REFERENCES


Recent work on coordination models has focused on nominal-price rigidity; however, the results are conflicting. Ball and Romer [1991], using a model that mixes menu costs with price coordination, show that nominal-price rigidity is associated with price-coordination failure and Pareto-inferior results. In contrast, Mischel [1998], using a model in which firms adjust price in response to a demand shock, and Bohn and Gorton [1993], using a demand externality model with money and nominal contracts, suggest that sticky nominal prices produce an equilibrium that is Pareto-preferred to that produced by flexible prices.

These contributions, however, have not addressed the relationship of real-price rigidity to coordination failure for one of a broad class of coordination models called the production-coordination model. This paper presents a production-coordination model in an explicit pricing context. This coordination model will then be used to examine how the incorporation of fixed and flexible real-price systems alters the operation of the model, specifically how the price regime impacts strategic behavior of suppliers and thus the level of production in the economy.

The following section begins with an overview of the development and importance of coordination failures and introduces the production-coordination model. We then develop and analyze a production-coordination model incorporating complementarities among intermediate-input and final-good producers. Although money could play an accounting and transactions role without substantively changing the results, production-coordination models are essentially real-exchange models. Therefore, the model is presented in real terms. Constraints on the degree of communication between intermediate-input suppliers and between final-good and intermediate-input suppliers may result in a sub-optimal level of production [Bryant, 1993].

This paper's model differs from Bryant's [1988, 1993] by having an explicit final-good production sector. In addition, this paper's model substitutes marketing trading and price setting firms for Bryant's assumed allocation system. Initially, Bryant uses a Walrasian price mechanism to show that the resulting Nash equilibrium will be zero output. This pathological result persuaded Bryant to substitute an equal-sharing feature to guarantee a nonzero Nash equilibrium. Using Bryant's institutionally-determined allocation system in this paper's model results in similar conclusions. Therefore, we connect the model to the existing literature and its conclusions when a pricing system is not employed. However, the following sections of the paper replace the institutional allocation system with an explicit trading/pricing system and develop the behavior and results under this market system.

We continue by presenting the trading mechanism and developing the behavior of the final-good and intermediate-input suppliers in a pricing system. That provides the basis for the first three propositions. Proposition 1 states that a flexible, real-price system fails to improve the welfare properties of the economy and can actually cause welfare to deteriorate by introducing serious production instabilities. In contrast, Proposition 2 states that downwardly inflexible prices as a response to demand constraints may result in sustainable, Pareto-preferred outcomes. Although both the context and application are different, the advantage of inflexible prices in stabilizing output in this production-coordination model complements a strand of the New Keynesian literature that proposes that inflexibility of wages and prices will reduce economic fluctuations [Greenswald and Stiglitz, 1993, 25].

Colander [1992] suggests that perfectly flexible prices will generate such instability that institutions will be created to substitute a rigid-price system. The model presented in this paper, however, indicates that formal institutions to limit price movements may not be necessary. Proposition 3 states that price rigidity is a rational, strategic mechanism adopted by the individual firm in a production-coordination environment.

We continue by developing the pricing conditions necessary to sustain a particular Nash equilibrium, including the pricing conditions associated with the maximum sustainable Nash. Although the adoption of rigid prices by intermediate firms in the context of strategic behavior prevents severe production instabilities, prices in the model represent a two-edged sword. Proposition 4 states that pricing limits the ability of the system to sustain coordinated equilibria. For example, the Pareto-optimal production level, $Q_0$, does not have a sustainable price set. In general, unless the sum of intermediate-input prices is less than an amount specific to the coordinated level of production, that production level, even though a potential Nash equilibrium, will not be sustained. Propositions 5 and 6 state that, with the exception of one coordinated, maximum sustainable production equilibrium called $Q_{max}$, the set of real prices associated with a particular equilibrium is not unique.

Proposition 7 states that given an initial equilibrium below $Q_{max}$, price coordination may be necessary even for coordinated-production expansions. The traditional literature suggests that government can play a critical role in attaining a Pareto-preferred equilibrium by supporting production level confidence. However, Proposition 7 indicates that successful expansion may require the government to undertake specific price coordination.

THE IMPORTANCE OF COORDINATION FAILURES IN MACROECONOMICS

The theoretical and experimental work on coordination-failure models, based on Bryant [1983], Diamond [1982], Hart [1982], and Weitzman [1982], has expanded sufficiently to justify an extended book survey of these models as a new sub-field of macroeconomics [Cooper, 1999]. In contrast to the New Classical Economics perspective in which unemployment arises from inter-temporal substitution of leisure or from failures to distinguish between relative and price level changes, and, in contrast to the Keynesian perspective in which wage-price rigidities play a prominent role, coordination-failure models generate underemployment equilibria from the inability of agents to coordinate their actions successfully in a many-person, decentralized economy [Cooper and John, 1998, 9].

A key feature of coordination-failure models is strategic complementarity, defined as interaction among economic agents in which an increase in the level of activity by one agent will induce other agents to behave similarly. Environments characterized by strategic complementarity have coordination failures because no one agent has an incentive to change activity level so that potential mutual gains from an overall change in agent behavior are not realized [Tesfatsion, 1994]. As Cooper states,
"Models of complementarities are really about life inside the production frontier. They distinguish possible for producing more of all goods or (if activities can be properly coordinated" (1999, 151-2). The concept of multiple equilibria has been applied in empirical studies to explain trade patterns and industrial sales patterns (van Beu and Garretson, 1992, 471-2). The concept has also been applied (with little support) to the Great Depression (Dugavsky and Jovanovic, 1994) and more successfully to British unemployment patterns (Manning, 1990, 160-1). Empirical work has, however, been limited because of geometric complexities (Silvestre, 1998, 128-30). In contrast to the embryonic empirical work, the concept of complementarity and coordination has been applied to macroeconomics through several theoretical models. Cooper (1999) provides an extensive review of the models; the following is only a sample.4

Multiple equilibria can occur in multiple sector models in which firms have market power and complementarity exists across sectors through income effects (Heller, 1986; 1990; 1992; 1993; Roberts, 1987). Expansions of other sectors that produce goods purchased by agents operating in the expanding sectors. Multiple equilibria also exist in models with market power and increasing returns to scale in production (Weitzman, 1982; Manning, 1990; Hahn and Gorlin, 1995). Both models generate Kuznetzian type multiplier effects as well as multiple equilibria that can be Pareto-ranked as illustrated in the models of Heller (1998) and Cooper (1994).

The trading externality model posits that efficiencies resulting from lower transaction (specifically search) costs are generated as markets become thicker with more participants (Diamond, 1982). Multiple equilibria occur that can be welfare-ranked based on the Pareto criterion. The production complementarity model— the focus of the present paper— assumes technical complementarity by introducing interaction of agents through the production function (Bryant, 1983; 1990; 1996). Because of imperfect information and the requirement that things have to fit together, the theoretical model generates multiple equilibria that can be Pareto-ranked. Laboratory experiments have indicated that coordination failures do occur in the production-complementarity environment, and often low-level equilibria are the norm (van Huyck, Battalio and Bell, 1990). Bryant (1993, 218-219; 1996, 167-9; 1992) suggests that the production-coordination model isolates the basic cost-production problem inherent in an integrated economic system, justifies the development of institutions, and focuses attention on the role of institutions (in contrast to market prices) as the coordinating mechanism in macroeconomics. The production-coordination model has also been adapted to economic growth and to economic development models (Bivona, Hinakopahia and Remer, 1998). Still other applications are to the business cycle (Baxter and King, 1991).

AN OVERVIEW OF THE PRODUCTION-COORDINATION MODEL

Assume a two-sector macro economy in which n independent, monopolistic suppliers trade specialized, intermediate inputs for a final good produced under competitive conditions in a final-good sector consisting of n firms.5 The final good is employed as a numeraire and has a price equal to one. Strict complementarity exists between the intermediate inputs themselves and between the intermediate inputs and the final good. In addition, intermediate-input and final-good suppliers are owner-manager-laborers. These owner-manager-laborers are the only suppliers of labor, have identical leisure goods, have final-good preferences, and have diminishing marginal rates of substitution between the final good and leisure.6

Formally, the owner’s utility functions can be written as:

\[ U_n = U(Y_n, t - L_n) \]
\[ U_n = U(Y_n, t - L_n) \]

where \( U, Y, L, \) and \( t \) refer to the utility level of the owner, the quantity of the final good available to the owner for consumption, the owner’s labor applied to production, and the time available to the owner, respectively. The subscript \( n \) denotes an owner \((n = 1 \text{ to } n)\) in the final-good sector, and subscript \( f \) denotes an owner \((j = 1 \text{ to } m)\) in the intermediate-input sector. Note that, in this real exchange model, \( Y_n \) is the amount of final good produced by final-good firm \( i \) from supplying \( L_n \) units of labor and its payments of the final good to its intermediate-input suppliers. Similarly, \( Y_n \) is the total amount of final good payment received by the \( j \text{th} \) intermediate-input supplier from all final-good suppliers.

The utility functions specified in equations (1) and (2) are assumed to have the usual “well-behaved” properties. In particular, each utility function implies a diminishing marginal rate of substitution of leisure for consumption of the final good. Each owner is assumed to operate under utility-maximizing behavior.

The production functions in each sector are such that: (1) each intermediate-input supplier, \( i \), requires one unit of labor for each unit of the intermediate input produced (raw materials used in the production of intermediate inputs are “free” gifts of nature); and (2) each final-good supplier, \( j \), requires one unit of each type of intermediate-input and one unit of labor per unit of the final good produced. For the intermediate-input firms, the production function can be written as:

\[ Q_n = L_n \]

where \( Q_n \) and \( L_n \) are the output of and the amount of labor supplied by the \( j \text{th} \) firm in the intermediate-input sector respectively. For each of the \( j \text{th} \) final-good firms, the production function can be written as:

\[ Q_n = \min (Q_{n1}, Q_{n2}, \ldots, Q_{nM}, L_n) \]

where \( Q_{nj} \) and \( L_n \) are the output of the \( j \text{th} \) final-good firm, the quantity of intermediate-input available to the \( j \text{th} \) final-good firm, and the labor time used in production by the owner of the \( j \text{th} \) final-good firm, respectively. In particular, under the trading system described below, unless the final-good firm is certain that a complementary unit of each intermediate input is available, the owner will not buy an additional unit of intermediate input from any firm \( j \) or supply an additional unit of labor.
Based on equations (1) and (2), define marginal cost, $MC$, either for an intermediate-input or final-good firm, as the extra unit of final good for consumption required to compensate for the production of one more unit of the intermediate input or final good and thereby supplying one more unit of labor into production. Since in what follows, labor (or forgone leisure) is either the only input in the production process (as in the intermediate-input sector) or the only input that is incorporated into the marginal cost curve (as in the final-good sector), the marginal cost curves for firm $i$ in the final-good sector and for firm $j$ in the intermediate-input sector are, respectively:

$$MC_i = \frac{dY}{dQ_i} = \frac{dU_i}{dt(-L_i)}(\frac{dU_i}{dY}) = dY(-d(-L_i))$$

$$MC_j = \frac{dY}{dQ_j} = \frac{dU_j}{dt(-L_j)}(\frac{dU_j}{dY}) = dY(-d(-L_j)).$$

A diminishing marginal rate of substitution of leisure for consumption of the final good implies an increasing marginal cost curve in both sectors.

The technology implied by production functions in equations (3) and (4) produces a continuum of coordinated production levels, $Q_i$, defined by conditions (A) and (B):

(A) $Q_i = Q_i/n = Q_i/n$, for each firm $j(i)$ in the intermediate-input (final-good) sector, and

(B) $Q_j = Q_j = Q_j$, for each firm $j$ in the intermediate-input sector,

where $Q_i$ is total production ($\sum Q_i$) in the final-good sector. The coordinated production levels can range from $Q_i = 0$ to a Pareto-optimal level of production, $Q_i = Q_j$, defined by the additional condition:

$$1 = \sum MC_i(Q_i) + MC_j(Q_j/n),$$

for each firm $i$ in the final-good sector and for each firm $j$ in the intermediate-input sector. However, these potential coordinated production levels may not be Nash equilibria.

For the economy to be at a coordinated production, Nash equilibrium, two critical equilibrium conditions must hold. First, for the production level to be coordinated, each intermediate-input supplier must produce a level of production, $Q_i$, equal to the common expected production of all other intermediate-input suppliers (conditions (A) and (B)). Second, for this coordinated level of production to be a Nash equilibrium, no firm should have an incentive to alter its production from the common expected level. At a given coordinated production level, whether a firm has an incentive to change production depends on the particular final-good allocation system.

In Bryant's formulation of the production coordination model, agents are allocated the final good by institutional agreement. As long as this institutional agreement allocates the final good in an amount at least equal to the agent's marginal cost, a similar institution in this model could establish any coordinated production equilibrium between zero and the Pareto optimal level of production (Bryant, 1993; 1998; 1999).

However, as will be discussed below, if the institutional allocation agreement is replaced by a pricing system, the pricing system may not coordinate suppliers adequately to ensure that the Pareto-optimal equilibrium will be chosen. The pricing system may not even ensure that any coordinated-production level, once initially obtained, is sustainable.

**THE TRADING SYSTEM**

The trading system underlying the pricing behavior in this model can be described in terms of the following island story. All final-good suppliers are located on a single island, and each intermediate-input supplier is located on a separate island. The intermediate-input suppliers do not communicate with each other during any stage of the trading process. The intermediate-input suppliers communicate with the final-good suppliers only on the trading day. Each period in the trading system consists of the following three stages.

In stage one, each intermediate-input supplier (on its own island) chooses both a production level and a price for its output. The intermediate-input supplier's choice of price and production level is based on the knowledge that (1) each final-good supplier (on the final-good island) will use the posted prices to determine the profit-maximizing level of each intermediate-input it desires to purchase; and (2) the aggregate level of demand for any intermediate input by final-good suppliers cannot collectively exceed the level of output produced by the intermediate-input supplier(s) choosing the smallest output level.

In stage two, the trading day, each intermediate-input supplier loads its production on a ship and delivers the cargo to the docks on the final-good production island. On delivery, each intermediate-input supplier initially posts its chosen price and begins negotiations with the final-good producers for the ship's cargo. The intermediate-input supplier may attempt to clear the ship's cargo by varying the posted price according to market conditions (flexible-price behavior) or hold the posted price constant (rigid-price behavior) even if the cargo is not completely sold.

During stage two, each final-good supplier visits the docks and observes the posted prices (specifically, the sum of the posted prices). Each final-good supplier adjusts its demand for each input to the profit-maximizing level; however, they realize that each ship holds only limited and possibly unequal supplies, and thus they are collectively subject to intermediate-input availability constraints. Therefore, to ensure delivery of each and all inputs in the desired quantities, each final-good supplier discloses input availability with the intermediate-input suppliers but places an order only if each supplier can guarantee a matching level of intermediate inputs.

In the final stage (stage three), the intermediate-input suppliers release their cargos to the contracting final-good firms at the market prices. The intermediate-input suppliers cautiously dispose of any intermediate-input stock not sold. The intermediate-input suppliers wait in port until the final-good producers transform the
intermediate-inputs into the final good and make the contracted payments. The intermediate-input suppliers then return to their own islands with these final-good payments.

BEHAVIOR UNDER FLEXIBLE AND RIGID REAL-PRICE REGIMES

Final-Good Supplier Behavior

To establish the behavior of final-good firms in stage two under this trading system, first define the marginal residual, \( MR_i \), for final-good firm \( i \) as the final-good output remaining for firm \( i \) to consume after it supplies one more unit of labor to produce one more unit of final good, and pays each intermediate-input firm for the one extra input unit required to complement this extra unit of labor. Therefore, employing the final good as the numerator:

\[
MR_i = 1 - \Sigma P_j
\]

where \( 1 \) is the price of the final good, \( P_j \) is the price of firm \( j \)'s intermediate input, and \( \Sigma \) is the summation over all \( j \).

Because of the competitive nature of the final-good industry, in stage two of the trading system each final-good supplier considers the posted prices of the intermediate-inputs as fixed. Given these fixed prices, each final-good firm, \( i \), attempts to expand its labor input, \( L_i \), its intermediate-input use, \( Q_{j(i)} \), and thus its final-good production, \( Q_i = L_i + Q_{j(i)} \), until its marginal residual from supplying an extra unit of labor (producing an extra unit of final good) is equal to its marginal cost. Therefore, condition (C):

\[
(\text{C}) \quad MR_i = 1 - \Sigma P_j = MC_{ij}
\]

for each firm \( i \) in the final-good sector, is added to conditions (A) and (B) described above. Using equations (B) and (7), condition (C) can be written as:

\[
1 - \Sigma P_j = \frac{dU_i}{dL_i} = \frac{dU_i}{dY_i} = \frac{dY_i}{dL_i} = \frac{dY_i}{dL_i} = 1 - L_i.
\]

Given the utility function of final-good supplier \( i \), its level of final goods supplied, its level of labor supplied, and its level of each intermediate-input demanded are each a function of the sum of the intermediate-input prices:

\[
Q_i = L_i = Q_{j(i)} = h(\Sigma P_j).
\]

Because each final-good firm equates \( MR_i = 1 - \Sigma P_j \) to \( MC_{ij} \), the supply curve for each final-good firm is identical to its marginal cost. In a manner identical to the procedure employed in a standard microeconomic theory course, the marginal cost curves of the individual final-good suppliers can be horizontally summed to obtain the desired aggregate (market) level of final good produced, the corresponding desired aggregate (market) level of labor supplied, and the corresponding aggregate (market) level of each intermediate input demanded by the final-good sector as a function of the sum of the intermediate-input prices:

\[
Q_i = L_i = Q_{j(i)} = h(\Sigma P_j).
\]

Given identical and linear marginal cost curves for the final-good suppliers, condition (C) and equation (10) are illustrated in Figure 1. Measuring vertically downward from the horizontal line \( DIP \) on the horizontal axis, \( DIP \) is the horizontal sum of the individual final-good firm's marginal cost curves. Therefore, if the final-good sector were producing \( Q_i \), \( K_i \) would be the marginal cost of a final-good supplier. If the sum of the intermediate-input prices were equal to \( P_i + P_j = Q_{j(i)} \), then the marginal residual for any final-good firm would be \( 1 - Q_{j(i)} \). The marginal residual would equal the marginal cost, \( K_i \), at aggregate final-good production level \( Q_i \), the optimal production level for the final-good sector given this sum of intermediate-input prices.

This analysis assumes that, in stage two, each final-good firm can obtain from each intermediate-input firm all the inputs the final-good firm desires at a given (posted) intermediate-input price. However, constraints imposed by the current coordinated output level by the intermediate-input sector may prevent final-good firms from obtaining desired quantities of intermediate inputs at fixed \( P_j \). Under these excess-demand conditions, at the current coordinated-output level, the intermediate-input suppliers will in stage two increase their prices until, in equilibrium, the sum of the intermediate-input prices is such that the excess of the marginal residual over the marginal cost for each firm in the final-good sector is zero. That is, the rule \( MR_i \).
will depend on whether real prices are flexible or rigid in stage two of the trading story.

First, consider the flexible real-price case. If, in stage one, firm $k$ decides to produce $Q_a$, where $Q_a > Q_e$ (equation (13)), because of complementarity, firm $k$'s intermediate-input would be in excess supply in stage two of the period (given the assumption that intermediate-input firms other than firm $k$ continue to produce $Q_a$). In a perfectly flexible price environment, the price of firm $k$'s intermediate input in stage two would drop toward zero. However, in contrast to the standard case in which a lower real price increases the quantity demanded, in a production-coordination model, a reduction in price by firm $k$ would not increase the quantity demanded of its product. That is, firm $k$'s demand curve in stage two is perfectly inelastic at output level $Q_a$ for increases in output above $Q_e$. Therefore, intermediate-input firm $k$ would have no incentive to produce a level of output greater than $Q_e$ in stage one.

On the other hand, given the assumption that other intermediate-input firms will leave their output unchanged, if, in stage one, firm $k$ decides to produce slightly less than the coordinated level $Q_e$, the only relatively scarce intermediate input in stage two of the trading story would be the output of firm $k$. The output of all non-$k$ intermediate-input suppliers would be in surplus. In a flexible-price environment, during stage two of the trading story firm $k$ would raise its real price while the non-$k$ firms would lower their real prices. Again, in contrast to the standard case in which a lower real price would increase the quantity demanded, in a production-coordination model, a lower real price by non-$k$ intermediate-input suppliers will have no impact on their quantity demanded as long as firm $k$ maintains its output below $Q_e$. In other words, the demand curve for non-$k$ suppliers is perfectly inelastic at level $Q_e$ as long as firm $k$ is the constraining (lowest production) firm.

More formally, under perfect price flexibility in which the prices of non-$k$ intermediate-input firms drop to zero, a slight reduction in output by firm $k$ equal to $j$ would enable firm $k$ to increase its price by $\Sigma P_j (=\sum_j)$ for all $j$ not equal to $k$ to $1 - MC_e(Q_e)$. In effect, under perfect price flexibility, by raising its real price as other intermediate-input firms lower theirs, firm $k$ is redistributing revenue, $(\Sigma P_j/Q_e)$, from other firms in the intermediate-input sector to itself. As $x \to 0$, the increase in firm $k$'s price to $1 - MC_e(Q_e)$ would, in the limit, occur at output $Q_e$. This increase in price shifts the demand curve for firm $k$ from $P_a^{\tau}$ to $1 - MC_e(Q_e)$ at output $Q_e$ and increases revenue in the limit by $(\Sigma P_j/Q_e)^{\tau}$.10

Because of the redistribution of revenue that results from the slight decrease in output, the higher price corresponding to the reduction of output at $Q_e$ is called the price-redistribution effect, which is illustrated in Figure 1. In Figure 1, holding firm $x$'s output constant at $Q_x$, let firm $k$ lower its output slightly below $Q_e$. As a result, firm $x$, responding to its excess supply condition, lowers its price toward zero, firm $k$ could raise its price toward $Q_e$, gaining the revenue, $P_a^{\tau}Q_e$, that firm $x$ loses. In addition to the shift in the demand curve for firm $k$ at $Q_e$ that results from any price-redistribution effect, by reducing output further, firm $k$ can move along its demand curve and increase its real price because of the rent-capture effect. This effect occurs because as firm $k$ decreases its output below $Q_e$, the marginal cost of the final-good sector falls.
More formally, the rent-capture effect operates as follows. Prior to the production reduction by firm k, each final-good firm i, equates its marginal residual to its marginal cost, \( 1 - \Sigma P_i = MC_i \) (evaluated at the set of intermediate-input prices that prevailed at \( Q_k \) prior to the reduction in output by firm k). Therefore, the excess of the marginal residual over final-good firm i’s marginal cost, \( (1 - \Sigma P_i - MC_i) \), is initially zero. Abstracting from the price-redistribution effect on the non-k firms’ prices, a reduction in firm k’s production below \( Q_k \) will increase this excess marginal residual by an amount equal to the derivative of the final-good marginal cost curve, \( dMC_i/Q_{k,\theta} \). Given that intermediate-input firm k reduces its production below \( Q_k \), this excess marginal residual is extracted by firm k through an increase in its real price. This power of firm k to further increase its price as it reduces production below the coordinated-output level \( Q_k \) is called the rent-capture effect.

In summary, given perfect real-price flexibility, as firm k lowers its supply, if prices by non-k intermediate-input firms drop to zero in the face of excess supply, k’s demand curve would first shift upward to \( P_{1k} = 1 - MC_i(Q_k) \) because of the price-redistribution effect and then would slope upward by \( dMC_i(Q_k)/dQ_k \), because of the rent-capture effect. That is, equations (11) through (13) (firm k’s inverse demand curve) would take the following form:

\[
\begin{align*}
(\text{i}) & \quad P_{1k} = 1 - MC_i(Q_k), \quad \text{for } Q_k < Q_{e}; \\
(\text{ii}) & \quad P_{1k} = P_{1e}^* \quad \text{for } Q_k = Q_{e}; \text{ and} \\
(\text{iii}) & \quad P_{1k} = 0, \quad \text{for } Q_k > Q_{e}. 
\end{align*}
\]

For example in Figure 1, after the price-redistribution effect has been completed, supplier k’s demand curve (reflecting the rent-capture effect) would be given by the line segment DI determined by the final-good sector’s marginal cost curve. Therefore, for \( 0 < Q_k < Q_{e} \) and the flexible price case (both the redistribution and rent-capture effect are operative), total revenue for firm k is:

\[
TR_{1k}(Q_k) = [1 - MC_i(Q_k)]Q_k, 
\]

and marginal revenue is:

\[
MR_{1k}(Q_k) = [1 - MC_i(Q_k)]dMC_i(Q)/dQ_k. 
\]

The non-constraining (non-k) intermediate-input firms would eventually learn that real-price reductions are ineffective in increasing quantity demanded and instead would use quantity adjustments, with downwardly-rigid real prices, when excess supply exists. Therefore, the degree to which firm k’s demand curve shifts in response to a slight reduction in its output depends on the degree of price flexibility, specifically, the price response of non-k firms to the excess supply that results from the reduction in output by firm k.

Now, consider the rigid real-price case. If, in stage two of the trading story, all intermediate-input firms hold their price rigid in the face of excess supply, firm k’s demand curve would not shift and points on it would be based only on the rent-capture effect. That is, equations (11) through (13) (firm k’s inverse demand curve) would take the following form:

\[
\begin{align*}
(\text{i}) & \quad P_{1k} = 1 - \Sigma P_i - MC_i(Q_k), \quad \text{for } 0 < Q_k < Q_{e}; \\
(\text{ii}) & \quad P_{1k} = P_{1e}^* \quad \text{for } Q_k = Q_{e}, \text{ and} \\
(\text{iii}) & \quad P_{1k} = P_{1e}^* \quad \text{for } Q_k > Q_{e}. 
\end{align*}
\]

The rent-capture effect in the rigid real-price case is illustrated in Figure 1. If no price-redistribution effect occurs, supplier k’s demand curve (reflecting the rent-capture effect) would be given by the line segment RN, whose slope is determined by the slope of the final-good sector’s marginal cost curve (along section DI).

For \( 0 < Q_k < Q_{e} \) and the fixed price case (only the rent-capture effect is operative), total revenue for firm k is:

\[
TR_{1k}(Q_k) = [1 - \Sigma P_i - MC_i(Q_k)]Q_{e}, 
\]

and marginal revenue is:

\[
MR_{1k}(Q_k) = [1 - \Sigma P_i - MC_i(Q_k)]dMC_i(Q)/dQ_{k}. 
\]

In Figure 1, supplier k’s marginal revenue would be given by RS.

**THE EFFECTS OF PRICE FLEXIBILITY AND PRICE RIGIDITY**

The first three propositions relating to the effect of flexible versus rigid pricing by intermediate-input firms follow directly from the analysis in the previous section.

**Proposition 1:** A flexible, real-price system fails to improve the welfare properties of the economy and can actually cause a deterioration of welfare by introducing serious production instabilities. With perfect real-price flexibility, the perceived increase in total revenue from the price-redistribution effect combined with the perceived decline in total cost will create an incentive for each intermediate-input firm to undertake a slight reduction in output below any coordinated output level. Because this incentive exists for any positive level of coordinated output, in the limit, output will fall to zero. Therefore, the price-redistribution effect resulting from price flexibility creates serious instability in a production-coordination model with real exchange.

**Proposition 2:** Downwardly inflexible prices as a response to demand constraints may result in sustainable, Pareto-preferred outcomes. In an economy with price-rigid flexibility, a reduction in output by firm k in the intermediate-input sector will generate only a rent-capture effect. Therefore, whether firm k has an incentive to reduce production depends on whether at \( Q_k = Q_{e} \), marginal revenue, described by equation (15a), is less than marginal cost,
If marginal revenue exceeds or equals marginal cost, firm k would have no incentive to reduce production below the coordinated output level. However, if marginal revenue were less than marginal cost for any firm k in the intermediate-input sector, firm k would reduce production. Therefore, a coordinated output level \( Q_k \) is a Nash equilibrium only if marginal revenue equals or exceeds marginal cost for all firms in the intermediate-input sector. That is, in addition to conditions (A) through (C), condition (D) must hold:

\[
(MR_k = MC_k) \text{ for each firm } j \text{ in the intermediate-input sector.}
\]

A stable Nash equilibrium under rigid real prices is illustrated in Figure 1. In Figure 1, if \( MC \) is the marginal cost for firm i (c), then at \( Q_i \) marginal revenue is equal to (greater than) marginal cost for firm i (c). Therefore, firm k (c) has no incentive to reduce production from the coordinated equilibrium at \( Q_k \).

Proposition 3: Price rigidity is a rational, strategic mechanism adopted by the individual firm in a production-coordination environment. In an economy with production-coordination problems, because a price reduction would not increase the level of sales for an intermediate-input supplier who is not the "shortest" supplier, rational behavior for an individual supplier facing excess supply is to refuse to lower its price. Maintaining rigid prices in the face of excess supply will prevent strategic behavior by other suppliers from reducing their share of any residual marginal price (i.e., rigid prices prevent the redistribution effect). Therefore, prices become downwardly rigid as a result of protective strategic behavior by individual suppliers.

MAXIMUM NASH EQUILIBRIUM, PRICE SETS, AND PARETO-OPTIMALITY

The complete conditions for coordination-production Nash equilibrium at \( Q_n \) can be expressed as follows:

\[
(A) \quad Q_n = Q_j/n = Q_j/n \quad \text{for each firm } j \text{ (i) in the intermediate-input (final-good) sector; \( n \) is the number of firms in the sector.}
\]

\[
(B) \quad Q_x = Q_x \quad \text{for each firm } x \text{ in the final-good sector;}
\]

\[
(C) \quad MR_x = 1 - (SP_x) = MC_x \quad \text{for each firm } x \text{ in the final-good sector; and}
\]

\[
(D) \quad MR_i = MC_i \quad \text{for each firm } i \text{ in the intermediate-input sector.}
\]

Each of these conditions was established in the previous sections.

Assume that the economy is producing a coordinated output level \( Q_n \) as described by the above conditions. This section will demonstrate Propositions 4 through 6.

Proposition 4: For \( Q_n \) above a critical level, \( Q_{max} \), no intermediate-input price set exists that satisfies conditions (A) through (D), and thus output levels above \( Q_{max} \) cannot be Nash equilibria. From equations (11b) and (10a), when all other firms are producing at coordinated-equilibrium \( Q_n \), the marginal revenue for an intermediate-input firm k that is producing \( Q_k < Q_n \) is \( MR_k = P_k - (dMC_k/Q_k)(dQ_k/Q_k) \). From condition (D) for a Nash equilibrium at \( Q_n \), \( MR_k(Q_n) = MC_k(Q_n) \) or \( P_k - (dMC_k/Q_k)(dQ_k/Q_k) = MC_k(Q_n) \) for each k. Rearranging:

\[
P_k > MC_k(Q_n) + (dMC_k/Q_k)(dQ_k/Q_k)Q_n = SQ_k(Q_n).
\]

In addition, from condition (C), for a Nash equilibrium at \( Q_n \), \( 1 - SP_x = MC_x(Q_x) \) or by rearranging:

\[
(17) \quad SP_x = 1 - MC_x(Q_x) = T(Q_x).
\]

As \( Q_n \) increases, \( T \) and therefore \( SP_x \) must increase. From equation (16), summing over all i in the intermediate-input sectors, \( SP_x \geq SQ_x(Q_x) \). Further as \( Q_n \) increases, \( SQ_x(Q_x) \) must also increase. Therefore, \( SQ_x(Q_x) \) is a monotonically increasing function of \( Q_n \) and \( SQ_x(Q_x) \) is a monotonically decreasing function of \( Q_x \). For output levels beyond the point where these two functions intersect, the condition \( SP_x \geq SQ_x(Q_x) \) is violated. Since this condition follows from condition (D), for output levels beyond this point, Nash equilibria cannot exist for these output levels. More specifically, the maximum output level \( Q_n \) for which a Nash equilibrium can exist is that output level, \( Q_{max} \), where the two functions intersect, i.e., \( SQ_x(Q_x) \leq SQ_x(Q_x) \).

Does \( Q_{max} \) correspond to Pareto-optimal production, \( Q^P \)? For a Pareto-optimal equilibrium, conditions (A) through (C) hold at the Pareto optimal output level \( Q^P \), and condition (D) is replaced by:

\[
(D) \quad P^P = MC_i(Q_i),
\]

for each firm, i in the intermediate-input sector.

Conditions (C) and (D') are analogous to the marginal conditions for Pareto-optimality in standard welfare economics.

Because intermediate-input firms operate in imperfectly competitive markets, at the maximum Nash equilibrium, \( P_k > MR_k = MC_k \). Intermediate firms will have no incentive to produce at a level where price is equal to their marginal costs (and marginal revenue is less than marginal cost). Therefore condition (D) does not hold even at the maximum Nash equilibrium.

Given the simplifying assumptions in Figure 1, Figure 2 illustrates both the maximum Nash at \( Q_{max} \) and the Pareto-optimal level of production at \( Q_n \) (also illustrated in Figure 1). Let both intermediate-input suppliers, \( x \) and \( k \), have price, demand, marginal revenue, and marginal cost equal to \( P_x, D_x, MR_x, \) and \( MC_x \), respectively. In addition, \( 1 - SP_x + 1 - Q_{max} \) is the marginal residual and \( 2Y \) is the marginal cost of a final-good firm. Therefore, \( Q_{max} \) is a Nash equilibrium. In addition, because conditions (C) and (D) are holding as strict equalities, no firm has an incentive to expand beyond \( Q_{max} \). However, given that \( 2MC \) is the vertical sum of the marginal cost of firms \( x \) and \( k \), Pareto-optimal production is at \( Q^P \), where \( 1 - MC_x - MC_k = MC_x \).
Even though a Pareto-optimal equilibrium is not feasible, equilibrium output levels can be Pareto-ranked by comparing various output levels to the level associated with Pareto-optimality. The closer any equilibrium level of output is to the Pareto-optimal level, the more Pareto preferred that output level would be. Since $Q_{ext}$ is the highest equilibrium output that is feasible, it is the Pareto-preferred coordinated-output level.

Proposition 5: At the unique, coordinated-output level, $Q_{ext}$ the set of intermediate-input prices will be unique, and $Q_{ext}$ will be the maximum, Nash equilibrium. From Proposition 4, at $Q_{ext}$, $\Sigma P_i = T(Q_i) = 1$. However, Nash equilibrium condition (C) implies that:

$$P_i = MC_i(Q_i) + [dMC_i(Q_i)/dQ_i]Q_i = S(Q_i).$$

If the strict inequality holds, the condition $\Sigma P_i = T(Q_i) = 1$ is violated. Therefore, $P_i = S(Q_i)$ at $Q_{ext}$, and $P_i$ for all $i$ is unique at $Q_{ext}$.

Proposition 6: Levels of $Q_i$ below $Q_{ext}$ can be Nash equilibria; however, the set of intermediate-input prices associated with the equilibrium will not be unique. This proposition is a corollary of Proposition 5. For output levels below $Q_{ext}$, equation (16) holds as an inequality for at least some firms in the intermediate-input sector. This inequality implies that the prices in the intermediate-input sector are not unique for $Q_i$ below $Q_{ext}$. For example in Figure 1, the price set of $P_4^s$ and $P_5^s$ are only two of many possible price sets compatible with the Nash equilibrium at $Q_4$. 

**PRICING CONDITIONS FOR A SUCCESSFUL COORDINATED EXPANSION**

Events promoting production coordination, whether the result of government policy or private sector developments, require suitable pricing conditions for the improved production coordination to result in economic expansion. Given an initial output level $Q_i$ below $Q_{ext}$, for a production-coordinating action to successfully expand output to a Pareto-preferred level, no intermediate-input firm, prior to the expansion, can satisfy condition (D) as a strict inequality (i.e., marginal revenue is equal to marginal cost). If such an intermediate-input firm did exist, its expansion would cause its marginal cost to exceed its marginal revenue. For example, in Figure 1 intermediate-input supplier $A$ has marginal revenue equal to marginal cost at $Q_a$. This firm would have no incentive to expand even if expansion was otherwise coordinated. Therefore, only as long as condition (D) is satisfied as a strict inequality for each intermediate-input supplier can a coordinated production expansion successfully move the system to a Pareto-preferred Nash equilibrium. The following proposition follows from this analysis.

Proposition 7: Given an initial equilibrium below $Q_{ext}$, price coordination may be necessary even for coordinated production advances. If, at the initial output level, marginal revenue equals marginal cost for some intermediate-input firm, this firm would expand regardless if it received an increase in its price. However, a rise in one intermediate-input firm’s price would require a reduction in the price of other intermediate-input firms. A coordinated production expansion would thus require accompanying price coordination. Specifically, if these respective increases and decreases in intermediate-input prices occur such that $\Sigma P_i$ remains at a level continuing to satisfy $1 - \Sigma P_i = MC_i(Q_i)$ for each final-good firm, then these real-price changes would provide an incentive for all firms in each sector to jointly expand.

For example, in Figure 1 at coordinated production equilibrium $Q_i$, marginal revenue exceeds marginal cost for intermediate supplier $x$, but marginal revenue equals marginal cost for supplier $h$. Therefore, intermediate-input supplier $h$ would be willing to expand to a higher coordinated production equilibrium only if its price increased (effectively its demand and marginal revenue curves shift up), thus forcing the price of supplier $x$ to fall (effectively its demand and marginal revenue curves shift down).

It is not clear, however, that market incentives would lead to such price-coordinating changes. Therefore, by Proposition 7, in a world characterized by production-coordination failure, pure production-coordinating policy actions that are intended to be expansionary will not succeed unless accompanied by incentives for economic units to engage in the appropriate price coordination. Specifically, intermediate-input firms that have marginal revenue greater than marginal cost must reduce their price in the face of expanding demand and production (even though as described above they would refuse to lower prices when subject to sales constraints) to allow intermediate-input firms that have marginal revenue equal to marginal cost to increase their price and marginal revenue.
CONCLUSION

This paper demonstrates that, in economics characterized by production coordination failures, price-setting firms, corresponding to excess supply resulting from the strategic behavior of other firms, will have an incentive to maintain real-price rigidity. This paper shows that such real-price rigidity results in Nash equilibria associated with nonzero output levels. Therefore, in contrast to Bryan's pathological result in a Walrasian market environment, this paper argues that a market mechanism can avoid a Nash equilibrium with zero production.

In addition, this paper concludes that markets with production-coordination failures and rigid real prices will not generate a configuration of real prices for which a Pareto-optimal level of output is a Nash equilibrium. Instead, a model with these features is associated with a Pareto-sub-optimal maximum level of output. This result is associated with the non-uniqueness of the price. At output levels below this maximum level of output, Nash equilibria are associated with a non-uniqueness of the price.

Finally, the pricing features of this model require a degree of pricing coordination to sustain output levels generated by production-coordination actions. Because of these features, this paper can be regarded as a contribution to both the price and production-coordination literature.

The strategic behavior in this production coordination model suggests an advantage for inelastic real prices (and price coordination) supports a strand of the New Keynesian literature that emphasizes real-price rigidity. The implicit contract, insider-outside, and efficiency-wage models generate real-wage rigidity by proposing a non-market-clearing function for real intermediate-input prices. This paper argues that price flexibility results in losses to the supplier that changes prices in response to excess supply. This result may offer an even stronger justification for fixed real prices than those models that indicate that price flexibility provides gains too small to offset the costs of price adjustment [Romer, 1995, 8-11].

NOTES

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1. Silvestre (1993) argues that, because of the assumption of instantaneous price adjustments in coordination models, these models do not formally belong to the sticky-price models of the New Keynesian. However, Colander (1990), van Leeuwen and Garrelsen (1992), and Mihalik (1990) believe that, although sticky prices do not cause coordination failure, coordination failure models are consistent with sticky prices. See Gordon (1990), Mihalik and Romer (1991), Colander (1990), van Leeuwen and Garrelsen (1992), and Davidson (1992) for various views on taxonomy and labeling. For example Colander (1990) prefers the Post Walrasian label for coordination models.


3. This paper follows Colander's (1990) suggestion to incorporate coordination into the production function.
INTRODUCTION AND OVERVIEW

The fall of the Berlin wall was the most symbolic event of the political and economic transition process in Central and Eastern Europe. Literally overnight the wall came down, opening the way for ideas, trade, people, and economic resources to flow completely unhindered across the former border. East Germany's adjustment from a closed economy to a completely open one was the most abrupt among all the transition economies. The initial stages of adjustment to such an extreme shock can teach important lessons about the dynamic and nonlinear processes that fully or partially restore international competitiveness.

A decade later, the adjustment path taken by the East German (GDR) economy and the determining factors leading down a particular high-tech, high-wage path have become clear. Paus (1998, 1840) concludes that overall East German manufacturing succeeded in becoming more competitive, in spite of adverse initial post-unification conditions. Nonetheless, while some sections and sectors of East German manufacturing managed quite successfully to increase their competitiveness sufficiently to survive, others did not. The existing literature doesn't clarify how this happened. This paper analyzes the dynamics of competitiveness using the case of East Germany to test a novel "evolutionary trade theory" with a novel East German data series. According to Amendola et al. (1993), such "evolutionary" approaches have the advantage of favoring variables that affect competitiveness and trade in the longer term, rather than macroeconomic variables such as exchange rates elicit short-term responses of trade flows.

This paper argues, on a theoretical basis, that structural changes, as indicated by export unit values and reflected in export shares at the manufacturing sector level, are indicative of the transition paths taken in East Germany. East Germany, because of its expensive currency and relatively high wages upon unification, may have moved early on to penetrate western markets on the basis of quality and technology content rather than on price only. East German competition and technology dynamics would, it is hypothesized, more rapidly and successfully make internationally competitive manufacturing sectors that are intensely competition and technology driven. Sectors that traditionally rely more on cheap, low-skilled labor for their inputs, or that use natural resources, including energy, intensively would be much less amenable to com-