SHORT-RUN COSTS OF FINANCIAL MARKET DEVELOPMENT IN INDUSTRIALIZED ECONOMIES

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INTRODUCTION

The financial sector share in most industrialized economies has more or less doubled in the last few decades. The expansion is associated with decreasing, rather than increasing, growth rates over most of this period. Only in the last decade do we see a trend towards increased productivity growth. This is puzzling, since there is a well-documented positive correlation between financial and economic development, (Goldsmith, 1969; 1986). Growth-enhancing effects of financial markets seem to show up only in the long run while the short-run effect is to temporarily inhibit growth.

A tentative explanation is offered here. An extension of the financial sector requires organization of new markets. This can only be done in discrete steps due to indivisibilities. A fixed cost associated with these indivisibilities temporarily raises the relative cost of financial services. Thus, resources are diverted from production causing a temporary fall in the growth rate. However, increased risk sharing favors specialization, and as the economy adapts growth recovers. In the growing economy the benefits of pooling risks sooner or later again outweigh the fixed cost of a further extension of the financial markets, and so forth.

The model developed in this paper reconcilest long-run growth-enhancing financial development with short-run slow-down effects. Since the mechanisms are embedded in an endogenous growth model, saving rates take on a much more important role than in the traditional Solow growth model, the latter only permits transitory effects on growth while the former allows for permanent effects. Saving is negatively affected by increasing costs in the transformation of saving into investment, and probably also by decreases in precautionary saving. This is consistent with another stylized fact: the postwar decrease in national saving rates.

This paper provides a consistent theoretical framework to explain the dynamics of a staggered development with regime shifts in the relation between the financial sector and the growth of output in industrialized countries. The paper does not attempt to empirically test this hypothesis. There is, however, empirical evidence of regime shifts in the relation between the financial sector on one hand, and saving and growth, on the other hand (for the United States see Jacobson et al. [1996] and for the G-7 countries Lindh and Lindström [1997]). The next section reviews and summarizes the empirical and theoretical background. The conjectured explanation is formally modeled. The final section concludes and summarizes.
FINANCIAL DEVELOPMENT, GROWTH AND SAVING

The first subsection briefly illustrates some empirical facts in U.S. data. Theoretical and empirical research that is relevant for the paper is surveyed in the following two subsections.

An Illustration: United States 1929-1996

The middle panel in Figure 1 shows the ratio of financial corporate GDP to total corporate GDP in the United States. The growth rates of non-financial corporate GDP are plotted in the upper panel and the gross saving share of GDP in the lower panel.1

The financial sector share shows quite different trend behavior in different periods. One can discern a division of roughly four periods between 1929 and 1991. The first, 1929-1943, was a period of contraction, including the Great Depression and ending at the outbreak of World War II. The financial sector share shrank to less than half of its 1929 value in 1943. Up to around 1968 the financial share rapidly expanded again, recovering to approximately its 1930 value. Then followed a period of more tranquil growth in the share 1959-1981. In the last fifteen years the financial share again expanded rapidly (from around 7 to 10.5 percent).

The correlation coefficient between the financial sector share and the growth rate over the whole period is -0.14. Although this overall estimate is statistically insignificant, the correlation coefficients are negative in each of the sub-periods marked in Figure 1. In the contraction phase 1929-1943 the correlation is -0.63 and is statistically significant at the 5 percent level. In the more tranquil phase 1959-1981 the correlation is -0.35 which is borderline significant. In the expansive phases of the financial sector share 1944-1958 and 1982-1996 the correlation is still negative but close to zero and statistically insignificant.

A correlation is no proof of a relation, but it is still surprising in view of standard results showing a positive relation between financial development and growth. How come? The value added in the financial sector would seem intuitively to be related to financial development. High average growth rates in the 1960s and 1970s follow the expansions of the financial share in the 1950s and 1980s with a quite considerable lag. The gross saving share in the lower panel also seems to go down initially as the financial sector expands.

Descriptive statistics in Table 1 show that average growth rates in expansionary phases are lower than in the preceding periods. The linear growth trend estimate is significantly positive only in the contraction phase while the hypothesis of a constant growth rate cannot be rejected for the other periods. The corresponding saving pattern is very volatile up to the 1950s and the first expansionary phase, but does show a tendency to decrease in the second financial expansion period. From these data it would thus be reasonable to conjecture that financial sector expansions are associated with lower growth and possibly also a decrease in saving rates.2
Theoretical Background

Financial development can influence growth in three distinct ways by raising the proportion of saving actually invested; by raising marginal productivity in the economy; and by influencing the private saving rate. The first mechanism depends on the efficiency of financial intermediation, i.e., the fraction of saving absorbed to pay for financial intermediation services. The second mechanism works by improving the allocation of capital through information pooling (Greenwood and Jovanovic, 1990) or risk pooling, e.g., liquidity risks (Benavenna and Smith, 1991). The third mechanism has ambiguous effects on growth since saving may go either way. If consumers have utility functions with a positive third derivative, precautionary saving decreases. When, as will be assumed in the model of this paper, the utility function is of the constant relative risk aversion (CRRA) type, the relative risk aversion parameter is then greater than one. Financial markets may also ease liquidity constraints by providing consumption credit. On the other hand, financial development may also increase the rate of return and therefore boost saving. Devereux and Smith (1994) and Obstfeld (1994) study models incorporating these mechanisms. Pagano (1992) provides a brief overview and summary of the literature about financial development and growth. Levine (1997) contains a more extensive survey.

If there are inherent fixed costs in establishing financial markets—as is very likely since it entails new information networks—extensions will take place discretely. Saint-Paul (1992a) models how fixed financial market costs that imply a trade-off between risky specialization and financial costs for diversifying income risks may induce a poverty trap in an endogenous growth context. The model developed in the next section takes Saint-Paul’s model as its starting point. Alternatively, indivisibilities in the production technology may require a threshold level of financial development in order to be insurable (Acemoglu and Zilibotti, 1997). Yet another source for scale effects caused by informational asymmetries is explored by Khan (1999).

Different theoretical approaches tend—as Pagano (1992) points out—to yield similar implications for the relations between finance, saving and growth on the macro level. Growth is enhanced by financial development through a variety of mechanisms that improve productive efficiency or increase the volume of savings. Mechanisms, such as screening and monitoring, higher transaction efficiency, etc. working through liquidity risks, informational pooling, and so forth have been proposed in the literature. Growth is negatively affected by the resource cost of financial markets if risk aversion is high enough, possibly also by a negative effect on precautionary savings. Moreover, growth tends to feed back into increased demand for financial insurance by increasing aggregate risks.

While most of the literature concerns a long-run steady state, what we observe is a variation in the relations over different periods. Saint-Paul (1992a) and Acemoglu and Zilibotti (1997) allow for a one-shot take-off relevant for the discussion of how developing countries may get out of poverty traps by tapping into the international financial markets. This paper extends this work to a sequential process of market extensions, which makes the same ideas relevant for developed countries. As illustrated in the U.S. time series there are clearly distinct periods of financial develop-
tive impact on growth contingent on a sufficient educational level. A high correlation between educational level and GDP level indicates a productivity feedback. De Gregorio and Guidotti (1990) report further empirical evidence pointing in that direction. They find that most of the growth effects of financial services are through the marginal productivity of capital and not through investment volume. They also find a positive relation to growth in a cross-section of about 100 countries from 1960 to 1985, but a negative relation in a sample of six-year panel data for 12 Latin American countries between 1960-1985. Although they explain the negative short-run relation by financial repression, it could also be interpreted as a short-run negative growth effect from financial market extensions, thus supporting the second conjecture.

Ajte and Jovanovic (1993) find effects on growth from stock markets in a broad sample cross-country regression based on Greenwood and Jovanovic (1990), but fail to find this for bank lending. A partial explanation might be found in dappelli and Pagano (1994) who find that household credit has a negative influence in mature countries, and attribute this to the liquidity constraints in less developed countries, which force higher saving for the young generation. This would also help explain the Fernandes and Galetonic (1994) result and be consistent with the fourth conjecture in developed countries.

Some direct firm evidence points to scale effects in the financial sector that would be consistent with fixed market costs. Suseman and Zeira (1993) and Harrison et al. (1999) find in U.S. cross-state data that banking costs are lower where income per capita is higher and financial centers are geographically closer. In Saint-Paul (1992b), a circular localization model based on the same idea of fixed financial costs as Saint-Paul (1992a) explains dual economies by the segmentation of financial markets, making it hard to diversify financially in some branches or regions of the economy. Empirical evidence from Japan (over branches) and Italy (over regions) is consistent with this.

Finally, regime shifts in the relations between saving, growth and financial sec-
tor shares are reported in Lindh and Lindstrom (1997) for the G-7 economies. The structural breaks can be tied to major institutional changes in the financial markets and regulations. Jacobson et al. (1998) apply Markov-switching regressions to U.S. data 1948-1996 and find evidence of several regime shifts that also can be connected to major institutional changes in the financial system. Short-run Granger causality analysis indicates that the financial share Granger causes saving with a negative effect but is otherwise inconclusive. All three variables combine to predict regime shifts, indicating that this is endogenously determined.

**A MODEL OF FINANCIAL MARKET INTERACTION WITH GROWTH AND SAVING**

The model modifies the conventional two-period overlapping generations (OLG) model in several dimensions. Agents are spatially fixed and one location requires an additional fixed cost. Technological choice is endogenous and made by agents, acting as entrepreneurs. Following the tradition, labor supply is assumed to be exogenous and normalized to unity at each location, with the additional restric-

**SHORT-RUN COSTS OF FINANCIAL MARKET DEVELOPMENT**

This allows us to view each local set of agents as a representative individual.

Insurance against the uncertainty of old age income is achieved either by flexible, less specialized technologies at the cost of lower productivity, or by diversifying in-
vestment to other locations at a fixed cost. Extensions of the financial markets are
discrete events. Specialization, on the other hand, is not immediate, due to convex adjust-
dment costs, since time-consuming investment, research, and learning are neces-
sary.

**Formal Specification**

Local production at location \( i \) per labor unit is \( x_i \), and \( k_i \) is the average capital services per labor unit. Without loss of generality it is assumed that all capital is consumed over the production period. The local technology is affected by positive externalities from \( k_i \) such that average production becomes

\[
x_i = A_i k_i^d \quad \text{and} \quad A_i = B_i k_i^e
\]

Since aggregate local production \( B_i \) is linear in \( k_i \), there are no decreasing re-
turns to capital. The technology factor \( B_i \) is a random variable with a distribution conditional on the degree of specialization—an arbitrary index \( x \). The choice of tech-
nology is restricted, such that the expected value of \( B_i \) and its variance both increase strictly with \( x \). The \( B_i \) in different locations are independently distributed and ident-
ical for given \( x \). Specialization will not be more explicitly modeled as this index could also be more broadly interpreted as any increase in expected productivity that is accompanied by increasing risk.

Financial markets link different locations by providing mutual diversification opportunities. The analysis here is conducted from the point of view of one specific location taking the rest of the world as given. At each time there are a number of possible connections to other locations, providing access to already established financial markets. The alternative extensions are ordered by a discrete index \( q \). This index measures the extent of the financial markets, such that, ceteris paribus, the expected utility of a given investment is increasing in the index \( q \).

It takes at least one period to start changing the degree of specialization. Agents therefore take the current distributions, as well as the realization of their stochastic labor income, as given. A competitive labor market ensures that labor income equals the marginal product of labor \( b_i k_i^{1-d} \) (skipping indices henceforth when unnecessary). Consequently capital income is \((1-\delta)e^{x} = (1-\delta)B_i\), which is consumed in the retirement period.

Let \( F_{x} \) be total financial costs at financial extension \( x \), such that \( F_{x} > F_{x'} \) for some fixed amount \( e \). Let \( \beta \) be the discount factor and \( \mu \) the saving share of disposable income after financial costs, \( \gamma = \gamma - F_{x} \). Young agents maximize expected utility over their lifetimes by choosing the share of disposable income to save and by adjusting their portfolios of investment shares \( q \) in different locations.
### Schematic Diagram of the Sequence of Decisions in the Model

<table>
<thead>
<tr>
<th>Period</th>
<th>t−1</th>
<th>t</th>
<th>t+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohort i−1:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohort i:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohort i+1:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrepreneurs:</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### FIGURE 2

| Cohort i−1: | | Contingent on $a_i$ |
| Cohort i: | | Contingent on $a_i$ |
| Cohort i+1: | | Contingent on $a_{i+1}$ |
| Entrepreneurs: | | Contingent on $a_i$ | Contingent on $a_{i+1}$ |

### Properties of the Equilibrium

Conditional on $g$ and $s$, the first-order necessary maximization conditions for equation (2) require that marginal utilities between periods are equalized by choosing $\beta$ as

$$y_{it}^{0} u(c_{it}) = \delta \frac{\partial E(u(c_{it}))}{\partial \beta},$$

and, simultaneously, that marginal utilities of each local investment are equalized by the portfolio choice

$$\frac{\partial E(u(c_{it}))}{\partial a_{it}} = m, \text{ for } i = 1, \ldots, n,$$

where $m$ is the shadow value of each investment unit. The first-order condition in equation (4) with respect to $\beta$ can be rewritten, using the CRRA specification (3) and dividing by $(y_{it}^0)^{1−p}$, as

$$\max_{\beta, a_{it}} (1 - \beta) y_{it}^{1−p} - \Delta E[(1 - \beta) P_{it} a_{it} (y_{it}^0)^p] \text{ subject to } \sum_{i} a_{it} = 1,$$

where $c$ is consumption, and $u(c)$ is a constant relative risk-aversion (CRRA) function.

$$u(c) = \frac{c^{1−p} - 1}{1−p}, \quad p > 0 \text{ and } p \neq 1.$$

If $p = 1$ then $u(c) = \log c$. The distribution of $B_t$ is Gaussian, or has finite support, so a unique maximum of expected utility of capital returns, $E(u((1 - \beta) a_t^{1−p})$, with respect to $s$, exists. A competitive capital market equilizes returns within locations. Entrepreneurs choose both capital intensity and specialization, but the latter is only gradually adjusted to its optimal value. This choice is not formally modeled since free entry for entrepreneurs will ensure that $s$ converges to its expected utility-maximizing value. The competitive factor markets ensure that the local realization of $B_t$ is the same for all local entrepreneurs, which is why we do not have to distinguish between a firm's capital intensity and the average capital intensity.

If there were no financial costs, risk-averse agents would prefer to insure against risk by spreading investments over many locations as possible, (i.e., integrating the financial markets maximally). The presence of a fixed cost implies that financial diversification will only be taken to the point where the incremental expected utility offsets the decline in utility from the incremental cost. The young agent must therefore also decide at which level of financial diversification his expected utility will be maximized.

This choice is made by comparing maximal utility in equation (2) for two financial extensions, while taking current specialization as given. The sequence and timing of decisions in the model are summarized in Figure 2.

In Figure 3 the LHS schedule is therefore fixed and the RHS schedule shifts up (down) as $\beta$ increases (decreases).
Financial Extensions and Specialization

With the degree of specialization given, there are three effects of an extension of the financial regime. First, it decreases variance in expected old age income; second, it decreases disposable labor income; third, the optimal β changes as the asset portfolio is reallocated. To analyze how β changes we only need to know how R changes.

Expand the integral in R in a second order Taylor approximation around the expected value, \( E(\Sigma_i B_i a_i) \).

\[
R \left( \sum_i B_i a_i \right)^{1-p} = \mu^{1-p} \left( 1 - \frac{1}{2} (1-p) \mu \right) \left( \sum_i B_i a_i \right)
\]

If expected returns are constant over the two regimes it is obvious that R decreases (increases) as variance is increased (p>1/2 and p<1/2) and the optimal savings share will decrease (increase) correspondingly. With low risk aversion the agent will shift disposable labor income to investment as risk decreases to increase future consumption. With higher risk aversion he will prefer to shift disposable income to current consumption instead. A slightly more complicated argument in the Appendix shows that this conclusion holds also when expected returns change.

To the agent an extension becomes favorable as the optimal expected utility in the current financial regime becomes less than in the alternative. Eliminating constants on both sides, the equilibrium at extension \( \eta \) becomes sub-optimal as

\[
\frac{c_t^{\eta} - p}{1-p} + \delta \int \frac{c_t^{\eta} - p}{1-p} d\Phi < \frac{c_t^{\eta} - p}{1-p} + \delta \int \frac{c_t^{\eta} - p}{1-p} d\Phi + \lambda \eta.
\]

The first order conditions in equation (4) imply that

\[
\frac{c_t^{\eta} - p}{1-p} + \delta \int \frac{c_t^{\eta} - p}{1-p} d\Phi < \frac{c_t^{\eta} - p}{1-p} + \delta \int \frac{c_t^{\eta} - p}{1-p} d\Phi + \lambda \eta.
\]

Using this and multiplying both sides with 1 + p we get, when p > 1

\[
\frac{c_t^{\eta} - p}{1-p} + \delta \int \frac{c_t^{\eta} - p}{1-p} d\Phi < \frac{c_t^{\eta} - p}{1-p} + \delta \int \frac{c_t^{\eta} - p}{1-p} d\Phi + \lambda \eta.
\]

If p < 1 the inequality is reversed. Since \( c = (1 - \beta)(\gamma - F) \) this implies

\[
\frac{y - F}{y - P_{q1}} < \frac{1-\beta_{q1}}{1-\beta_{q2}}.
\]

This condition will eventually hold, if disposable income growth is strictly positive. Since p > 1 the savings share \( \beta_{q1} > \beta_{q2} \), so the RHS in equation (12) is greater than unity and the LHS converges to unity from above as \( \gamma \) grows. Reversing inequalities when p < 1 the condition will again hold eventually. Then \( \beta_{q2} > \beta_{q1} \), so the RHS in equation (12) is less than one while the LHS goes to one from below.

Total savings, \( \beta_{q2} \), will obviously fall when p > 1 at the financial market is extended. For p < 1 total savings will decrease in some cases, too, since disposable labor income decreases. As entrepreneurs adapt s to the new financial regime, risk increases in production, but is balanced by increases in the expected value of B. Entrepreneurs will only change s to increase the expected utility of agents, but, as above, the behavior of R depends on risk aversion. It is shown in the Appendix that the R again decreases (increases) when p > 1 (p < 1). So when specialization has adapted, the savings share will be even lower (higher).

If \( F \) was increasing with output or investment, specialization might ultimately de-stabilize the new financial extension and lead to a contraction of the financial markets. Depending on the rate of growth, this may even give rise to cyclical behavior. Such cost behavior could for example arise if costs are tied to the volume of trading, as they often are.

However, in the real world we would also expect some offsetting factors, e.g., economies of scale in information handling, a downward drift in financial costs due to improved transaction technologies, and external effects from neighboring financial markets.
TABLE 2
Summary of the Expected Effects in the Transition to an Extended Financial Regime and the Following Increase in Specialization if \( F/y \) Increases

<table>
<thead>
<tr>
<th>Transition</th>
<th>( p=1 )</th>
<th>( p=1 )</th>
<th>( p&lt;1 )</th>
<th>Specialization</th>
<th>( p=1 )</th>
<th>( p )</th>
<th>( p&lt;1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>( \beta )</td>
<td>0</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>( \nu/y )</td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>( \nu/y )</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-</td>
<td>-</td>
<td>?</td>
<td>( \gamma )</td>
<td>*</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>( F/y )</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>( F/y )</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

**Growth Rates**

For a tractable analysis of growth rates we simplify further in this subsection by assuming that the \( F_{i} \) are identically distributed at all locations encompassed by the current financial markets. This implies that investment in local production equals the total investment of local agents. Then the growth rate of output is

\[
g_{x} = \frac{x_{t+1}}{x_{t}} - 1 = F_{B} b_{t+1} - 1 = F_{B}^{\beta} x_{t+1} - 1 = b F_{t} + \frac{F_{t}}{x_{t}} - 1.
\]

The growth rate of local disposable income (for the young) is higher

\[
g_{y} = \frac{y_{t+1}}{y_{t}} - 1 = \frac{b h_{t+1} - F_{t}}{y_{t}} - 1 = \frac{b F_{t} - F_{t}}{y_{t}} - 1
\]

as can be seen by taking the difference

\[
g_{x} - g_{y} = \frac{F_{t}}{y_{t}^{2}} = \frac{B F_{t} x_{t}}{x_{t}^{2}} = \frac{F_{t}}{y_{t}^{2}} = \frac{B F_{t}}{x_{t}} - \frac{F_{t}}{y_{t}^{2}} - 1.
\]

With positive growth in \( y, x, F_{t} \) converges to \( g_{x} \), which in turn converges to \( b b F - 1 \), if financial costs decrease relative to income. The financial regime associated with a higher \( g \) will on average have an initial slump in the growth rate since \( b F_{t}^{\beta} \) decreases for empirically plausible values of the risk aversion parameter. As entrepreneurs adjust specialization to the new regime, the expected value of \( F_{t} \) will increase, and hence the growth rate also decreases while \( b \) decreases further. Recall that a decreasing saving rate refers to \( y^{s} \). Since the relative leakage through financial costs decreases, the effect on savings as a share of \( y^{s} \) is a prior ambiguous. Expected utility increases with specialization and variance increases. The increase in risk is balanced by an increase in the expected growth rate. Of course, the higher the risk aversion, the more sluggish will the increase in the growth rate be.

Whether growth rates increase relative to the previous equilibrium depends on the parameters. The pattern of growth to expect from the model would be a saw-tooth diagram of the growth rate, collapsing as a new financial regime was introduced and then slowly recovering again. The financial cost share would be expected to show sharp increases at transitions. Whether this share tends to grow or decrease between transitions would depend on exactly how costs are related to the scale of production.

In any case the relationship between growth and the financial share would not be linear over time. Savings as a share of gross product would decrease at the transition and is indeterminate between transitions.

Table 2 summarizes the effects predicted by the model for different values of the risk-aversion parameter. These predictions are, however, contingent on the assumption that \( F/y \) decreases in the period following an extension. In Figure 4 the dashed lines presume that financial costs increase with production fast enough to dominate the scale economies generated by the fixed costs. This gives an impression of the differences. These graphs are not formally derived from the model but illustrate possible patterns that are consistent with the model.

In the real world there are interactions between countries, changes in institutional and political regimes, and so forth, that will cause patterns to differ over time. Moreover, there are adjustment costs, which are likely to cause considerable delays.
could, in principle, be extended to any form of financial insurance, including social welfare and other institutions for income insurance.

A natural extension would be to consider insurance against variability in labor income by a social welfare state or other institutions like long-term labor contracts. One would expect that differences in cost structure between different insurance devices, i.e., social institutions, would become crucial to long-term growth.

Finally, it could be noted that this kind of model is a natural complement to the literature on technological shifts. As David [1991] shows very convincingly with regard to the electric dynamo there is a quite considerable lag of 20 to 30 years before new general purpose technologies actually start to yield a productivity payoff. This is because new intermediate components, new organizations, and new distribution channels need to be developed and not least the labor force needs to be trained, before the potential in the new technology can be realized. Many believe that the so-called “new economy” is a sign of the computer technology finally starting to yield productivity gains, which previously have been hard to find. A quite extensive literature—both theoretical and empirical—now studies such technology shifts, see Holmström (1988) for a sample. This relates to the paper in that the initial phase of development and training calls for quite extensive investment to replace old technology, thus producing a demand for increased financial intermediation. In Eriksson and Lindahl [2000] a formal model of this process is shown to produce patterns very similar to those implied by the financial development model in this paper.

APPENDIX

In the section entitled, “Financial extensions and specialization” it was asserted, that the saving rate increases (decrease) as the risk-aversion parameter is below (above) unity and also whenever expected returns change. This assertion is formally proved here.

Proposition:

a) The saving rate $\beta$ increases (decrease) at the transition to a more extended financial market when the risk-aversion parameter $p < 1$ ($p > 1$). If $p = 1$ the saving rate is constant.

b) The saving rate $\beta$ increases (decrease) as production becomes more specialized when the risk-aversion parameter $p < 1$ ($p > 1$). If $p = 1$ the savings rate is constant.

Proof:

a) First note that equation (7) can be written $(\beta(1-\beta))p = R$. Taking the differential confirms that $\beta$ increases monotonically with $R$. Second, note that the difference in expected utility at a transition must be positive:
\[
\frac{(1 - b)\gamma_{\text{eq}} \sum_{i=1}^{n} \alpha_i B_i \bar{a}_i^{2} \phi_{i} \phi_{i+1} - (1 - b)\gamma_{\text{eq}} \sum_{i=1}^{n} \alpha_i B_i \bar{a}_i^{2} \phi_{i} \phi_{i+1}}{\bar{a}_i} > 0
\]

\[0 < p < 1: \text{Equation (A.1) implies that } (\beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}) > (\beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}) \text{. Hence if } R_{\text{eq}} < R_{\text{eq}} \text{, it follows that } \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} > \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \text{. Since } \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1} = \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1} \text{, thus contradicting that } \beta \text{ increases monotonically with } R \text{. Therefore it must be that } R_{\text{eq}} < R_{\text{eq}} \text{ and } \beta_{\text{eq}} > \beta_{\text{eq}}.\]

(ii) \( p > 1 \): Equation (A.1) in this case implies that \( (\beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}) < (\beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}) \). By equation (7) we get

\[
\frac{\beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}}{\beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}} < 1 \Rightarrow \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1} < \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1} \frac{1 - \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}}{\beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}} \frac{1}{\beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1}}
\]

so we can divide away the \( \beta_{\text{eq}} \). Since \( \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1} \geq 1 - \beta_{\text{eq}} > 1 - \beta_{\text{eq}} \), we obtain a contradiction and conclude that \( \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1} \leq 1 - \beta_{\text{eq}} \). Then \( \log(1 - \beta_{\text{eq}}) \leq 0 \) and the first order condition w.r.t. \( \beta \) reduces to \( \beta \frac{1}{1 - \beta} = 0 \) so the savings rate will not depend on a financial extension.

b) In this case no fixed cost need be considered, so it is sufficient to review the case where specialization increases between two periods with given disposable income. If expected utility is increased it must hold that

\[
\frac{\left( \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1} \left( 1 - b \right) \right) \sum_{i=1}^{n} \alpha_i B_i \bar{a}_i^{2} \phi_{i} \phi_{i+1} - \left( \beta_{\text{eq}} \gamma_{\text{eq}} \phi_{\text{eq}} \phi_{\text{eq}+1} \left( 1 - b \right) \right) \sum_{i=1}^{n} \alpha_i B_i \bar{a}_i^{2} \phi_{i} \phi_{i+1}}{\bar{a}_i} > 0
\]

(i) \( 0 < p < 1 \): Essentially the same argument as in a) (ii) remains true only with the modification that we do not need the argument that disposable income has decreased.

(ii) \( p > 1 \): Again we proceed as in case a) but in this case the middle part of the first inequality in equation (A.2) is not needed. Alternatively note that the change in the distribution as specialization increases will, by assumption, cause more density to be concentrated at higher outcomes on average. The fact that the integrand is decreasing in outcome will then directly imply that \( R_{\text{eq}} < R_{\text{eq}} \) and \( \beta_{\text{eq}} = \beta_{\text{eq}} \).

(iii) \( p = 1 \): Exactly the same argument as in a). QED
REFERENCES


