AGENTS’ MODELS AND PARTICIPATION IN A GAMBLING MARKET

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A gambler was hopelessly hooked on football betting. Nothing else interested him. Unfortunately, he lost almost every bet he made. Finally, even his bookie felt sorry for him. "You lose all your bets," said the bookie. "Why don't you bet on hockey instead of football?"

"Hockey?" said the gambler in dismay. "But I don't know anything about hockey!"

Mike Orkin, Can You Win? [1991]

The old adage that it takes a difference of opinion to make a horse race is particularly relevant in explaining why individuals participate in gambling markets. Although the role of agents’ opinions has been applied to other speculative markets [Smith, 1971; Grossman, 1976; and Varian, 1985; 1986], the literature is remarkably silent on applying agent opinions to gambling markets. This paper addresses the importance of an individual’s beliefs on his decision to participate in a pointspread gambling market. New insights include a description of the equilibrium in a gambling market, how agents form their subjective probabilities from their models and why heterogeneity in beliefs opens the gambling market to less risk-loving individuals.

THE GAMBLING MARKET

Gambling, like any speculative activity, allows participants to act on their subjective forecasts of some future event. Since gambling often involves a sporting contest in which opponents are usually unequally matched, some type of pointspread or line is attached to the bet to even out the odds.1 The potential gambler compares the announced line to his own forecast, then bets on the favorite (underdog) if he believes this pointspread underestimates (overestimates) the relative strength of the favorite.

Every game presents the gambler with two inversely-related lotteries: betting on team A (designated lottery \( G_A \)) or betting on team B (lottery \( G_B \)). To formalize the notion of a gambling market equilibrium, I define the demand side of the bet as the amount of money gamblers are willing to bet that the favorite, team A, will beat the underdog, team B, by at least the announced pointspread. The supply side of the market is defined as the amount gamblers are willing to bet that team A will win by less than that spread (i.e., betting on team B). In other words, demanders of the bet participate in \( G_A \) while suppliers participate in \( G_B \).


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The betting period begins when the opening line is announced and ends at game time, at which point the bookmaker ("bookie") ceases to take bets. Within the betting period gamblers take the current line \( (L_t) \) as given and decide whether to gamble, and if so, whether to take the favorite or underdog. These decisions determine the quantity demanded \( (B_t^A) \) and the quantity supplied \( (B_t^B) \), with \( \Delta B_t = (B_t^A - B_t^B) \) denoting the excess demand or amount by which the book is unbalanced during the betting period. At the close of the betting period, these values have been determined: the final line \( (L_f) \), the final amounts bet on both sides \( (B_f^A \text{ and } B_f^B) \), and the amount by which the book is unbalanced \( (\Delta B_f = B_f^A - B_f^B) \). Bets taken during the betting period contractually determine the conditions under which the gambler wins or loses. Whereas the payoff in horse racing is determined by the final odds, payoffs in a pointspread gambling market are determined by the line agreed on during the betting period (which may not coincide with the final line).

It is generally assumed that the bookie brokers the market, allowing individuals with different expectations to act on their beliefs. Payoffs typically follow some variation of the "11-to-10" rule: a gambler receives $10 from the bookie if he wins and pays $11 if he loses. The extra dollar, equal to 10 percent of the bet, is the brokerage fee or "vigorous" for using the bookie's service. The bookie brokers the market by adjusting the line during the betting period to balance the final amounts bet on each team (i.e., \( \Delta B_f = 0 \)), allowing him to collect his 10 percent brokerage fee risklessly. For example, if the public feels that \( L_t \) underestimates the strength of the favorite, too much money will be bet on this team and \( B_f^A > B_f^B \). To even out the amounts bet on both teams, the bookie would adjust the line upward, making the favorite a less desirable choice. This adjustment process continues until a line is announced that sufficiently evens out the money bet on each team.²

Economic models of behavior under uncertainty assume that the individual chooses among alternatives \( X \) in order to maximize expected utility: \( EU^X = P_X^t U^X (X) \). The gambler calculates his subjective probabilities \( (P_X^t) \) in a manner similar to interval estimation in classical statistics. For example, let the \( i^\text{th} \) gambler's exogenously-determined forecast of the game outcome \( (F^i) \) be 10 points, (i.e., his best guess is that team \( A \) should beat team \( B \) by 10 points). It is reasonable to assume that he would implicitly recognize the uncertainty of his forecast by stating that team \( A \) should win by somewhere between 8 to 12 points. In classical statistics, we would call this a (say) 95 percent forecast interval given by 10 plus or minus 2 points. Said differently, the gambler feels there is a 2/2 percent probability that the actual spread will be below 8 points, hence a 97/2 percent probability that the actual spread will end up larger than 8 points. If the current line is 8 points on team \( A \), this gambler's subjective probability of winning \( G_A \) (denoted \( P_A^i \)) is 97/2 percent. Hence, \( P_A^i \) comes from this agent's calculation that the ex post realization of the spread \( (S) \) will be greater than \( L_t \) given that his point-estimate forecast is 10 points; i.e., \( P_A^i = Pr(S > L_{t} | F^i = 10) \).

Of course, the gambler has the option of taking the other side of the bet, \( G_B \), which pays off if team \( A \) wins by less than 8 points. The probability of winning \( G_B \), \( P_B^i \), is the \( i^\text{th} \) gambler's assessment of the probability that \( S \) will be less than \( L_t \), given
and ends at game time. Within the betting lines, sports bettors differ to gamble, and the odds the bookies set determine the amount bet. The spread, $\Delta B_i = (B_i^A - B_i^B)$, is balanced during the betting period, and the amount bet is $E_B = P_i \cdot B_i$. If the gambler wins or loses. The betting odds, payoffs in a given outcome are bet on during the betting period.

Following individuals that typically follow some house rules. If the gambler wins or loses the bet, the bookie annotate the market by changing the spread. If the gambler wins, the market bet on each side of the bet is balanced. The spread bet risklessly.

The spread is an expression of the strength of the favorite, but the market bet size is determined by the favorite a less chance to win. The bookie's spread is announced that

that the individual gambler's utility: $EU^i (X) = \frac{1}{2} (P_i^A + P_i^B)$ in a manner that maximizes his utility. Let the $i^{th}$ gambler's spread at bet $X$, $P_i^A$, is his best estimate of the probability that team $A$ will win. Let the spread be $2$ points, i.e., his best estimate of $P_i^A$. Suppose the bookie is not able to assume that the spread is that the actual spread $\Delta B_i$ is not the actual spread $\Delta B_i$ is not the actual spread $\Delta B_i$. The spread will end being $2$ points, i.e., $P_i^A = \frac{1}{2}$. Hence, $P_i^A$ comes from the spread $(S)$ will be $2$ points, i.e., $P_i^A = \frac{1}{2}$. $P_i^A = \frac{1}{2}$.

by $P_i^A = Pr(S < L, F = 10) = (1 - P_i^A)$. In this example, the gambler would infer $P_i^A$ to be $2\%$ percent. By monotonicity, an 8-point line would lead the gambler to prefer $G_A$ to $G_B$, and this gambler would be a demander of the bet.

Clotfelter and Cook [1990] define the implicit price of a spread as the cost of purchasing $1$ of expected value, derived by dividing the cost of the bet by its expected value. To illustrate, consider the above example where $L = 8$ and $F = 10$, implying that the agent calculates $P_i^A$ to be $97\%$ percent. This gambler would evaluate the expected value of $G_A$ as $2.05$, and its price as $0.54$. At a line of $10$ points, implying $P_i^A$ equals $50$ percent, this expected value is $1.05$ and its price is $1.05$. In short, as the line increases, the gambler's implicit price of $G_A$ rises (and the price of $G_B$ falls.) As such, fewer gamblers are willing to bet on the favorite (quantity demanded falls) and more will want to take the underdog (quantity supplied rises). The final line, $L_F$, is determined by the condition that quantity demanded equals quantity supplied.

So far, we have been assuming that agents' beliefs are independent of the tattlement process. But an increase in the line during the betting period may signal to a gambler that the general public believes that the favorite is currently underrated. This may cause agents to re-evaluate their own beliefs, in which case the equilibrium must clear the market given that the pointspread is constant.

The importance of this “double fixed point” problem to the existence of an equilibrium is mitigated by the fact that the lines typically change very little within the betting period. Of 133 NFL games played in 1993 in which an opening and closing line existed, the average change (positive or negative) between the opening and closing lines was only 0.75. With such meager changes in the line it is doubtful that gamblers would update their expectations significantly within the betting period.

**EQUILIBRIUM UNDER OBJECTIVE MODELS**

It is an agent's model that formulates his beliefs or understandings of reality. Thus, predictions or forecasts are generated from one's model. If everyone agrees on the process generating chance events, we say this common model used for creating expectations or beliefs is objective. This is one of the cases considered by Hakansson, Kunkel and Ohlson [1982] where the likelihood functions used by agents are said to be “essentially homogeneous.” Games of pure chance, such as blackjack, are gambles in which the probabilities of winning are widely known and might be viewed as gambles in which agents' models are objective. Even games of uncertainty, such as betting on the outcome of a NFL game, could be viewed in this fashion if the determinants of success on the field became widely accepted. On the other hand, when agents use their personal interpretations of chance events, we say the models used are subjective. All agents act on a common forecast when the models used are objective; differences in expectations arise if agents use subjective models.

We begin by examining the market equilibrium when all gamblers utilize the same model; hence, all are acting upon a common forecast, $F$. If this collective forecast of a particular game is $10$ points, an 8-point line could not be an equilibrium spread because the typical gambler would infer that $P_i^A > 50\%$ percent and $P_i^A < 50\%$ percent.
percent. This would create excess demand in this market and the line would need to be adjusted upward. By similar reasoning, 12 points could not be an equilibrium spread, for everyone would be taking $G_B$; the line would need to be adjusted downwards.

At a point spread of 10 points, the collective forecast, all gamblers would infer the probabilities of winning $G_A$ and $G_B$ as 50 percent each. Given identical payoffs and equal probabilities of winning, gamblers would be indifferent between the two lotteries; only then could equal amounts of money be bet on both sides of the game. Thus the final line, $L$, must make the representative gambler indifferent between $G_A$ and $G_B$; this occurs if and only if $L = F$.

If the final line converges to the common point-estimate forecast, however, why would anyone want to take the wager? This paradox, known as the No-Trade Theorem [Rubinstein, 1975; Milgrom and Stokey, 1982], arises because agents

should realize there is a mismatch of complete capitalization of the information because they get paid for the difference. We now turn to the willingness to engage in the risk.

Let $M_o$ denote the amounts paid if he provisioned his income levels in the case where a lottery the vigor of the participants in this market is unfair (i.e., neutral nor risk-averse). In case where $E(U(G_A) > U(M_o))$ for risk-avoiding individuals, can affect the viability of the market, which case the implications for risk-loving individuals?

**EQUILIBRIUM UNDER RISK**

In this section, consider an equilibrium in a game where the gamblers, each with their own independent point-estimate forecast.

Assume that the market assigns a probability that the outcome will be greater than 50 percent if $L > F$. Calculate the probability that the suppliers of this bet would assign a greater value. Each team would consider the consequence between 5 and 6 points.

This simple example highlights the price of a risky market. In general, the market would be driven by some distribution that reflects identical attitudes to risk. If $L_i$ would assess $P_i$
should realize there is no potential for gain in this transaction since the market has completely capitalized the "common wisdom." Gamblers might still participate because they get pleasure from the very act of gambling, i.e., if they are risk-lovers. We now turn to the effect that an individual's attitude towards risk has on his willingness to engage in a gamble.

Let $M_o$ denote the representative gambler's fixed income and $W$ and $L$ be the amounts paid if he wins or loses; hence $M_w = (M_o + W)$ and $M_l = (M_o - L)$ represent income levels in these two states. Because the bookie expends resources to provide his service, the vigorish is positive, implying $|L - M_o| > |M_o - W|$.

Since the equilibrium must equate $L$ to $F$, only risk-loving individuals will participate in this market. This is because when $L = F$, the bookie's vigorish makes the bet unfair (i.e., its expected value is less than its cost). As such, neither risk-neutral nor risk-averse individuals would engage in this activity. Figure 1(a) shows the case where a risk-loving gambler would be willing to take either bet since $EU(G_j) > U(M_0)$ for $j = A, B$. But transaction costs, in the form of the bookie's vigorish, can affect the viability of this market. A large vigorish can cause $U(M_0) > EU(G_j)$, in which case the implicit price of the bet could rise so high that it rations even the most risk-loving individuals out of the market (see Figure 1(b)).

**EQUILIBRUM UNDER SUBJECTIVE MODELS**

In this section, we show how differences in individuals' opinions affect the equilibrium in a gambling market. Suppose a wagering market consists of 10 gamblers, each with his own subjective forecasts of the spread. Let these subjective point-estimate forecasts, $F^i$, be given by

$$\{F^1, F^2, ..., F^{10}\} = \{3, 3, 4, 4, 5, 6, 6, 7, 7, 7\}.$$ 

Assume that the amount bet by each gambler is constant across gamblers and that $L$ is announced as 4½ points. Recall that a gambler's assessment of the probability that the favorite will cover the line is exactly 50 percent if $L = F^i$ (and less than 50 percent if $L > F^i$). The four gamblers with forecasts of 4 points or less would calculate the probability of winning $G_A$ at less than 50 percent, hence would be suppliers of this bet. By the same reasoning, the 6 gamblers with forecasts of 5 or greater would be demanders of this bet. This imbalance between the amounts bet on each team would cause bookies to revise the line upward until it reaches an equilibrium between 5 and 6 points.

This simple example points out the similarity of the line in gambling markets to the price of a risky asset — it reflects the median belief of all participants in the market. In general, we could summarize all agents' subjective forecasts of the spread by some distribution with median $f$. We begin by assuming that everyone has identical attitudes towards risk and gambles a fixed amount. All gamblers with $F^i < L$ would assess $P^i_A > P^i_B$ and be inclined to take $G_A$; all gamblers with $F^i > L$ would
tend to take $G_A$. Since $\Delta B_{z} = 0$ only when $L_i = f$, the final line ($L_{w}^*)$ must converge to the median forecast $f$ for quantity demanded to equal quantity supplied.\footnote{It was shown above that under objective models a risk-neutral individual would not participate in a gambling market. With subjective forecasts, however, the announced line may deviate sufficiently from the forecast of a risk-neutral individual that he is enticed into the gamble. For example, consider two risk-neutral gamblers $Y$ and $Z$ with subjective forecasts $F^Y$ and $F^Z$, and assume that $F^Z > F^Y$. If the announced line, $L_{w}$, is set so that $F^Z > L_{w} > F^Y$, then gambler $Y$'s calculation of $P_{z}^Y$ will be greater than 50 percent, perhaps enough to make $EU^Y(G_{z}) > U^Y(M_{o})$. At the same time, gambler $Z$'s calculation of $P_{z}^Z$ will be greater than 50 percent, possibly making $EU^Z(G_{z}) > U^Z(M_{o})$. With money on both sides of the bet, an equilibrium with risk-neutral participants could exist.}

The important implication here is that differences in forecasts created by subjective models open up the market to less risk-loving individuals. Crucial to this argument is that an individual's subjective probability of winning the $j$th lottery is determined by the difference between the announced line and $F^i$. We can ascertain the point at which an individual is just willing to enter a wager by solving for his certainty-equivalence pointspread. Dropping the superscript designating the $i$th agent, let $L_A$ be the certainty-equivalent pointspread, and $\bar{P} = Pr(S > L_A)$ be the resulting certainty-equivalent probability, which makes the gambler just indifferent between participating or not in $G_A$, defined implicitly as

$$U(M_o) = \bar{P} U(M_w) + (1 - \bar{P}) U(M_L).$$

Approximating the utility of each outcome by a Taylor series expansion:

$$U(M_w) = U(M_o + W) = U(M_o) + WU'(M_o) + 1/2W^2U''(M_o) + \epsilon_1,$$

$$U(M_L) = U(M_o - L) = U(M_o) - LU'(M_o) + 1/2L^2U''(M_o) + \epsilon_2,$$

where $\epsilon_1$ and $\epsilon_2$ are error terms in higher powers of $L$ and $W$ that become insignificant. Substituting (2) into (1), and ignoring the error terms, we get the certainty-equivalent probability:

$$\bar{P} = [L/(L + W)][2U' - LU''](2U' - (L - W)U'').$$

Equation (3) shows that if the individual is risk-neutral (RN), hence $U'' = 0$, the certainty equivalent probability is $\bar{P}_{RN} = L/(L + W)$, which is greater than 50 percent. If the individual is risk averse (RA), $U'' < 0$, making $\bar{P}_{RA} > \bar{P}_{RN}$, the deviation between the spread and one's forecast must be even greater. An even clearer demonstration of the relationship between $\bar{P}$ and risk aversion can be seen with a zero vigorish. In this case, $L = W$, and we have

$$\bar{P} = \frac{L}{2L} = \frac{1}{2}.$$
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\[
\overline{P}_A = 1/2 - [(W/4)(U''/U')] = 1/2 + (W/4)R,
\]

where \( R = -(U''/U') \) is the gambler's Arrow-Pratt measure of absolute risk aversion. From (4) we see how this probability is directly related to the individual's degree of risk aversion; i.e.,

\[
(\frac{\partial \overline{P}_A}{\partial R}) > 0.
\]

Since \( R \) is positive, negative or zero for an individual who is risk-averse, risk-loving or risk-neutral, Equation (5) shows us that the certainty-equivalent probability is greater for more risk-averse individuals. This parallels the condition in other speculative markets that a greater aversion towards risk necessitates a higher rate of return. In this market such a condition arises from a greater deviation between \( F' \) and the announced line.

CONCLUSION

As pointed out by Figlewski [1979], betting on games of pure chance where the probabilities are widely known (a game of risk) is fundamentally different than betting on games where the probabilities cannot be known with certainty (a game of uncertainty). This paper has attempted to explain the economics of gambling behavior in a framework that explicitly incorporates agents' models. New insights developed include the process by which individuals create their subjective probabilities, how the divergence between one's belief and the market's belief affects the decision to participate in a speculative market and why differences in opinion open gambling markets to more risk-averse individuals.

NOTES

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1. The pointspread determines the number of points by which team A (the "favorite") must beat team B (the "underdog") for the gambler to win.
2. Arbitrage would induce convergence of all gambling houses to a market-wide price, although slight differences could arise due to asymmetric information and other transaction costs.
3. Since \( P_A' \) and \( P_B' \) are derived from the deviation between \( L_i \) and the individual's point-estimate forecast, \( F' \), then \( P_A' = P_B' = 0.50 \) if and only if \( L_i = F' \). Further, we ignore \( Pr(S = L_i) \) since this probability is zero.
4. This bet can be specified in the following manner. The gambler pays $1.10 to play a game with payoffs equal to $2.10 if he wins ($1.00 plus his $1.10 entry fee), and $0 if he loses. For \( L_i = 8 \) points, the agent's subjective probability of winning is 97.4 percent and probability of losing is 2.6 percent. Thus, \( EV = 0.974 \times ($2.10) + 0.026 \times ($0) = $2.0475. Since the cost of entering is $1.10, this lottery's price is given by $1.10/$2.0475 = $0.5373.
5. We would like to thank an anonymous referee for directing our attention to this point. To the extent that this "double fixed point" problem is important, gamblers' forecasts should be considered conditional on the current pointspread.
6. This is most likely when everyone is utilizing the same handicapping services, reading the same sports column, or simply using similar rules of thumb (like "a good defense beats a good offense").
REFERENCES


WHAT IT IS

Column 1

INTRODUCTION

Few would dispute the existence as a result of services. Such change is substantial and unique features of the

1. It is evolutionary and not organic.
2. It utilizes business rather than person.
3. It relies on documents.
4. It applies a range based on the given the average
5. Less rigorous and more constrained.
6. It may serve technological generating power.

A great deal of change through the physical characteristics of the hedonic price literature: estimates of the price applications and hedonic techniques of the first, and most influential of these was Cagan, 1965; and Trivedi, 1979, the number and variety.

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