WHEN ARE STATE LOTTERIES A GOOD BET (REVISITED)?

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INTRODUCTION

It is generally conceded that among games of chance state lotteries have among the worst expected payoffs. State lotteries return on average only 40 to 60 percent of the ticket price to the bettors, while by comparison, craps returns 98.6 percent, blackjack returns 99.5 percent, American roulette returns 94.74 percent, slot machines average approximately a 95 percent return, and pari-mutuel sports betting returns 91 percent [Eadington, 1999]. Under specific theoretical conditions, however, many researchers have hypothesized that certain types of lottery games can have payoffs exceeding 100 percent. This paper examines the conditions that would make the purchase of a lottery ticket a "fair bet," (i.e., the conditions necessary for the expected return of a lottery ticket to be higher than its price).

The so-called "Lotto" game, which consists of a player choosing a group of 6 numbers out of 35 to 55 possibilities, is among the most popular games offered by state lottery associations.[1] As of 1997, every U.S. lottery association offered some version of the game, and lotto games accounted for 28 percent of total lottery revenues nationwide [U.S. Census Bureau, 1999]. Lotto generally has two payoff components. First, individuals who match three to five of the winning numbers but do not match all six receive smaller prizes with a fixed dollar payout or a pari-mutuel payout based on the current period ticket sales and the number of current period winners. This lower-tier prize component generally amounts to 15 to 30 percent of the price of the ticket.

The second component is the jackpot prize. A portion of ticket sales, ranging from 20 to 40 percent of gross ticket sales, is diverted into the jackpot prize fund. A player who matches all six numbers exactly wins the amount in the fund. If more than one ticket matches all the numbers, the money in the fund is divided equally among each of the winning tickets. If no ticket matches the winning numbers, the money in the fund is added on to the ticket sales in the next period. (This amount is referred to as the "roll-over" in the literature.) If no ticket exactly matches the winning numbers in a large number of successive drawings the jackpot can potentially become quite large. Hence, it is reasonable to assume that the purchase of a lottery ticket may become a fair bet in the presence of large jackpots. See Table 1 for a sample of some of the largest jackpots to date in U.S. lotteries.

It is quite common for lottery tickets to be purchased by groups of bettors who form betting pools in order to increase their odds of winning the jackpot. For example,

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TABLE 1
15 Largest U.S. Lotto Jackpots through June 30, 2000

Jackpot	Lottery Drawing	Date
$363 million	Big Game	5/19/00
296 million	Powerball	7/29/98
195 million	Powerball	5/20/98
190 million	Big Game	4/06/99
160 million	Powerball	6/30/99
150 million	Powerball	3/04/00
119 million	California Super Lotto	4/17/91
116 million	Pennsylvania Super 7	4/26/89
111 million	Powerball	7/07/93
107 million	Florida Lotto	9/15/90
102 million	Powerball	3/04/95
102 million	California Super Lotto	4/08/98
101 million	Powerball	11/30/94
90 million	Florida Lotto	10/26/91
90 million	Powerball	12/22/93

the first jackpot won in Pennsylvania's Super 7 jackpot was won by the "Shippers 10," ten coworkers from York, PA. If the jackpot were to rise high enough, it is reasonable that a coalition of bettors may wish to attempt to purchase every possible combination of numbers to guarantee winning the jackpot. This strategy has been dubbed the "Trump Ticket" by Krautmann and Ciecka [1993]. The purchase of a Trump Ticket has been tried at least once in U.S. lotto history when an Australian consortium attempted to purchase every number combination in a 1992 drawing of the Virginia Lotto.

An understanding of how lottery ticket buyers react to large jackpots allows state lottery associations to design games that maximize the number of tickets sold and thereby maximize the amount of revenue transferred to state governments. In particular, if lottery ticket sales remain steady despite returns that approach a positive expected payoff, then lottery associations are clearly not operating games in which ticket sales are maximized.

The remainder of this paper will examine whether lotteries ever provide bettors with a positive net expected return. The first section examines previous attempts in the literature to address this issue. While the problem of the expected return for lottery games has been modeled extensively in the past, previous attempts to ascertain whether lotteries ever provide a positive net expected value have omitted certain variables necessary to determine the true expected value. Particular omissions are the taxation of lottery winnings and the tax deductibility of lottery ticket purchases. In addition, past models have based their conclusions on estimates of individual drawing ticket sales rather than actual ticket sales as reported by the state lottery associations. The second section corrects for the deficiencies of previous studies and uses a revised model to examine two lottery drawings, the $27 million drawing of the Virginia Lotto in 1992 mentioned previously and the $106.5 million drawing of the Florida Lotto on September 15, 1990, both which have been examined in previous papers.

The differences between this model and the previous models are large enough to change the conclusions as to whether certain lotto drawings have provided a fair bet to the participants.

Next, each of these drawings will be examined to discover whether these drawings would have provided a "good bet" for a coalition of buyers purchasing the Trump Ticket. From the results in this paper, I draw exactly the opposite conclusion regarding the profitability of the purchase of the Trump Ticket as those in previous analyses. The final section examines two more lottery drawings, the $296 million Powerball jackpot in July 1998, which is of interest because it was the largest recorded lottery jackpot at the time, and an $18 million drawing of the Oregon Lotto in February 1999, which is notable because of the high expected value of the game. The paper closes with conclusions and recommendations for lottery associations.

PAYOFFS FROM PURCHASING A SINGLE LOTTERY TICKET

Since the price of a lotto ticket and the odds of winning remain fixed regardless of the size of the jackpot, it is natural to assume that the expected return of purchasing a lotto ticket will simply increase with the size of the jackpot. However, the complicating factor is that as the advertised jackpot grows, the number of ticket buyers increases as well, which increases the probability that the winning numbers will be shared by two or more tickets. As noted by Clotfelter and Cook [1993], the increase in expected return due to the increase in the size of the jackpot is tempered by the prospect of potentially having to share this larger jackpot among several winners. In order to truly estimate the expected return from the purchase of a lotto ticket, one must not only know the size of the prizes and the odds of winning but also the odds of sharing the jackpot prize with one or more additional winning tickets.

Many studies including Clotfelter and Cook [1989, 1990], Theil [1991], Papachristou and Karamanis [1998], and Thaler and Ziemba [1988] have mentioned the possibility of lotto games that provide a fair bet. Krautmann and Ciecka [1993], Gulley and Scott [1993], Scott and Gulley [1995], Clotfelter and Cook [1993], and Ciecka, et al. [1996] present the most detailed attempts to describe the necessary conditions for the purchase of a lottery ticket to be a fair bet. Following their formulations, the expected return, ER_t, from the purchase of a single lottery ticket is directly derived from the definition of expected value which states that the expected return from a lottery ticket is simply the probability of winning a particular prize times the value of the prize won summed over all prize levels.

$$ER_t = \sum^{i} w_i V_{it} + w_j DV_{jt} \sum_{m=0}^{B_t} [p_m/(m+1)] \qquad (1)$$

where w_i is the probability of winning lower-tier prize i, V_{it} is the cash value of lower-tier prize i at time t,w_j is the probability of winning the jackpot prize, DV_{jt} is the discounted present value of the jackpot prize at time t,m is the number of tickets bought by competing players matching the jackpot prize, p_m is the probability that exactly m other tickets match the jackpot prize, B_t is the number of other ticket buyers for the drawing in period t.

The w_i's and w_j can be calculated in straightforward manner for any lotto based on the rules of the specific lotto [Packel, 1981]. For lower-tier prizes with a fixed prize value, V_{it} is a fixed dollar amount set by the lottery association and no further calculations or assumptions are necessary. Roughly one-third of lottery associations, including those of the large multi-state lotteries, Powerball and Big Game, use fixed dollar amounts for all lower-tier prizes.[2] For lower-tier prizes where the payout is pari-mutuel, it is convenient to assume that the expected payout from the prize will be the average expected payout. This assumes that other lottery players are equally likely to choose any combination of numbers.[3] Using this assumption, V_{it} is simply equal to α_i where α_i is the percentage of the ticket price allocated to lower-tier prize i. The majority of state lotto games, including those in both Florida and Virginia examined in depth later, use pari-mutuel payoffs for lower-tier prizes. For both fixed and pari-mutuel prize structures, the V_{it}'s are paid immediately to the winner on presentation and verification of the ticket. Therefore, the time value of money does not need to be considered when examining the lower-tier prizes.

On the other hand, state lotteries traditionally require winners to take jackpot winnings in annuity payments over an extended time period, usually between 20 and 30 years. A handful of small cash lottos (such as the Delaware All-Cash Lotto, Kansas Cash!, Minnesota Gopher 5, and the multi-state Wild Card Lotto) are exceptions. The jackpot prize fund is invested in interest bearing accounts from which the winner receives annuity payments over a specified number of years. Lottery associations announce the jackpot prize as the undiscounted nominal sum of these annuity payments. It is possible to convert an advertised jackpot AV_{jt}, paid in equal amounts of (AV_{jt}/n) over n years given a discount rate of δ into a discounted present value, DV_{jt}, using equation (2).

(2) $$DV_{jt} = (AV_{jt}/n)\sum_{k=0}^{n-1}(1+\delta)^{-k}$$

It should be noted that lottery players always receive their first annuity payment immediately upon presenting the winning ticket so that the first annuity payment is not discounted. This is in contrast to Krautmann and Ciecka [1993] who discount this first payment. For discount rates of 8 percent, DV_{jt} is approximately 53 percent of AV_{jt} for a 20-year annuity and approximately 40 percent of AV_{jt} for a 30-year annuity.

Typically, solving equation (2) would require knowing the lottery winner's personal time preference for current earnings. While personal time preference is subjective for each winner, several features of the lottery make it a relatively simple matter to objectively find the appropriate discount rate. First of all, almost all lottery associations have in recent times begun to allow lottery winners to take the jackpot in a lump sum. Of course the jackpot is still advertised as the sum of annuity payments in an effort to make the jackpot appear larger. Because the various lottery associations typically invest the jackpot fund in zero-coupon Treasury bonds, the cash value of the advertised jackpot can be calculated by discounting the future payments by the interest rates for zero-coupon Treasury instruments of the appropriate maturity.

Alternately, lottery winners should be able to borrow today in available liquid capital markets against future winnings. Indeed, numerous finance companies will

purchase a lottery winner's annuity for a current cash payment. Since the lottery payments that serve as the collateral for the loan are guaranteed by the state lottery associations, these loans can essentially be seen as risk free loans for the length of the lottery annuity. The effective interest rate that a lottery winner would receive on such a loan would be approximately the same as that of Treasury bonds or AAA-rated corporate bonds of the same duration. Since any lottery winner with a high time preference for money can convert the annuity stream into cash at prevailing market rates, the current market rates represent the upper bound for the appropriate discount rates.

The taxation of lottery winnings makes it preferable to borrow against future lottery payments rather than to take the cash payment unless the entire cash payment will be spent immediately or alternative investments are available that have an after-tax return that at least exceeds the available interest rate at which the lottery winner can borrow funds. The reason for this is that the annuity option allows lottery winners to defer taxation on their winnings until the annuity payment is actually made, as well as the fact that a single large payment puts the lottery winner in a higher average tax braket due to the progressive income tax rates. See Atkins and Dyl [1995] for more details.

Because of the issue of taxation, the current market rates represent only an upper bound for the appropriate discount rate. A winner with a low time value of money below that of the prevailing market interest rates would value the jackpot at a higher value than the sum of the annuity payments discounted at current market interest rates. However, we believe this issue to be of minor importance to our findings.

The binomial function is used to calculate the probability that exactly m tickets purchased by other bettors match the winning jackpot numbers and is a direct function of B_t. Equation (3) describes this function.

(3) $$p_m = [B_t!\,w_j^m(1-w_j)^{(B_t-m)}]/[m!(B_t-m)!$$

Using the Poisson distribution as an approximation to the binomial distribution, Gulley and Scott [1993], Scott and Gulley [1995], and Clotfelter and Cook [1993] combine equations (1) and (3) into equation (4):

(4) $$ER_t = \sum^{i} w_i V_{it} + \frac{DV_{jt}}{B_t}(1-e^{-B_t w_j})$$

It should be noted that the payoff to the holder of a winning lottery ticket does not depend on the average expected number of winning tickets as claimed by both Krautmann and Ciecka [1993] and Ciecka et al. [1996]. Instead the payoff depends on the expected number of winning tickets besides the one held by the particular owner of a winning ticket.

The final consideration necessary for the proper calculation of the expected return of the purchase of a lottery ticket is the issue of taxation. While Gulley and Scott [1993], Scott and Gulley [1995], and Clotfelter and Cook [1993] ignore the issue of taxation, both Krautmann and Ciecka [1993] and Ciecka et al. [1996] subtract applicable taxes from the expected return of a lottery ticket noting that, at least at the Federal level, lottery winnings are fully taxable as income. However, both Krautmann

Federal level, lottery winnings are fully taxable as income. However, both Krautmann and Ciecka [1993] and Ciecka et al.[1996] fail to include the fact that the purchase price of any lottery tickets are tax deductible to the extent of any lottery winnings in their formulation of expected return. For the purchase of a single ticket, this essentially means that all winnings are taxable but that the price of the ticket is tax deductible if you win a prize. Even the smallest lower-tier prizes are generally larger than the price of the ticket so any time a prize is won the price of the ticket can be fully deducted. The inclusion of taxes changes equation (4) to

$$ER_t = [\sum^{i} w_i V_{it} + \frac{DV_{jt}}{B_t}(1-e^{-B_t w_j})](1-t) + [\sum^{i} w_i + w_j]t\tau \qquad (5)$$

where t is the tax rate and τ is the price of a ticket.

In practice, the effects of taxation may be less than this since lottery winners who win small prizes are unlikely to report their winnings on their income taxes and the state lottery associations are not required to report the winnings to the IRS unless they exceed a certain amount, currently $600. In the case that winnings are unreported to the tax authorities, the term $(1-t)$ in equation (5) may not apply to every number within the first set of brackets.

When only considering the purchase of a single lottery ticket, it is clear why all previous estimates of the expected return of a ticket have ignored the fact that lottery ticket purchases are tax deductible to the extent of winnings. First, it is unlikely that a lottery player will take the time to report the purchase of a single $1 ticket on his taxes. Second, since the ticket price is deductible only as an offset to earnings, the player must win a prize to deduct the ticket. The overall odds of winning are less than 5 percent for all lotto games in the United States, and therefore the rightmost term in equation (5) is nearly zero for any single ticket purchase. However, while the deductibility of lottery ticket purchases may make an insignificant impact on the profitability of the purchase of a single lottery ticket, it makes a substantial impact in the case of the Trump Ticket, where a coalition may purchase anywhere from 3 to 80 million tickets.

A lottery ticket represents a fair bet if the expected return, ER_t, exceeds the price of the ticket, τ. Whether a past lotto drawing represented a fair bet can be determined *ex post* by examining the size of the jackpot and the actual number of tickets sold. Whether a particular drawing *ex ante* presents a fair bet can be determined in a couple of ways. First, expected ticket sales for a particular drawing could be estimated based on past ticket sales and their relationship to such factors as time trends, advertised jackpot size or the square of the jackpot size, and the day of the week of the drawing. For furthers details on how this may be done see Matheson and Grote [1999] among others.

Krautmann and Ciecka [1993] devise a second way to estimate ticket sales for a particular drawing. Since lottery associations typically allocate a fixed portion of ticket sales to the jackpot prize pool, the change in the advertised jackpot from one period to the next implies a certain number of ticket sales in the current drawing period. If the jackpot prize is not won in the preceding period so that the prize pool already contains $DV_{j(t-1)}$ of rolled-over money, and the percentage of current period

ticket sales allocated to the jackpot pool is α_j, then the number of tickets sold, B_t, can be found by equation (6):

$$B_t = [(AV_{jt} - AV_{j(t-1)})/n\alpha_j]\sum_{k=0}^{n-1}(1+\delta_k)^{-k} \qquad (6)$$

Although this is an intriguing attempt to estimate ticket sales in the absence of actual ticket sales data, this method should be seen at best as a rough estimate of ticket sales. First and foremost, the lottery's advertised jackpot is in and of itself an estimate of the expected jackpot based on historical trends. Any method that uses the current period advertised jackpot as a predictor is simply an estimate based on an estimate. Next, the change in the advertised jackpot may be affected by factors such as rounding of the jackpot to an even dollar amount or interest rate changes. Finally, this method is likely to be highly inaccurate during the first few drawings of a new cycle because many lottery associations guarantee an initial jackpot amount and a minimum drawing-by-drawing increase in the jackpot. For example, since 1997 the multi-state Powerball Lotto has guaranteed a minimum jackpot of $10 million, with minimum increases of $5 million per drawing. Therefore early in the draw cycle, the increase in the jackpot is based on defined lottery rules and not upon actual ticket sales.

EMPIRICAL IMPLICATIONS: THE FLORIDA LOTTO

As mentioned by Krautmann and Ciecka [1993] and Theil [1991], during September, 1990, following a string of four consecutive drawings in which the jackpot was not won, the Florida Lotto jackpot reached an advertised value of $106.5 million. At the time, this was the second largest jackpot in any U.S. lotto game. The previous drawing had an advertised estimated jackpot of $50 million. Krautmann and Ciecka's method of estimating ticket sales turns out to be a very good estimator of the actual number of tickets sold. Equation (6) predicts that with 25 percent of gross ticket sales allocated to the jackpot prize pool, a 20-year annuity period, and prevailing interest rates of approximately 8.79 percent, a jackpot increase of $56.5 million implies ticket sales of $113.9 million. Florida lottery officials reported actual total sales of 109,163,978 tickets.

Assuming that a lottery winner could borrow against future annuity payments at roughly the same interest rate as is paid by U.S. Constant Maturity Series bonds with a 20-year maturity, the appropriate interest rate to use to discount the advertised jackpot would be about 9 percent. During September 1990 the interest rate was 8.89 percent on 10-year bonds and 9.03 percent on 30-year bonds. At an interest rate of 9 percent, the discounted present value of the $106.5 million advertised jackpot was $52.98 million.

Accounting for multiple winners using equation (5) with B_t=109,163,978 and w_j = 1/13,983,816, the expected value for a single player holding a winning lottery ticket is $52.98 million divided by 8.04 or $6.59 million. Krautmann and Ciecka's method of dividing the jackpot prize by the expected number of winning tickets (equal to 109,163,978 divided by 13,983,816 or 7.806) overestimates the value of a winning ticket to the holder of such a ticket. Using equation (5), combined with the expected returns from the lower-tier prizes of 25 percent, the value of the purchase of a single $1.00 lottery ticket was $0.72 before taxes. Contrary to Krautmann and Ciecka's findings, even with a $106.5 million jackpot, the Florida Lotto did not represent a fair bet to the players.[4] In fact, the drawing the week before with a mere $50 million jackpot was a better deal for players because the number of ticket buyers was only 44 million. The lower jackpot combined with a significantly lower number of persons with whom a jackpot winner would expect to have to share the prize led to an expected discounted jackpot of $7.56 million and an expected value of a $1.00 ticket of $0.79 before taxes.

The choice of the proper tax rate to use is problematic since lottery winners face different average tax rates depending on factors such as marital status, the size of the jackpot (based on the number of other winners), and other income and deductions. Assuming the winner takes the annuity, a jackpot winner will have taxable income from the jackpot between $5 million a year (if he is the sole winner) to $250,000 per year (if he shares the jackpot with 20 winners). At $5 million per year, the winner will face average federal income tax rates that approach the maximum marginal tax rate of 39.6 percent. At $250,000 per year, average tax rates will be more like 25-30 percent based on other factors such as other income or deductions and marital status. As an estimate, a tax rate of 35 percent will be used. (Note that Florida has no state income tax.)

Including taxes at a rate of t = 35 percent yields an expected after-tax return from the purchase of a single ticket of $0.47 (= 0.72×0.65 + 0.019×0.35) for the $106.5 million drawing since the probability of winning a prize by matching three or more of the numbers was 1.9 percent. For the previous week's $50 million drawing, the expected return was $0.52 (= 0.79×0.65 + 0.019×0.35). If we are slightly more realistic and assume that the winner will only pay taxes on prizes over $600 and will not deduct the price of a single ticket, the expected after-tax return becomes $0.53 (= 0.535×0.65 + 0.185×1) for the $106.5 million drawing and $0.58 for the previous week's drawing (= 0.60530.65 + 0.18531).

EMPIRICAL IMPLICATIONS: THE VIRGINIA LOTTO

The $27 million Virginia Lotto drawing on February 15, 1992, also analyzed by Krautmann and Ciecka is, however, a different story. The final advertised jackpot for the drawing was $25 million. The advertised jackpot in the previous period was $15.5 million. So Krautmann and Cieska's method estimated that with 39 percent of gross ticket sales allocated to the prize pool, a 20-year annuity period, and prevailing interest rates averaging approximately 7.6 percent, a jackpot increase of $9.5 million ticket sales would be $13.6 million. In fact, Virginia lottery officials reported total sales of 14,879,779 tickets, a 10 percent difference. In fact, the final advertised jackpot underestimated the true jackpot by $2 million illustrating one of the problems with using the differences in advertised jackpots as a estimate for current period ticket sales.

Following the steps used for examining the Florida Lotto drawing above, the $27 million final jackpot had a discounted present value of $14.7 million with a discount rate of 7.6 percent and an expected value of $6.1 million after accounting for the

possibility of multiple winners. The Virginia Lotto awarded a free Lotto ticket to players that match 3 of the 6 numbers drawn. Assigning a value of $0.50 to each free ticket, the lower-tier prizes award $0.122 per dollar played. Combining the lower-tier prize return with the expected value of the jackpot prize, the expected return of a $1.00 Virginia Lotto ticket was $0.989 during this particular drawing and therefore nearly provided a fair bet to the players before taxes are considered.

After taxes, however, the Virginia lottery certainly becomes an unfair bet. Here the calculation of the appropriate taxes and deductions is more complex than is shown in equation (5) because the prize for matching 3 of 6 numbers is a free lottery ticket. In addition, Virginia has a state income tax with a top marginal rate of 5.75 percent. If we assume that the winner of a free lottery ticket can value the ticket at $1.00 for tax purposes and t = 40 percent, then equation (5) can be applied, and the expected return becomes roughly 60.3 percent. Even when prizes below $600 are not reported as income, the expected return is still only 63.3 percent.

THE TRUMP TICKET

What makes this drawing of the Virginia Lotto particularly interesting is the fact that an Australian consortium attempted to purchase every possible numeric combination for the Lotto drawing. In fact, the consortium was able to purchase 2.4 million of the 7,059,052 possible combinations before time ran out. Krautmann and Ciecka conclude that the consortium's actions were not sound and suggest that the conditions necessary for the purchase of a Trump Ticket to be a fair bet are more restrictive than those for the purchase of a single ticket. Furthermore they state that any time the Trump Ticket is a fair bet, so too will any bet of a lesser amount. In fact, the true results are just the opposite. The conditions for the purchase of the Trump Ticket to be a fair bet are less restrictive than those for the purchase of a single ticket and the purchase of the Trump Ticket, while not always profitable, always has a greater return per dollar compared to the purchase of a single ticket.

Krautmann and Ciecka's conclusions regarding the more restrictive conditions needed for a profitable Trump Ticket are based on the fact that the purchase of a large number of additional tickets increases the expected number of winners and therefore reduces the average expected jackpot because the jackpot is shared among more winners. Ciecka et al. [1996] extend these findings and demonstrate that depending on the size of the jackpot pool rolled over from the previous period and the number of current ticket buyers, the purchase of the Trump Ticket can either increase or decrease the average expected jackpot.

However, both of these formulations miss the basic point that the purchasers of the Trump Ticket are not concerned with the how their actions affect the average jackpot winner, only how they will affect their own winnings. Assuming that a consortium's decision to purchase every combination has no effect on the buying decisions of other players, (i.e. B_t is independent of the decision to purchase the Trump Ticket), then the decision to purchase the Trump Ticket does not affect the probabilities that other individuals will have winning tickets. The number of tickets bought by competing players matching the jackpot prize, m, and the p_m's, the probabilities that exactly m other tickets match the jackpot prize, are fixed regardless of the number of tickets the consortium buys.

While the purchase of the Trump Ticket does not affect the probability of any single ticket winning the jackpot nor does it change the probability that the purchaser of the Trump Ticket will have to share the jackpot with another player, the purchase does increase the size of the winners' jackpot. Since the purchase of the Trump Ticket necessitates the purchase of a large number of tickets, if a specific portion of ticket sales is allocated to the jackpot prize pool, the purchase of the Trump Ticket will cause a significant increase in the size of the jackpot. Mathematically, $DV_{jt}^{TT} = DV_{jt} + \tau\alpha_j/w_j$ where DV_{jt}^{TT} is the discounted present value of the jackpot after the purchase of the Trump Ticket, DV_{jt} is the discounted present value of the jackpot before the purchase of the Trump Ticket, τ is the price of a single lottery ticket, α_j is the percentage of gross sales allocated to the jackpot pool, and w_j is the probability of winning the jackpot prize.

The situation above can be illustrated using the example of the \$27 million Virginia drawing. Had the Australian consortium not attempted to purchase the Trump Ticket, 12.5 million tickets would have been sold for the drawing. Since 39 percent of ticket sales go into the jackpot pool, the loss of the 2.4 million tickets purchased by the consortium would have reduced the discounted present value of the jackpot by (0.39×2.4) or \$936,000 down to \$13.8 million.

Now suppose the consortium buys a single ticket. With 7,059,056 possible combinations and 12.5 million tickets sold to other players, if the single ticket is a winner, equation (3) tells us that there is a 17.0 percent chance that no other winning tickets were purchased, a 30.1 percent chance that exactly one other winning ticket was purchased, and 26.7 percent chance that exactly two other winning tickets were purchased, etc.

Now suppose the consortium goes ahead and buys a partial Trump Ticket of 2.4 million combinations. The probabilities that there are other winners among the 12.5 million competing tickets remains unchanged. However, the size of the jackpot increases to \$14.7 million because of the purchase of the additional tickets. The purchase of the full Trump Ticket would increase the jackpot to \$16.5 million. Following the purchase of the Trump Ticket, the probabilities of winning the jackpot with any particular ticket remain unchanged, the probability of winning a lower-tier prize remains unchanged, and the probability that one or more other tickets also matches all six numbers remains unchanged. The only change in the expected return for the purchase of a lottery ticket is that the jackpot, DV_{jt} increases. Therefore, the purchase of the Trump Ticket can only increase the expected return per ticket for the consortium purchasing the Trump Ticket.

Table 2 shows the probability that there are exactly m other winners among the 12.5 million tickets purchased by competing players as well as the value of the consortium's share of the jackpot given m other winners with the purchase of a single ticket, a partial Trump Ticket (2.4 million tickets), and the full Trump Ticket. The values shown in the columns under the expected value of the jackpot are simply the probability multiplied by the jackpot share. While these numbers are meaningless individually, the sum of these numbers represents the expected value of holding a winning ticket as a member of the consortium under each strategy.

TABLE 2
Expected value calculations for Trump Ticket in the Virginia Lotto (before taxation)
(Jackpot values in millions of dollars)

# of Other Winners	Probability	Value of Jackpot Single	Value of Jackpot Partial	Value of Jackpot Full	Expected Value of Jackpot Single	Expected Value of Jackpot Partial	Expected Value of Jackpot Full
0	17.0 %	\$13.8	\$14.7	\$16.5	\$2.34	\$2.50	\$2.81
1	30.1	6.9	7.3	8.3	2.07	2.21	2.48
2	26.7	4.6	4.9	5.5	1.22	1.31	1.47
3	15.8	3.4	3.7	4.1	0.54	0.58	0.65
4	7.0	2.8	3.0	3.4	0.19	0.21	0.23
5	2.5	2.3	2.4	2.8	0.06	0.06	0.07
6	0.7	2.0	2.1	2.4	0.01	0.01	0.02
7	0.2	1.7	1.8	2.1	0.00	0.00	0.00
Total	99.9				6.44	6.88	7.73

Given a 12.2 percent return from lower-tier prizes, the expected payoff from the purchase of a \$1.00 lottery ticket is ($0.122 + w_j \times 6.44$ million) = \$1.034 if the consortium purchases a single ticket, ($0.122 + w_j \times 7.11$ million) = \$1.096 per ticket for the purchase of the partial Trump Ticket of 2.4 million combinations, and ($0.122 + w_j \times$ 7.96 million) = \$1.217 per ticket for the purchase of the full Trump Ticket. All of these numbers represent expected returns before taxes.

The final piece to be considered is again taxation. As before, any winnings are fully taxable at the rate t, but the consortium may deduct the cost of the tickets purchased. For the purchase of a single ticket, equation (5) can be applied. With a tax rate of $t = 40$ percent, the after-tax expected return for a purchase of a single ticket by the consortium is $ER_t = 1.034(1-0.4) + 0.024\times0.4 = \$0.630$.

The purchase of the partial Trump Ticket requires a few more calculations but the basic premise is the same. The consortium's winnings are taxable, but the consortium can deduct the full cost of the 2.4 million tickets purchased only if they have at least \$2.4 million in winnings. Roughly, this means that they can deduct the full \$2.4 million if they win the jackpot since there is over a 99.5 percent chance that their share of winnings from the jackpot and the lower-tier prizes will exceed \$2.4 million. If they do not hit the jackpot, they can still deduct 12.2 percent of the \$2.4 million ticket costs because on average they will win 12.2 percent of their ticket price in lower-tier prizes. Assuming the consortium bought no duplicate tickets, the probability that they would win the jackpot was 2.4 million/w_j = 34 percent. Given an average return of \$1.096 per dollar spent on the partial Trump Ticket before taxes, the expected return after taxes was $ER_t = (1.096\times2{,}400{,}000)\times(0.6) + [(2{,}400{,}000\times0.34) + (0.122\times2{,}400{,}000(0.66)]\times0.4$ = \$1.982 million. Because the expected return of the partial Trump Ticket was less than the \$2.4 million cost, the purchase of the partial Trump Ticket did not offer a fair bet. The return per \$1.00 ticket purchased was \$0.826, still below the fair bet threshold.

However, had the consortium been able to purchase the full Trump Ticket, expected returns should be higher. As discussed above, the expected jackpot to the Trump Ticket purchaser rises as a greater percentage of the total combinations is purchased. In addition, because they are guaranteed a share of the jackpot by purchasing every combination, there is an increased chance that they will be able to deduct the full cost of the Trump Ticket (47.1 percent for the Virginia Lotto drawing) while the purchaser of the partial Trump Ticket or a single ticket was able to deduct the full cost of the ticket(s) only a small percentage of the time (34 percent and 12.2 percent respectively).

Table 3 shows the probability of m other winners given B_t other players using equation (3), the expected winnings, including lower-tier prizes, from the purchase of the full Trump Ticket given m other winners of the jackpot, the net value of the Trump Ticket which is the expected winnings minus the cost of the Trump Ticket, the after-tax value of the Trump Ticket, which is $(1-t)$ times the net value if the net value is positive and simply the net value if it is negative, and the expected value of the Trump Ticket, which is the after-tax value times the probability. Again, the expected value numbers are meaningless individually, but the sum of these numbers represents the expected value of purchasing the full Trump Ticket. The tax rate is again set at $t = 40$ percent and B_t is 12.5 million.

After taxes, the net expected value of the purchase of the Trump Ticket was \$587 thousand or \$1.083 per dollar of tickets purchased. Contrary to the conclusions of Krautmann and Ciecka, the \$27 million Virginia Lotto drawing was a fair bet for the purchaser of the Trump Ticket, and Australian consortium members were acting rationally as risk neutral investors in the face of a lottery with a positive expected payoff. The consortium's problem was not that they were facing an unfair bet, but that they were unable to complete their purchase of the Trump Ticket before time ran out.

While the purchase of the Trump Ticket may have a positive expected value, the purchase of the Trump Ticket is far from a guarantee of riches. To begin with, the Trump Ticket does not necessarily have a positive expected payoff. In the Florida Lotto drawing examined earlier, the purchase of the Trump Ticket would have increased the after-tax expected return per dollar played above the single-ticket figure of \$0.470 but only up to \$0.687. After taxes, the purchaser of a single ticket would have expected to lose only \$0.53 on average while the average loss to the purchaser of the Trump Ticket would be just under \$4.4 million.

Even when the expected return is positive, the purchase of the Trump Ticket does not guarantee a profit. In the example of the Virginia Lotto, the purchase of the Trump Ticket had an expected profit of 8.3 percent. However, the Trump Ticket only had a positive return when 0 or 1 competing tickets matched the winning jackpot numbers. The probability that 1 or fewer tickets among the 12.5 million additional competing tickets matched the winning numbers was only 47.1 percent. Thus, even in the presence of a large expected profit, actual profits from the Trump Ticket were positive less than one-half of the time. In general the purchase of the Trump Ticket will not be profitable except in the presence of very high jackpots combined with a relatively small number of other ticket buyers.

TABLE 3
Expected Value Calculations for Trump Ticket in the Virginia Lotto (after taxation)
(All numbers in millions of dollars)

# of Other Winners	Probability of m Winners	Expected T.T. Winnings	Net Value of Trump Ticket	After-tax Value of T.T.	Expected Value of T.T
0	17.0 %	\$ 17.37	\$ 10.31	\$ 6.19	\$ 1.05
1	30.1	9.12	2.06	1.23	0.37
2	26.7	6.36	−0.69	0.69	−0.19
3	15.8	4.99	−2.07	−2.07	−0.33
4	7.0	4.16	−2.90	−2.90	−0.20
5	2.5	3.61	−3.45	−3.45	−0.09
6	0.7	3.22	−3.84	−3.84	−0.03
7	0.2	2.91	−4.13	−4.13	−0.01
Total	99.9				0.587

EMPIRICAL IMPLICATIONS: POWERBALL AND THE OREGON LOTTO

Two other lottery drawing are interesting from an empirical standpoint: the \$296 million Powerball drawing in July 1998, because of its size and the \$18 million Oregon Lotto drawing in February 1999, because of its profitability.

In July of 1998, following a string of 18 straight drawings in which the jackpot rolled over, the Powerball Lotto, a game run by the 15-state Multi-State Lottery Association, reached an advertised jackpot of \$296 million, the highest jackpot ever recorded in the United States at the time. (This record has since been eclipsed by a \$363 million drawing of the Big Game in May 2000.) The huge jackpot set off a wave of ticket buying with tickets sales for the four-day drawing period reaching just over \$210 million, roughly 25 times the sales for the median jackpot drawing. With such a large jackpot, did this drawing provide a fair bet to an individual ticket buyer? With prevailing interest rates of roughly 5.5 percent, the 25-year, \$296 million annuity had a present value of \$166 million. Applying equation (5) with an assumed tax rate of 35 percent, the expected after-tax value of \$1 Powerball ticket was only \$0.612. Even if small prizes went unreported as income, the expected return is only \$0.654. The purchase of the Trump Ticket, which would require a significant effort since over 80 million number combinations are possible, would have returned an expected after-tax value of \$0.967 per ticket. While the lure of a \$296 million jackpot is tempting, the huge number of other ticket buyers makes even this huge payout a losing proposition.

In February of 1999, following over five months of rollovers, the Oregon Lotto hit an advertised jackpot of \$18 million. However, unlike the Powerball drawing, this jackpot, which was extremely large compared to the average jackpot offered by the Oregon Lotto, generated only minimal interest among the Lotto ticket buying public. Only 741,729 one-dollar tickets were sold, an amount only three times the typical sales level for a Oregon Lotto drawing. In Oregon each one-dollar ticket allows the player to pick two sets of numbers. Applying equation (5) with a discount rate of 5.5 percent for a 20-year annuity, B_t = 1,483,458, w_j = 1/7,059,052, and lower-tier prizes

TABLE 4
A Comparison of Four Lotto Drawings

Lotto	Date	Advertised Jackpot	Expected after-tax return per dollar played (Krautmann and Ciecka's [1993] values, ER_t, in parantheses.) Single Ticket	Trump Ticket
Florida	9/15/1990	$106.5 million	$0.470 ($ER_t$ > $1.00)	$0.687 ($ER_t$ < $1.00)
Virginia	2/15/1992	$27.0 million	$0.603 ($ER_t$ < $1.00)	$1.083 ($ER_t$ < $1.00)
Powerball	7/29/1998	$296.0 million	$0.612	$0.967
Oregon	2/10/1999	$18.0 million	$1.698	$2.134

returning 13 percent, the expected return before taxes was $1.515 for a $0.50 ticket. Even after taxes with an assumed rate of $t = 44$ percent (Oregon's highest marginal state income tax bracket is 9 percent for all income over $50,000 per year), the expected return was $0.849 for a $0.50 ticket, or a 69.8 percent rate of return. It should be noted that this particular drawing was not a one-time anomaly caused by unexpectedly low ticket sales in a single period. In fact, the Oregon Lotto presented the individual ticket buyer with a positive after-tax expected rate of return for each of the 12 previous drawings.

The purchase of the Trump Ticket is even more profitable. Even without the addition to the jackpot prize that the purchase of the Trump Ticket would make (the Oregon lottery association does not designate a specified percentage of each period's ticket sales to the jackpot pool), the purchase of the Trump Ticket would still have an after-tax expected return of $1.067 per $0.50 ticket purchased for a total expected profit of just over $4 million on the purchase of the Trump Ticket. See Table 4 for a comparison of the four lotto drawings studied in this paper.

CONCLUSION

This paper derives the necessary conditions for a lotto to have positive net expected payoffs and advances previous attempts in the literature to derive such conditions, which have either been incorrect or have omitted significant variables such as taxation and deductibility of lottery ticket purchases. Furthermore, the new findings regarding the expected payoffs from the purchase of the Trump Ticket directly contradict previous research. Applying these new findings to previously examined drawings of the Florida and Virginia Lotto games as well as newer drawings of the multi-state Powerball Lottery and the Oregon Lotto gives new understanding of the rationality of single-ticket Lotto players and the actions of ticket buying consortiums.

Of particular interest empirically was the finding that the purchase of a single lottery ticket in the February 10, 1999 drawing of the Oregon Lotto provided the buyer with a positive net expected return. In a situation like this, the state lottery association should be able to significantly increase ticket sales by exploiting the promise of an almost unheard-of phenomenon in the gambling industry: a more than fair bet.

Furthermore, the February 15, 1992 drawing of the Virginia Lotto and the February 10, 1999 drawing of the Oregon Lotto provided the purchaser of the Trump Ticket with more than a fair bet even after accounting for multiple winners and taxation. If drawings where the Trump Ticket provides a positive expected return are common, lottery associations may be able to increase annual ticket sales by selling Trump Tickets directly to buying consortiums who are unwilling to attempt to purchase Trump Tickets through retail outlets due to the high transaction costs. Further research is necessary to determine whether drawings like this are simply a rare anomaly due to unexpectedly low ticket sales or whether lottery drawings where the Trump Ticket has a positive expected payoff are a common occurrence.

NOTES

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1. A recent marketing tweak by several lottery associations has been to have the individual pick five numbers from a group of approximately 50 and an additional number, from a group of approximately 50. This change has been instituted to reduce the chance of individuals winning the grand prize and therefore leads to larger jackpots. This strategy is particularly pursued by larger lotteries, and the phenomenon is discussed in greater detail by Clotfelter and Cook [1990].
2. It has been pointed out by a helpful referee that even fixed dollar prizes may become pari-mutuel if the number of winners exceeds a certain level. This type of rule limits the liability of the lottery association in the event that a very popular number combination is picked as a lower-tier prize. For example, several days after the TWA Flight 800 disaster in July 1996, hundreds of people picked 8-0-0 in the Connecticut Daily 3 game. When the number 8-0-0 was actually drawn, the lottery association faced a huge loss on the drawing which did not have a provision that limited prizes to a pari-mutuel amount in the face of an excessive number of winners. While this situation will result in the expected payoffs from lower-tier prizes being lower than estimated in the model, we consider this situation to be remote enough to allow us to ignore such a possibility in our model.
3. It is a fact that certain combinations of numbers (birthdays, vertical or diagonal columns on the play slip, etc.) are more commonly played than other combinations and therefore by playing rarer combinations (such as numbers all above 28 or 31) a ticket buyer can earn an expected return on the lower-tier prizes above the average expected payout. For example, an examination of 668 drawings in the Texas Lotto (a 6 of 50 Lotto) show that the average payout for choosing 5 out of 6 numbers correctly was $1,661 and $105 for choosing 4 of 6 correctly. However, in the 44 drawings where the smallest number drawn was 29 or higher, the average payouts were $2,122 and $131 respectively while in the 106 drawings where the highest number drawn was 28 or lower, the average payouts were $1,303 and $86 on average. See Clotfelter and Cook [1989, 81], MacLean et al. [1992], Thaler and Ziemba [1988], or Papachristou and Karamanis [1998] for further discussion.
4. To be fair to Krautmann and Ciecka, it must be said that they were misinformed by a Miami Herald article that stated that the advertised jackpot in the previous period was $75 million instead of the true value of $50 million. Their methods would have led them to conclude that the $106.5 million Florida Lotto drawing was not a fair bet to players if they had the correct value for the previous jackpot.

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