THE NEO-PASINETTI THEOREM IN CAMBRIDGE AND KALECKIAN MODELS OF GROWTH AND DISTRIBUTION

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The neoclassical theory of distribution is usually associated with diminishing marginal physical products, technical substitution, competition, and optimizing, where factor prices are determined by their relative scarcities. In Solow's neoclassical growth model, the long-run equilibrium growth rate is exogenous, set by the rate of growth of the labor force (and the exogenous rate of technical progress). The rate of profit is then endogenous, determined by this rate of growth, technology, and the rate of saving of the economy. In the so-called *new* neoclassical growth models, these relationships are reversed. When these models are reduced to their bare essentials, the steady-state rate of profit is the exogenous variable, while the rate of growth becomes an endogenous variable, entirely determined by the product of the profit rate and the given savings rate [Kurz, 1997].

In contrast to the above neoclassical view of growth and distribution, the so-called Cambridge school of thought emerged in the mid-1950s, with an emphasis on constant or increasing returns to scale, fixed technical coefficients, imperfect competition, behavioristic "rules of thumb", and a macroeconomic theory of income distribution [Hacche, 1979, 182]. In the Cambridge models of growth and distribution, the rate of profit is a macroeconomic variable, thus eschewing the drawbacks of assuming, as did the earlier neoclassical models of growth, that the rate of profit depends on the scarcity of capital relative to labor — a hypothesis that was shown to be based on dubious grounds during the capital controversies of the 1960s.[1] The other crucial feature of Cambridge models is the presence of an independent investment function, which insures that growth rates are endogenous variables. By contrast, neoclassical growth models generally assume that investment is driven by savings, even when steady-state growth rates are endogenous, while the old neoclassical models of the Solow type assumed more specifically that *full-employment* savings were the driving force.

In the models first proposed by Joan Robinson and Nicholas Kaldor, when savings out of wages are assumed away, the rate of profit of an economy only depends on its rate of accumulation and the propensity to save out of profits, to the exclusion of any other variable. Pasinetti [1962] generalized this result by noting that workers could earn both wages and profits, thus distinguishing between workers on the one hand and capitalists on the other, the latter earning only profits. Various neoclassical authors, such as Meade and Hahn [1965] and Samuelson and Modigliani [1966], objected to what became known as Pasinetti's paradox, that the long-run rate of profit of an economy depends only on its rate of economic growth and the propensity to save of the capitalist class, to the exclusion of any other variable, including technological coefficients and worker's propensity to save. Neoclassical authors countered Pasinetti's paradox by showing that Pasinetti's result was just one of two possible regimes. They derived a second long-run equilibrium, where the propensity to save of the working class compared to that of the capitalist class would be such that the latter class would eventually disappear. With this anti-Pasinetti regime, as it was called by Kaldor, a way seemed open for reinstating marginal productivity analysis based on the flexibility of technical coefficients.

In response to the arguments by Samuelson and Modigliani [1966], Kaldor produced a model which he called a neo-Pasinetti theorem. The model had four connected purposes. First Kaldor wanted to produce a model, similar in spirit to that of Pasinetti, which would achieve similar results without requiring assumptions valid only in the long run. Secondly, Kaldor wanted to build a model for which no dual regime could be offered as a counterpoint. Thirdly, Kaldor's neo-Pasinetti theorem was meant as an attack on those neoclassical models where "savings governs investment." Finally, and perhaps most important, Kaldor [1978, xv] wanted "to clarify the reasons why the savings propensity out of profits must be considerably greater than the savings propensity out of wages and salaries, or of household incomes in general".

Whereas Kaldor's [1966] article is generally regarded as a neat reply to the empirical arguments supporting the likelihood of the anti-Pasinetti regime in long-run models of balanced growth, few authors have attempted to incorporate the results of the neo-Pasinetti theorem within a post-Keynesian framework.[2] Post-Keynesians have been of two minds when evaluating the importance of Kaldor's neo-Pasinetti theorem. On the one hand, it is regarded as a clear demonstration of a fundamental post-Keynesian thesis, (i.e., post-Keynesian models "are designed to project into the long period the central thesis of the *General Theory*, that firms are free, within wide limits, to accumulate as they please, and that the rate of saving of the economy as a whole accommodates itself to the rate of investment that they decree" [Robinson, 1962, 82-3]).[3] On the other hand, Kaldor's neo-Pasinetti theorem is also seen as a compromising concession to the neoclassical camp, because it incorporates an adjusting mechanism between savings and investment — the valuation ratio — which appears to be similar to the neoclassical interest rate mechanism.[4] In fact, for all practical purposes, Kaldor's *valuation ratio* is no different from Tobin's better-known *q-ratio*, which appears in many neoclassical models.[5]

This may explain why, with a few exceptions, the neo-Pasinetti theorem has been relegated to obscurity. The aim of the present paper is to offer a remedy for this lacuna, by providing a heuristic representation of the neo-Pasinetti model within the new framework developed by Kaleckians over the last fifteen years. Thus, in contrast to previous variants of the neo-Pasinetti theorem, we shall extend the model to situations where *both* the rate of accumulation and the rate of capacity utilization are *endogenous* variables. By doing so, we shall incorporate what is considered by many to be the main feature of Keynesian models in contrast to neoclassical models: the presence of an explicit and independent investment function. Furthermore, as will be shown, the incorporation of an endogenious rate of capacity utilization within the neo-Pasinetti model allows the model to retain its Keynesian results, even when investment is sensitive to the values taken by the valuation ratio. This is in sharp contrast to what occurred in previous models where the rate of capacity utilization was assumed to be constant.

The outline of the paper is the following. First, we shall recall the algebra and the graphics of the neo-Pasinetti theorem, with its two main variables, the rate of profit and the valuation ratio, when the rate of growth and the rate of capacity utilization are assumed to be exogenous. We then distinguish two different routes by which the rate of profit may adjust to its equilibrium value: through profit margins, as in the standard Cambridge models *à la* Kaldor and Robinson, and through rates of capacity utilization, as in the new Kaleckian models of growth and distribution. Just as in the new neoclassical growth models, we endogenize the rate of accumulation. We verify whether such a neo-Pasinetti model retains the properties of Kaleckian models of growth, while assuming away any feedback of the valuation ratio on the desired rate of growth. In the final section, we shall look more closely at the determinants of the valuation ratio when rates of capacity utilization are endogenous, and we shall consider in turn the impact of the valuation ratio as a determinant of investment decisions.

THE STANDARD VIEW OF THE NEO-PASINETTI THEOREM

Although Kaldor's neo-Pasinetti theorem has generated contributions from just over a dozen authors, Araujo [1995] has recently reinterpreted it graphically.[6] While we follow Araujo in his graphics, we shall suppose that households have a single propensity to save on all income, as briefly assumed by Kaldor [1966, 318, fn. 1]. In doing so, we follow Robinson when she says that the "most important distinction between types of income is that between firms and households" [1962, 38]. In a sense, the assumption is more faithful to Kaldor since this distinction was at the core of his belief that the propensity to save out of profits is higher than the propensity to save out of wages, an assumption which drives the mechanics of most post-Keynesian models.[7] With the neo-Pasinetti theorem, Kaldor's intent is to free the determination of the profit rate from its classical dependency on a class of capitalists with a high savings propensity. By doing so, Kaldor provides additional institutional reality, introducing corporations, with leverage and retention ratios, as institutions with a role going beyond the preferences of their shareowners.[8]

The innovation in Kaldor's model is the explicit introduction of capital gains. These gains arise from the retained earnings accumulated by firms. The precise value of these capital gains depends on the valuation ratio. The valuation ratio is defined as the ratio between the financial value and the replacement value of capital. Kaldor's valuation ratio is indeed akin to Tobin's q-ratio, which is sometimes defined as the ratio between the market value of equity and the value of the stock of capital at replacement cost [Brainard and Tobin, 1968]. The higher the spread between the financial value of a unit of capital and the replacement cost of such a unit, the greater the capital gain per additional unit of investment. When dealing with the savings of households, one must account for the dissavings out of capital gains. In growth terms, equilibrium in the product market requires the following equality between investment and overall savings:

(1) $$g = g^s = s_h[(u/k - r) + (1 - s_f)r)] - (1 - s_h)(v - x)g + s_f r.$$

The term associated with square brackets represents the savings out of wage income and out of distributed dividends; the next term stands for the dissavings out of capital gains, while the last term represents the retained earnings of the firm.[9] The notations are the following: g is the rate of growth of the economy; s_h the propensity to save of households; k the capital to capacity ratio; r the rate of profit; v the valuation ratio; s_f the retention ratio of firms; x the proportion of investment which is financed by the issue of shares, or the external finance ratio. The latter two parameters are assumed to be given. The only innovation here is the introduction of u, the rate of capacity utilization, which is usually assumed constant and equal to unity.[10] In the present section we also assume that the rate of capacity utilization is a constant, but we make it an endogenous variable in the following sections.

Similarly, we may derive an equation expressing equilibrium in the financial market: the flow of supply of shares must equate the demand for shares, i.e., the supply arising from the issue of new shares by firms plus the sales of shares by households who desire to realise capital gains must equate the demand arising from the current savings of households out of ordinary income (wages and dividends). The supply flow is on the left-hand side of equation (2), while the demand flow is on the right-hand side:

(2) $$xg + (1 - s_h)(v - x)g = s_h[(u/k - r) + (1 - s_f)r)].$$

As Araujo [1995] notes, since the first condition describes the equilibrium in the product market, it is akin to an IS curve. We shall call it an ED curve, since it represents the locus of profit rates and valuation ratios for which the effective demand conditions are fulfilled. The second condition, which describes the equilibrium in the financial market, is akin to an LM curve. As a consequence we shall call it an FM curve. Equations (1) and (2) can be rewritten respectively as:

(3) $$r = \{g[1 + (1 - s_h)(v - x)] - s_h u/k\}/[s_f(1 - s_h)],$$

(4) $$r = \{-g[(1 - s_h)v + xs_h)] + s_h u/k\}/(s_h s_f).$$

With the rate of profit r expressed as a function of the valuation ratio, the ED curve has a positive slope. In the product market, a higher profit rate, associated with higher savings, must be compensated by a higher valuation ratio, associated with larger dissavings out of capital gains. On the other hand the FM curve has a negative slope.[11] An increase in the rate of profit diminishes the share of output left to households: hence it diminishes the demand for equities. This must be compensated by a decrease in the supply of equities, a decrease that will need to be accommodated by a fall in the value of the shares sold by existing shareowners, and hence by a fall in the valuation ratio. The rate of profit plays the role of the output level in Hicksian IS/LM models, whereas the valuation ratio, as already said, plays the inverse of the role of the interest rate in such models, which is why the slopes of the ED and FM curves here are inverted compared to the standard Hicksian IS and LM curves.

The equilibrium values of the rate of profit and of the valuation ratio are given by:[12]

(5) $$r^* = g(1 - x)/s_f.$$

(6) $$v^* = s_h(u/gk - 1)/(1 - s_h).$$

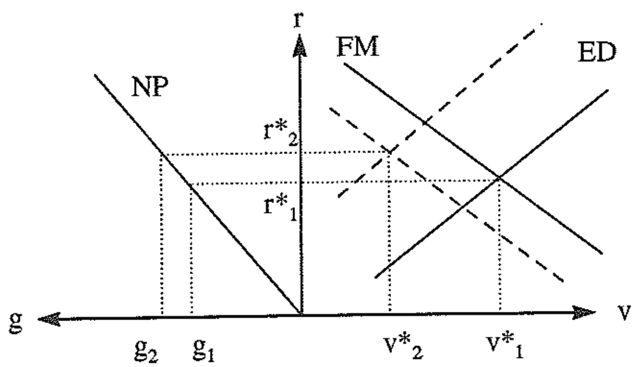
While the product market is assumed to clear through variations in the rate of profit and the stock market is assumed to clear through variations in the valuation ratio, we have a two-equation system with two variables, the variations of which affect both markets as Rimmer [1993, 55] and Araujo [1995] have pointed out. What is remarkable about the neo-Pasinetti theorem is that once these two adjusting mechanisms are taken for granted, the rate of profit and the rate of growth are linked through a relationship — given by equation (5) — which is independent of the propensity to save of households. This yields the main feature of Pasinetti's model — savings of workers have no impact on the rate of profit.[13] In that context, until we examine more closely the possible determinants of the rate of accumulation, it would seem indeed that firms are free to accumulate at the rate that they wish, provided the rate of profit (and the valuation ratio) can adjust. If this is the case, "demand will be brought in line with capacity, no matter what the rate of growth of capacity", and hence "the growth rate becomes indeterminate" [Wood, 1975, 118].

In light of this indeterminacy, several commentators on Kaldor's neo-Pasinetti model have interpreted the rate of accumulation to be the natural rate of growth, i.e., the full employment rate of growth [Davidson, 1968, 259; 1972, 300; Harcourt, 1972, 220]. This need not be the case, however, as Léonard [1980, 135] and Lavoie [1987, 189] have pointed out, and as Kaldor himself acknowledged.[14] More specifically, we may introduce an independent investment function which will help determine the actual rate of growth set by the entrepreneurs.

But before we do so, let us examine what happens in the neo-Pasinetti model as firms set a higher rate of accumulation. Let us assume, as is usually done in the interpretation of the neo-Pasinetti theorem, that the rate of utilization u is a constant. First, taking the derivatives of equations (3) and (4) with respect to g, we see that the ED curve would shift upwards while the FM curve would shift downwards. This implies, as can be seen immediately from the right-hand side of Figure 1, that the valuation ratio v must necessarily be lower with a higher rate of accumulation. This is confirmed by taking the derivative of equation (6) with respect to g ($dv^*/dg < 0$). Now what about the impact on the rate of profit? Once all the proper adjustments are made, the impact is necessarily positive, as can be verified by taking the derivative of equation (5) ($dr^*/dg = (1 - x)/s_f > 0$). This can also be seen on the left-hand side of Figure 1, reading along the NP curve, which represents the key feature of the neo-Pasinetti model, and which shows how accumulation is financed, in accordance with equation (5).

While it is easily understood that, when accumulation is accelerated, an excess supply of shares in the financial markets will reduce the price of shares and hence the valuation ratio, what is the mechanism that drives up the rate of profit along the NP curve? This is the topic of the next section.

FIGURE 1



ADJUSTMENT MECHANISMS IN THE CAMBRIDGE AND KALECKIAN FRAMEWORKS

Two adjustment mechanisms may help explain rising profit rates when accumulation is accelerated. According to the first mechanism — the Cambridge case, based either on a competitive model with full capacity utilization, or on the maintenance of normal rates of capacity utilization within an oligopolistic framework — prices and profit margins will rise when aggregate demand speeds up. This mechanism has been explored by almost all authors dealing with the neo-Pasinetti model. It is associated with the other well-known growth models of Kaldor and Robinson, but also with the more modern ones, set explicitly within an oligopolistic framework, where mark-ups set by price leaders are proportional to growth trends, just as they would be in models of growth based on assumptions of pure competition.[15]

A second possible mechanism is based on the endogeneity of the rate of capacity utilization which has been endorsed by both Kaleckians and Sraffians.[16] According to this second mechanism — the Kaleckian case, based on an oligopolistic environment with excess capacity, subjected to labor market pressures — profit margins need not be modified when growth rates rise. But the higher rates of capacity utilization will nevertheless allow for higher profit rates. We shall consider each of these mechanisms in turn.

As was pointed out in the previous section, the rate of accumulation is assumed to be given in the neo-Pasinetti model. Let us maintain this assumption for now, and consider how the rate of profit would adjust to the new rate of accumulation of the economy. First, consider the following identity, remembering that the profit rate is the ratio of profits to the value of capital:

$$r = P/K = (P/Y)(Y/C)(C/K),$$

where P stands for profits, while Y and C are respectively output and full-capacity output. The rate of profit can thus be broken down into three components:

$$r = mu/k, \tag{7}$$

where m is the share of profits in national income, while u and k are defined as before as the rate of capacity utilization and the capital to full-capacity output ratio.

We call this relationship the profits-cost function since profits are seen from the supply-side angle. In the simplified version of the Kaleckian model which we consider here, where overhead costs arising from labor or capital use are assumed away, the share of profits m is identical to the profit margin set by firms through their pricing procedure.[17] For instance, within a simple mark-up procedure, the mark-up rate would be θ, where θ is such that prices are defined as $p = (1 + \theta)UVC$, where UVC are the unit variable costs. In such a case, the share of profit, or the profit margin, is $m = \theta/(1 + \theta)$.

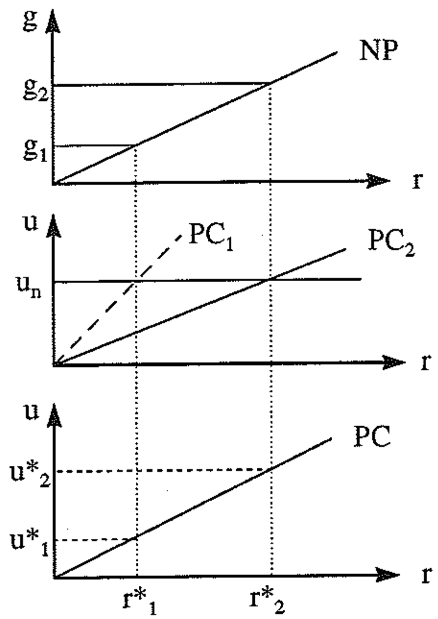
Let us first consider the Cambridge case, where the rate of utilization is assumed to be constant at its normal level, called u_n, while profit margins are flexible.[18] The profit margin must vary in line with the required rate of profit, as given by its solution found in equation (5). Making use of equation (7), we see that the profit margin must become equal to:

$$m^* = gk(1 - x)/s_f u_n. \tag{8}$$

Any increase in the rate of accumulation will thus require a higher rate of profit, and this higher rate of profit will be accommodated by an increase in the profit margin (i.e., a fall in real wages).[19] On the other hand, as is illustrated in the middle of Figure 2, a higher rate of accumulation and a higher rate of profit will be accommodated by a rise in profit margins, and a downwards shift of the profits cost function, shown here by the PC curves, while the rate of utilization remains at its normal value u_n. This is the mechanism sometimes contemplated by early Cambridge authors: "But let us suppose that competition ... is sufficiently keen to keep prices at the level at which normal capacity output can be sold" [Robinson 1962, 46].[20] The other mechanism, also contemplated by Robinson and Kaldor, but mainly made explicit by their followers, is that oligopolistic firms set profit margins to provide the retained earnings required to finance the secular rate of accumulation. What we have is a "long run model of determination of the share of profits at normal full capacity use" [Wood, 1975, 129].

We need not suppose however that the rate of capacity utilization is always at its normal level or equal to unity. Kaleckian authors have recently underlined the flexibility of the rate of capacity utilization, even in the long run, arguing either that no mechanism could automatically bring back the rate of utilization to its normal level, or that if such a mechanism did exist, one had to take into account the fact that the normal rate of utilization could itself become an endogenous variable.[21] As an alternative closure to the neo-Pasinetti theorem, one may consider the profit margin m to be a given, while the rate of utilization is flexible and accommodates any change in de-

FIGURE 2



mand.[22] This corresponds to what one could expect in an oligopolistic economy with reserves of capacity. Under these conditions, the rate of utilization would vary in line with the required rate of profit. Taking again into account equations (5) and (7), the rate of utilization needs to be:

(9) $$u^* = gk(1 - x)/s_f m.$$

This case is illustrated on the bottom part of Figure 2. An increase in the rate of accumulation and hence the rise in the required rate of profit are accommodated by an increase in the rate of utilization, at given profit margins (a fixed PC curve). The neo-Pasinetti theorem is perfectly compatible with rigid prices and profit margins. Here the valuation ratio and the rate of capacity utilization are the ultimate variables accommodating changes in the values of the parameters.

AN ENDOGENOUS RATE OF ACCUMULATION

Let us now relax the hypothesis of an exogenously given rate of accumulation and take into account an explicit investment function. This will remove the indeterminacy of the growth rate. In the standard Cambridge investment function, higher expected profits are assumed to induce entrepreneurs to enter into more investment projects. For a given stock of capital, this implies that the rate of accumulation ($g^i = I/K$) decided by entrepreneurs depends on the expected rate of profit, and hence, in long-run equilibrium, on the realized rate of profit ($r = P/K$). In linear form, we have:

$$g^i = \gamma + g_r r$$

where γ and g_r are parameters that represent the animal spirits of the entrepreneurs and those of bankers who grant credit allowing firms to go ahead with their investment plans.

The advantage of such a formulation is that it provides us with a recursive model, where the rate of accumulation is fully independent of profit margins [Lavoie 1992, 302]. The drawback is that, for the rest of the model, it is as if the rate of growth g were still a given variable. The results obtained with such an equation would not be any different from the ones presented in the previous section. The steady-state rate of growth would be determined in the upper graph of Figure 2, at the point where the NP curve intersects with the investment function given above.

Let us then consider an equation where investment is a function of the rate of utilization instead of the rate of profit, as many Kaleckians would have it when assuming that the rate of capacity utilization is an endogenous variable even in the long run. The logic of the argument is the following: the larger the current rate of capacity utilization, the higher the probability that some firms would hit full-capacity output as a result of the fluctuations of demand, and hence be unable to respond fully to demand and retain their share of the market. High rates of capacity utilization would thus induce more firms to enter into new investment projects, and hence, for a historically given level of capacity, induce a higher rate of capital accumulation. The investment equation in growth terms now is:

(10) $$g^i = \gamma' + g_u u$$

where γ and g_u are again parameters that represent animal spirits in the economy.

If equation (10) is the investment function, what then is the equivalent of the savings function? The savings function is given by equation (5), the function showing how accumulation is financed. The profit rate, however, must be given in terms of the rate of capacity utilization, which is done by making use of equation (7), the profits-cost function. These two equations yield the g^s curve of Figure 3 and the following savings equation:

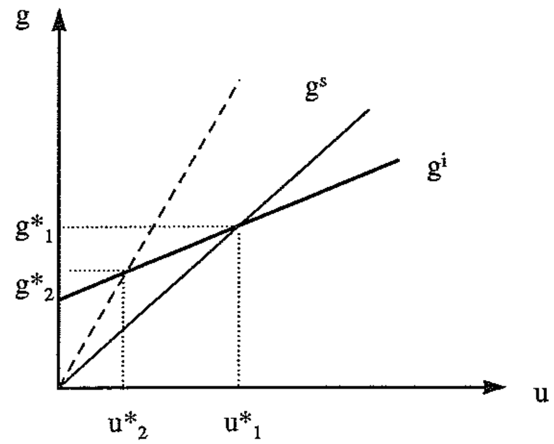
(11) $$g^s = s_f mu/k(1 - x).$$

Putting together the investment and the savings function — equations (10) and (11) — yields the steady-state rate of accumulation:

(12) $$g^* = (\gamma s_f)/[s_f - (k/m)(1 - x)g_u].$$

As is usual with models of this type, equilibria are stable if the slope of the investment function is smaller than that of the savings function. This implies that the denominator of equation (12) is positive (s_f must be sufficiently large), and hence that the constant term γ must be positive for g^* to have a positive solution, as is obvious from the inspection of Figure 3.

FIGURE 3



Since an increase in the retention ratio s_f or in the external finance ratio x will rotate the savings curve g^s towards the left, such changes in parameters will lead to a fall in the steady-state rate of accumulation. Both of these changes lead to a fall in effective demand. In addition the rate of accumulation g^* depends negatively on the profit margin m. An increase in the profit margin would again rotate the g^s curve to the left, thus inducing lower rates of utilization and of accumulation, as shown in Figure 3. As a result, the rate of profit is lowered. Standard Kaleckian results (the paradox of costs: higher real wage costs lead to higher realized profit rates) are thus vitiated within the framework of the neo-Pasinetti theorem.[23] Note again that in the neo-Pasinetti model, changes in the propensity to save of households (s_h) have no impact whatsoever on the equilibrium rate of accumulation, as long as investment does not depend on the valuation ratio.

VALUATION RATIOS AND GROWTH RATES

This leads us to an issue that has been emphasized by previous students of the neo-Pasinetti theorem, but which had been left aside to clear up the discussion. The issue is whether firms are truly free to accumulate as they wish, whatever the savings rate of households. Two observations have been made in the past: first, recall as we saw above by looking at equation (6), that a higher rate of accumulation, *ceteris paribus*, is associated with a lower valuation ratio; second, it can easily be shown, again by inspection of equation (6), that an increase in the propensity to save of households s_h is associated with a higher valuation ratio.[24] The decisions by the managers of firms to step up growth, when the savings decisions of households remain unchanged, may thus lead to lower valuation ratios and hence lower share prices.

The first question to tackle is whether or not valuation ratios will have any impact on the investment behavior of firms. Three positions may be taken. The first is to deny that valuation ratios (and hence implicit rates of return on shares) have any impact on the real investment decisions of firms. This would particularly be the case of managerial firms living in a world of fundamental uncertainty, where neither corporate managers nor shareholders have sufficient information and knowledge to influence the behavior of each other [Moss, 1976; Crotty, 1990]. That the valuation ratio is irrelevant to the investment decisions of entrepreneurs is also the opinion of New Keynesian authors. They argue that firms hardly ever sell shares to raise capital funds and hence that the price of shares is of no significance when compared to sales expectations [Greenwald and Stiglitz, 1987, 130]. This is also Kaldor's own position on the matter:[25]

> The stock exchange value of a company can fall to say one half of the value of the assets employed in the business. But this does not change the decision as to whether it is worthwhile to undertake some investment or not; the implicit rate of return would only become relevant to the firm's decisions if the normal method of financing investment were to be the issue of ordinary shares for cash — which in fact plays a very small role. Most of the finance comes from ploughed back profits, in which case the expected internal rate of return is relevant and not the implicit rate of return.

The second position is to argue, as in the first position, that real investment does not depend on the values taken by the valuation ratio, but only under the proviso that the latter exceeds unity ($v > 1$). Otherwise, the rate of accumulation would fall to zero. This is a position taken by several authors [Moss, 1978, 317; Lavoie 1987, 176; Abraham-Frois, 1991, 198]. They argue that when the valuation ratio falls below unity, plants can be purchased through financial markets at a fraction of production cost. As Keynes put it, "there is no sense in building up a new enterprise at a cost greater than that at which a similar enterprise can be purchased" [1936, 151]. In the neo-Pasinetti model with a fixed rate of capacity utilization, as can be derived from equation (6), the rate of accumulation has clear limits. A higher than unity valuation ratio requires the fulfilment of the following inequality:

$$s_h u_n > gk.$$

Finally, there is a third position: that the rate of accumulation will be continuously affected by the valuation ratio, as it would be in Tobin's q-theory of investment [Brainard and Tobin, 1968]. Rimmer [1993, 112] explicitly and Moore [1973; 1975] implicitly state this position.[26] While the first position was implicitly taken in the previous section, we now explore this third position.

Suppose that firms behave partly in a Kaleckian manner, and partly in a Tobinian manner; i.e., they speed up accumulation when the rate of capacity utilization is below its normal level, but they also speed up accumulation when the valuation ratio is above unity (i.e., when the internal rate of profit on capital is higher than the implicit rate of return on shares). The investment function would then be:

(13) $$g^i = \gamma + g_u u + g_v v,$$

or more precisely, it could be rewritten as:

(14) $$g^i = \gamma + g_u(u - u_n) + g_v(v - 1).$$

Whether one adopts the second or the third position, the implication is the same: firms are not quite free to accumulate as they please. If the valuation ratio must be above unity or some other value, it could happen that the efforts of firms to speed up accumulation would be hampered and even counter-productive, unless households are persuaded to save at a higher rate.[27] If the investment function must include the valuation ratio, then the neo-Pasinetti model loses part of its recursivity.

Let us check, however, whether an increase in the rate of accumulation still leads to a fall in the valuation ratio *when the rate of capacity utilization is free to vary*. We shall find that output growth and the valuation ratio are usually *positively* related, in contrast to what occurs in the standard formalization of Kaldor's neo-Pasinetti model. In this standard interpretation, it is assumed that the rate of utilization of capacity is a given, equal either to unity or to its normal level. The variable u which appears in equation (6) is thus a constant, and the negative impact of a higher rate of accumulation on the valuation ratio is thus certain, as we saw above. The contribution of the newer Kaleckian models is that these models, in contrast to the Cambridge models, old or new, do not assume that the rate of utilization is a given, even in the long run. The consequence is that, as the rate of accumulation increases, the rate of capacity utilization increases as well, as can be read from equation (9), thus providing a compensating positive effect on the valuation ratio, as can be seen from equation (6).

Let us then reconsider the equilibrium value of the valuation ratio, as given by equation (6), taking into account the flexibility of rates of capacity utilization by making use of equation (12). This valuation ratio is now given by:

(15) $$v^* = [(s_h)/(1 - s_h)][(1 - x)/ms_f - 1].$$

A higher than unity valuation ratio would thus require the following condition:

(16) $$s_h(1 - x) > ms_f.$$

The above two equations clearly show that any increase in the rate of accumulation induced by a change in one of the parameters of the investment function will have no impact whatsoever on the valuation ratio. On the other hand, an increase in the household propensity to save induces an increase in the valuation ratio, as was the case when rates of utilization were constant. It also turns out that higher external finance ratios, retention ratios, and profit margins all lead to lower valuation ratios. These results and those pertaining to the effects of these parameters on the endogenous rate of accumulation, when that rate is assumed to be impervious to changes in the valuation ratio, are presented in the table below. A higher rate of accumulation induced by a lower profit margin m — as would occur in the Kaleckian model and its solution given by equation (12) — would lead to a higher valuation ratio. Similarly, a fall in the retention ratio s_f or in the external finance ratio x, while inducing an increase in the rate of accumulation, would lead to a higher valuation ratio.[28] Finally, an increase in the household propensity to save, s_h, would have no direct impact on the rate of accumulation but would push up the valuation ratio.

Impact of changes in various parameters on the equilibrium values of the valuation ratio and the rate of accumulation, as given by equations (15) and (12)

	s_h	s_f	x	m	γ	g_u
v^* (15)	+	–	–	–	0	0
g^* (12)	0	–	–	–	+	+

The consequences of the above for a model in which the investment function would incorporate the valuation ratio, as in equations (13) and (14), are now obvious. Such an investment function would not alter the fundamental result of the neo-Pasinetti theorem, (i.e., it would have no impact on the claim that firms are free to accumulate as they please). Higher rates of accumulation resulting from higher animal spirits have no impact on valuation ratios, while the effect of lower retention ratios and lower profit margins on rates of accumulation and valuation ratios would be self-reinforcing: both would be increasing. The Keynesian causal mechanism — investment causes savings — and the Kaleckian paradox of costs are thus retained, even in a model where firms have a Tobinian behavior. However, an increase in the propensity to save of households would induce higher rates of accumulation, because of its positive effect on the valuation ratio. This is where introducing the valuation ratio in the investment function, as in equations (13) and (14), yields counter-intuitive results from a post-Keynesian perspective.[29]

CONCLUSION

Kaldor's neo-Pasinetti theorem can easily be integrated into Cambridge and Kaleckian frameworks. The adjustment of household savings to the investment decisions of corporations, based on their desired rate of accumulation, is done through the valuation ratio and the profit rate, as in the original neo-Pasinetti theorem. Whereas in the Cambridge model the profit rate had to adjust through changes in profit margins, in the Kaleckian variant the profit rate adjusts through changes in the rate of capacity utilization. The validity of the neo-Pasinetti theorem thus clearly extends to under-employment situations with excess reserves of capacity; it is not limited to full-employment or full-capacity economies.

An interesting feature of this modernized version of the neo-Pasinetti theorem, in contrast to its full-capacity version, is that higher rates of accumulation need not be associated with lower valuation ratios. Indeed, in many instances a higher rate of accumulation will induce a higher valuation ratio, because higher rates of growth will generally be associated with higher rates of capacity utilization, which have a highly favourable effect on valuation ratios. The belief, expressed by Joan Robinson, that firms are free to accumulate as they please within wide limits, is thus confirmed, even in an economy where the managers of firms take into account the implicit view of financial markets. This is an important result, because it has often been argued in the past that the Keynesian aspects of the neo-Pasinetti theorem do not hold when entrepreneurs incorporate the level of the valuation ratio in their investment decisions. The domain of validity for such an objection is narrower than previously thought.

It should be noted however that no attempt has been made to take into account the main critique that has been addressed against Kaldor's neo-Pasinetti model. As was pointed out by Davidson [1968; 1972], Kaldor assumes that shares are the only available asset: households must purchase shares with their entire savings, and hence they never try to acquire money balances. An alternative position is to suppose that "when banks provide deposits to satisfy the liquidity preference of rentiers, they hold shares of firms" [Robinson, 1971, 117]. A more adequate model would have to assume that firms issue shares and take on bank loans at a fixed interest rate, while households make a portfolio choice between bank deposits (or bonds) and shares. Such a generalized neo-Pasinetti model, dealing both with stocks and flows, has been proposed by Skott [1981; 1988], showing essentially that some of Kaldor's conclusions hold even in this more realistic case.[30]

APPENDIX

This appendix is devoted to the more general case with overhead costs. It turns out that some of the conclusions reached with the more simple case without overhead costs must be questioned. Let us first reassess the profits cost function. In the more general case with overhead costs, we may still interpret m as the margin of profit over direct costs, but the profits cost relationship now becomes more complex and can be written as:

$$r = [mu - (1 - m)f - \delta k]/k \tag{7'}$$

where f is the ratio of fixed to variable labor at full capacity utilization, and where δ is the rate of depreciation of capital [Rowthorn, 1981; Lavoie, 1992].[31]

Let us now consider what happens when this more complex profits cost function is used to compute the equilibrium value of the valuation ratio, i.e., when we combine the Kaleckian investment function (10) to the NP curve given by equation (5). From equation (6), we obtain:

$$v^* = \{(s_h)/[s_f k(1 - s_h)]\}\{\gamma(1 - x)k + s_f[(1 - m)f + \delta k]\}/\{m\gamma + g_u[(1 - m)f + \delta k]\} - (s_h)/(1 - s_h). \tag{15'}$$

Taking the partial derivatives of the equation above, it can be shown that, as before, a lower retention ratio, a lower external finance ratio, and a lower profit margin will induce a higher valuation ratio (and a higher rate of accumulation). In addition, a higher rate of depreciation δ and a higher ratio of fixed labor f will also induce both a higher valuation ratio and a higher rate of accumulation. For these variables, the introduction of the Tobinian q-theory investment equation makes no difference: the effects induced by the valuation ratio are self-reinforcing. On the other hand, in contrast to the simple case without overhead costs, an increase in the parameters representing the animal spirits of the entrepreneurs, γ and g_u, will induce a fall in the valuation ratio. If there are sufficiently strong feedback effects (a high g_v), an increase in the animal spirits of entrepreneurs may thus lead to some paradoxical results.

NOTES

This paper was presented at the inaugural conference of the Ph.D. program of the University of Brasilia, on April 2-4 1997. The conference was organized by Joanilio Rodolpho Teixeira on the theme "Money, growth, distribution and structural change". I wish to thank the participants at the confererence, as well as Mario Seccareccia, Steve Pressman and the anonymous referees of the journal for their useful comments. The usual disclaimers apply.

1. An account of these debates can be found in Harcourt [1972]. Articles are still being published about the Pasinetti and the anti-Pasinetti regimes: see for instance Samuelson [1991] and Baranzini [1991].
2. According to Thirlwall [1987, 169], Kaldor's neo-Pasinetti theorem "was left unchallenged" by the Cambridge, Massachusetts school. In a letter to Kaldor, Samuelson and Modigliani promised that they would respond to his model, but they never did.
3. This is clearly the interpretation of Robinson [1971, 117-25] in her discussion of investment financing and of Kaldor's neo-Pasinetti model. Shapiro [1977] argues that the revolutionary character of post-Keynesian models is that investment is independent of the intertemporal consumption decisions of households.
4. See Davidson [1968, 259; 1972, 300]. A higher valuation ratio is associated with a lower rate of return on shares — a lower rate of interest. This can be shown with the help of Richard Kahn's formula, established in 1964 but published only in Kahn [1972, 214], where the valuation ratio is given by $v = (r - g)/(i - g)$, where g, r and i are respectively the rate of growth, the rate of profit and the rate of return on shares. In a letter to me, dated 9th November 1983, Kaldor says that "in fact I did not realise that the valuation ratio indicates an implicit rate of return on shares."
5. The q-ratio usually comes down to be equal to $q = y_k/i$, where y_k is the marginal productivity of capital and i is the rate of return on equity [Rogers, 1989, 121]. Since the rate of profit (r) and the marginal productivity of capital are presumed to be equated in neoclassical theory, omitting the rate of growth of capital (g), the above relationship is similar to that established by Kahn, as shown in the previous footnote.
6. These authors are: Pettenati [1967], Davidson [1968, 1972], Robinson [1971], Marris [1972], Moore [1973; 1975], Moss [1976; 1978], Léonard [1980], Odagiri [1981], Skott [1981; 1988; 1989a; 1989b], Lavoie [1987; 1990], Rimmer [1989; 1993], Abraham-Frois [1989; 1991], Araujo [1995], Panico [1997].
7. The cause of this inequality is pointed out in his very first Cambridgian model of growth and distribution: "This may be assumed independently of any skewness in distribution of property, simply as a consequence of the fact that the bulk of profits accrues in the form of company profits and a high proportion of companies' marginal profits is put to reserve" [1956, 95]. Kaldor reasserted this ten years later: "I have always regarded the high savings propensity out of profits as something which attaches to the nature of business income, and not to the wealth (or other peculiarities) of the individuals who own property" [1966, 310].
8. Kaldor's views on savings propensities are given empirical substance in Marglin [1984, 409-411].
9. Savings in growth terms are given by S/K, where K is the replacement value of capital. u/k represents output per unit of capital and r is profit per unit of capital; therefore $(u/k - r)$ represents wages per unit of capital. Also $(1 - s_f)r$ are distributed dividends per unit of capital while $s_f r$ are the non-distributed dividends per unit of capital, i.e., the retained earnings of the firms. In a steady-state, an investment to the amount of $I = gK$ induces an increase in the value of all shares equal to vI. If however xI shares are being sold in the period, the capital gain *per share* is only $(v - x)I = (v - x)gK$. The capital gain per share per unit of capital is thus $(v - x)g$. A proportion $(1 - s_h)$ of this capital gain is assumed to be realized for consumption purposes, i.e., for dissaving.

10. The only exception is Skott [1988; 1989a, 71]. In other models, the capital/output ratio *k/u* is assumed to be constant. In neoclassical models, firms are usually assumed to be atomistic price takers and hence firms are assumed to produce at full capacity ($u = 1$), since their owners believe they can sell any amount at the going price [Dutt, 1990a, 16]. In Kaldorian models of the late fifties and early sixties, Kaldor assumed or tried to demonstrate that the only relevant equilibrium for the representative firm was one of full employment and full capacity utilization. Most of those who developed the neo-Pasinetti theorem thus kept these convenient assumptions [Skott, 1989b, 77-93].
11. Thus in the simple case where the propensity to save is the same whatever the source of income, the signs of the slopes of the two curves are the same as those arising from the case where savings out of dividends are excluded, which is the case considered by Araujo [1995] and by Kaldor [1966] in his main text. Thus the stability proof supplied by Araujo applies here as well.
12. The numerator of v^*, and hence v^* itself, is always positive, since $u > gk$ implies that total output exceeds output from the investment industry — a condition that would necessarily have to be realized.
13. In a neo-Pasinetti model akin to that of Moss [1978], with two classes of households, workers and capitalists, where the latter have a single propensity to save, Odagiri [1981, 104] demonstrates another variant of the neo-Pasinetti theorem: *i*, the *rate of return on stock market shares* only depends on the rate of accumulation and the propensity to save of the capitalists: $i = g/s_c$. This result is obtained with the help of Kahn's relation, as defined in footnote 4.
14. "It is not *necessary* to assume full employment in order that the 'Keynesian' mechanism of adjusting the saving ratio to the investment coefficient should operate" [Kaldor 1964, xvii]. In a letter to me, dated 30th October 1982, Kaldor says that "in retrospect I much regret that I used the full employment assumption which was the cause of great confusion."
15. The more modern Cambridgian models can be found in Eichner [1973], Wood [1975], and Harcourt and Kenyon [1976].
16. See Dutt [1990a] and Lavoie [1992] for the former, and Kurz [1994] for the latter.
17. The more general case with overhead labor costs is treated in the appendix.
18. Those who wonder why firms would want to keep capacity utilization at its normal rate can assume instead that utilization is at its full capacity level, meaning then that $u = 1$.
19. Wood [1975, 109] arrives precisely at this equation, but with the assumption that $u_n = 1$.
20. Davidson caught this assumption early on: "In Joan Robinson's model... it is assumed that competition brings down market prices (and profit margins) *at the normal or standard volume of output*" [1972, 125].
21. For various formalizations of the latter case, see Lavoie [1996].
22. Changes in output levels are also identified by Davidson [1972, 308] as a possible adjustment mechanism within the framework of Kaldor's model.
23. This effect can be linked to a similar result, achieved by Dutt [1990b] in a two-class model *à la* Pasinetti, to which savings by firms has been added. The paradox of costs is also sustained when rates of capacity utilization are flexible. Naturally, a drop in the animal spirits, given by γ or g_a, would also lead to a fall in the rates of accumulation and of capacity utilization, except that the g^i curve would shift down instead.
24. See the discussions around these two partial derivatives in Moore [1973; 1975], Lavoie [1990], Abraham-Frois [1991].
25. This is his position as expressed in a letter to me, dated 9th November 1983.
26. Delli Gatti, Gallegati and Gardini [1990, 107] provide a mix of the second and third positions, which they call the Keynes-Davidson-Minsky theory of investment determination, by assuming that the rate of accumulation is continuously affected by the valuation ratio as long as it exceeds unity, at which point a discontinuity arises with accumulation dropping to zero.
27. According to Moore such an argument "leads back to the neoclassical conclusions of the control of the rate of accumulation by saver preferences, albeit through a quite different mechanism. A reward to property must be paid ... to induce wealth owners to hold voluntarily, and not to spend on current consumption, the wealth accumulation that results from business investment" [1973, 543].
28. Note that the retention ratio s_f did not have an impact on the valuation ratio when the rate of accumulation *g* was seen as an exogenous variable (such a result requires however that households have a single propensity to save on all sources of income, as shown by Abraham-Frois [1991, 194]). When the realized rate of accumulation depends on one of the investment functions — equations (7) or (14) — this independence vanishes.

29. As argued by both Skott [1989a, 124] and Lavoie [1990, 130], this may be Asimakopulos's best defence when he made the statement — surprising from a post-Keynesian perspective — to the effect that "the independence of investment ... from saving is not as robust as Keynes stated. The investment market *can* become 'congested through shortage of saving', even in a closed economy" [Asimakopulos 1983, 230].
30. Another related critique is that the valuation ratio may often be determined by speculative forces rather than by non-speculative ones, as is implicit in Kaldor's neo-Pasinetti theorem. This is an objection which can be found in Davidson [1968, 258; 1972, 336] and Wood [1975, 118]. Finally, linked to this, it has been argued that capital gains might not induce consumption expenditures, in contrast to Kaldor's required mechanism which has been likened to a variant of the real balance effect [Davidson, 1972, 313; Mott, 1985-86, 227].

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