SAMPLE?

Here is an additional empirical finding that has to do with understanding the pitfalls involved in using quotations as your evidential data. Necessarily quotations are *incomplete* samples. *Context* can be significantly lost.

Thus, consider the Debreu quotation. I do not have his Presidential Address at hand, so I cannot judge exactly how to interpret his words. The authors perhaps expect their readers to interpret Debreu to be complaining about the overuse of mathematics. And indeed he may have meant precisely that. However, as I read the literal words of their selection, I could be forgiven for construing those words to say: "Without prejudice to the actual worthwhileness of mathematics in economics, it is a pity that so much of today's economics cannot be readily understood by so many of today's economists."

I am on safer ground when I refer to the authors' quotations from my 1951 AEA Boston Convention address, Samuelson [1952]. The authors seem to interpret my warning to young economists of the day (that they are likely to be handicapped without mathematics) as implying that "much of the criticism of mathematical applications ... can be explained as an irrational reaction ..." [1994, 252]. I confirmed, when I reread my text, that its actual purpose that day was "... not to praise mathematics, but slightly to debunk its use in economics" [1952, 56].

"Aha," a reader may be tempted to declare, "this means the authors could have coupled Samuelson's name with those of Allais, Hicks, Georgescu-Roegen, Frisch, and their other nay-sayers."

You will reckon I am a hard person to please when my retort to that would be that this inference would equally damn the use of quotation snippets as evidential data in the search for testable truth of what was said and what was intended to be meant.[2]

A FINAL QUOTATION

As some sage has said,

"Science advances funeral by funeral."

NOTES

1. See the paper by Haim Barkai [1993] entitled, "The *Methodenstreit* and the Emergence of Mathematical Economics" for documentation of the fractal property: the nineteenth century and *every* time period witnesses a similar debate about mathematics. My only difference with Professor Barkai's fine paper is this: a dichotomy between fact gatherers and theory spinners omits a third of the debate; there are mathematics-mongerers like me who are positivistically fact-obsessed and vigorous opponents of *a priori* deductionists. Don't put us in bed with Menger, von Mises, and Robbins.
2. After I delivered my AEA address in Boston, Robert Bishop gave our venerable teacher Edward H. Chamberlin a lift back to Harvard. "How did you like Paul's lecture?" Bob asked. Our mutual teacher replied: "I didn't like it." "What did you find to object to in it?" "Well, it wasn't what Paul said, as what I knew he was thinking." There's context for you! Even republishing the full text of a paper can be problematic for Aesopian writers.

REFERENCES

Barkai, H. The *Methodenstreit* and Emergence of Mathematical Economics. Working Paper #266, Hebrew University, March 1993. Original English version of the German publication, Der Methodenstreit und das Aufkommen der Mathematischen Ökonomie, in *Vademecum zu einem Klassiker der Preistheorie*, edited by B. Schefold. Klassiker der Nationalökonomie Series, Düsseldorf: Die Handelsblatt Bibliothek, 1993, 61-84.

Debreu, G. The Mathematization of Economic Theory. *American Economic Review*, March 1991, 1-7.

Ingram, J. K. in *The New Palgrave Dictionary of Economics*, edited by J. Eatwell, M. Millgate and P. Newman, Vol. 2. London: Macmillan, 1987, 851.

Quddus, M. and Rashid, S. The Overuse of Mathematics in Economics: Nobel Resistance. *Eastern Economic Journal*, Summer 1994, 251-66

Samuelson, P. A. Economic Theory and Mathematics: An Appraisal. *American Economic Review*, 1952, 56-66. Reproduced as Chapter 126 in *Collected Scientific Papers of Paul A. Samuelson*, Vol. 2, Cambridge, MA: The MIT Press, 1966, 1751-61.

Viner, J. Introduction: Professor Taussig's Contribution to the Theory of International Trade. Chapter I in *Explorations in Economics: Notes and Essays Contributed in Honor of F. W. Taussig*. New York: McGraw-Hill Book Company, 1936, 3-12, particularly 11-12.