AGGREGATE PRODUCTION FUNCTIONS AND THE MEASUREMENT OF INFRASTRUCTURE PRODUCTIVITY:

A REASSESSMENT

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INTRODUCTION

Since the publication of Aschauer's [1989a] seminal paper, the question of the quantification of the effect of government expenditures on private sector output and on productivity has been a subject of debate.[1] The concern is that the estimated elasticities of infrastructure, in particular those using time series, seem to be too high to be believable [Aaron, 1990; Rubin, 1991]. In a survey of the literature Munnell concluded: "In my view, the implied impact of public infrastructure investment on private sector output emerging from the aggregate time series studies is too large to be credible" [1992, 191]. Most of this literature has used econometric estimation, and the most common framework has been an extended aggregate production function (APF), often Cobb-Douglas, with a proxy for public infrastructure as a third factor of production. This has been estimated, also in most cases, using OLS. Critics suggest that the high elasticities are the result of problematic econometrics; that is, the estimated parameter of infrastructure is biased upward because the regressions have not taken into account the possibility of unit roots in time-series estimations; or the possible endogeneity of the regressors [Aaron, 1990; Berndt and Hansson, 1992; Gramlich, 1994].

Ford and Poret, using data for eleven OECD countries, concluded: "...time series regressions tend to yield non-robust and sometimes implausible parameter estimates, *suggesting a fundamental problem with the underlying methodology*" [1991, 65; italics added]. This paper argues that the issue at stake is not econometric, and that it is indeed methodological. The problem is much more serious than that of how to properly estimate an aggregate production function. Indeed, there is a large body of literature that some time ago questioned the notion of APF on theoretical grounds. Fisher [1965; 1969a; 1971; 1983] derived the theoretical conditions for successful aggregation. His conclusion was that the conditions under which the production possibilities of an economy can be represented by an APF are so stringent that one can hardly believe that actual economies satisfy them.[2] And in the three-factor production function, the case at hand here, the conditions for successful construction of a sub-aggre-

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gate of capital goods such as public infrastructure are even less reasonable [Fisher, 1965; 1983].[3] On top of this, and related, there is the problem of using "deflated values" as measures of capital in the aggregate production function (as is the case in the literature at hand). As Brown [1980, 397-98] shows, unless the restrictive Gorman aggregation conditions are satisfied, the deflation process will not yield a "real value." (See also Robinson [1971].)

The issue discussed in this paper can be encapsulated in the following rhetorical questions: if Fisher proved that the APF could not be derived from micro production functions, except under very restrictive assumptions, and that even the construction of sub-aggregates of capital requires heroic restrictions, what do economists obtain when they estimate the APF with public expenditures? And in particular, is the parameter of infrastructure in such regressions a measure of its productivity? What is wrong with the estimates advanced so far in the literature? Is it just a matter of running regressions until more "reasonable" estimates appear? Of course one could argue, following Samuelson [1962], that aggregate production functions are parables that can be used to illustrate important truths about the production process. This argument, however, encounters two problems. First, at the theoretical level, Garegnani [1970] proved that Samuelson's arguments could not be extended to the more realistic case of heterogeneous goods, (i.e., the one-commodity-model results do not hold in heterogeneous commodity models). Furthermore, Fisher [1969b] also proved that the conditions for the existence of aggregate production functions are unlikely to hold even as approximations. Second, it is not clear what the parable argument means in the context of applied work. Once one decides to estimate a production function at the aggregate level with actual data, one has to ask questions such as "What for?"; "What do we expect to obtain?"; "How are we going to interpret the results" (e.g., estimated parameters)?; and "Are we going to derive (policy) implications from the estimates?" The arguments in this paper relate to this second set of questions.

To gain insight into the issues at hand, Table 1 shows the estimates of the standard Cobb-Douglas augmented with infrastructure for the U.S. private sector. I use the data set provided by Ford and Poret [1991] for the United States from 1960 to 1989, taken from the OECD's Analytical Database. The OECD database provides two measures of infrastructure in constant prices: the so-called "narrow definition," which is the capital stock of producers of government services (denoted here by $G1$ in levels and $g1$ in growth rates); and the "broad definition," which consists of the components of the narrow definition plus equipment and structures in electricity, gas and water, and structures in transport and communication (these are subtracted from the private-sector capital stock. Ford and Poret [1991] provide details of these measures.

The first two equations are in growth rates, and the following two in levels. The results shown in this table are quite standard. The problem is how to interpret the estimates, in particular the negative sign of the stock of private capital. If this regression were indeed an aggregate production function, what should we make of it?

The paper develops a general argument, with direct implications for the discussion of the empirical results observed in the literature, to show why estimation of APFs yields, in general, contentious estimates of the productivity of infrastructure. Furthermore, the argument questions the estimates of APFs as meaningful even if

TABLE 1
Cobb-Douglas Production Function with Infrastructure

A. (Growth rates. Narrow measure) $q_t = \delta g1_t + \alpha \ell_t + \beta k_t$

Constant	δ	α	β	R^2	D.W.
0.045	1.04	1.33	−1.95	0.80	2.22
(3.63)	(5.16)	(10.43)	(−5.04)		

B. (Growth rates. Broad measure) $q_t = \delta g2_t + \alpha \ell_t + \beta k_t$

Constant	δ	α	β	R^2	D.W.
0.051	1.67	1.37	−2.52	0.81	2.14
(4.12)	(4.88)	(10.24)	(−5.61)		

C. (Levels. Narrow measure) $\ln Q_t = \delta \ln G1_t + \alpha \ln L_t + \beta \ln K_t$

Constant	δ	α	β	R^2	D.W.
-6.82	0.89	1.50	−0.68	0.99	0.53
(-6.84)	(8.04)	(6.22)	(−3.81)		

D. (Levels. Broad measure) $\ln Q_t = \delta \ln G2_t + \alpha \ln L_t + \beta \ln K_t$

Constant	δ	α	β	R^2	D.W.
-8.53	1.31	1.49	−0.91	0.99	0.40
(-5.65)	(5.94)	(5.67)	(−3.37)		

t-tests in parenthesis. Data for the U.S. private sector for 1960-89 (Ford and Poret 1991).

one could prove that the APF existed. This argument is that underlying every production function there is the income accounting identity that relates output to the sum of the wage bill and total profits. This identity can always be rewritten as a mathematical form that resembles a production function. Thus, if what has been estimated is, in fact, an accounting identity, we should be able to explain rather easily why estimation of aggregate production functions tends to yield very high fits (potentially $R^2 = 1$), as the results in Table 1 indicate. The argument also explains the expected size of the estimated parameters.

The rest of the paper is structured as follows. The next section develops the general argument in the context of time series, and explains why the existence of the underlying accounting identity matters for the interpretation of the estimates of the production function. We then discuss the specific case of the production function with infrastructure followed by a presentation of the empirical evidence. We continue by developing the argument further, and discuss the case of the cross-section production function with infrastructure expenditures. We then offer a discussion of the productivity puzzle observed by Aschauer and others.

THE AGGREGATE PRODUCTION FUNCTION AND THE INCOME ACCOUNTING IDENTITY

This section explains why estimation of APFs cannot yield estimates of the technological parameters in the context of time series analysis. The essence of the argument lies in the underlying relationship that exists between the APF and the income accounting identity.[4] To see this, let's write this identity for value-added of the private sector, namely, the sum of the wage bill plus the operating surplus (a similar argument can be developed with gross output),

$$Q_t \equiv w_t L_t + r_t K_t \tag{1}$$

where Q, w, r, L, and K denote output (real value added), the real average wage rate, the average accounting profit rate, employment, and stock of capital, respectively. Q, w, and K are monetary values deflated, and L is a quantity (number of hours or number of workers).[5] Equation (1) holds as an identity (i.e., without behavioral assumptions) for every period. It states that income equals the sum of the wage bill plus all types of profits. It does not assume constant returns to scale or perfect competition. It holds in every type of market, and it is compatible with any (aggregate) technology, or with the non-existence of any technology. The operating surplus (i.e., all profits on all types of capital goods) is written as the *ex-post* or accounting average profit rate (not the user cost of capital) times the stock of capital. The series Q, L, and K in equation (1) are the same as those used to estimate the APF. Now let's rewrite equation (1) in growth rates as

$$q_t = a_t\varphi_{wt} + (1-a_t)\varphi_{rt} + a_t\ell_t + (1-a_t)k_t = \varphi_t + a_t\ell_t + (1-a_t)k_t \tag{2}$$

where q, ℓ, and k are the growth rates of output, employment, and capital, respectively; φ_{wt} and φ_{rt} are the growth rates of the wage and profit rates, respectively; $a_t = (w_tL_t)/Q_t$ and $1-a_t = (r_tK_t)/Q_t$ are the labor and capital shares in total output; and $\varphi_t = a_t\varphi_{wt} + (1-a_t)\varphi_{rt}$. This derivation does not involve any behavioral assumption.

Without affecting the substance of the argument, assume that factor shares in this economy are constant over time, (i.e., $a_t = a$). (This assumption will be relaxed later on.) Integrate equation (2). This yields:

$$Q_t = A_0 w_t^a r_t^{1-a} L_t^a K_t^{1-a} \tag{3}$$

where A_0 is the constant of integration. It should be obvious that equation (3) is the income accounting identity, equation (1), rewritten under the assumption that factor shares are constant. In other words, if factor shares in a given economy were constant and one were to estimate equation (3) unrestricted, one would certainly obtain a perfect fit, and estimates equal to the factor shares. Expression (3) is very important for purposes of this paper because it clearly resembles a Cobb-Douglas function. The difference is, of course, that equation (3) includes the wage and profit rates. But suppose equation (3) is written as $Q_t = A_0A(t)L_t^a K_t^{1-a}$ where $A(t) = w_t^a r_t^{1-a}$. Now this expression certainly looks like a Cobb-Douglas function. What this indicates is that if we find a function of time that describes the motion of the product $A(t) = w_t^a r_t^{1-a}$, the last expression will continue yielding a perfect fit and estimates of the coefficients of labor and capital equal to the factor shares. Suppose, for example, that in this economy the growth rates of the wage and profit rates are constant, (i.e., their paths are $\varphi_{wt} = \varphi_w$, i.e., $\omega_t = \exp(\varphi_w t)$ and $\varphi_{rt} = \varphi_r$, i.e., $r_t = \exp(\varphi_r t)$). Under these circumstances, equation $Q_t = A_0 w_t^a r_t^{1-a} L_t^a K_t^{1-a} = A_0A(t) L_t^a K_t^{1-a}$ becomes $Q_t = A_0 e^{\varphi t} L_t^a K_t^{1-a}$, where e is the exponential number, "t" is a time trend, and $\varphi = a\varphi_w + (1-a)\varphi_r$. This form resembles the standard Cobb-Douglas with a time trend; but remember that here no reference has been made to a production function, and thus there is no reason why it should be interpreted as such. All these results follow simply because of the underlying accounting identity [Samuelson, 1979, 933].

What this argument shows is that if in this economy factor shares and the growth rates of the wage and profit rates are constant, then the equation $Q_t = A_0 e^{\varphi t} L_t^a K_t^{1-a}$ will provide a perfect fit to the data. And of course, if these assumptions are not true, the standard Cobb-Douglas with a linear trend will not work empirically. Certainly these two assumptions have nothing to do with the existence of an aggregate production function. Factor shares can be constant for a myriad of reasons that have nothing to do with a Cobb-Douglas production function, (e.g., a constant markup) [Kaldor, 1956]. Furthermore, in his seminal paper, Fisher concluded, "the view that the constancy of labor's share is due to the presence of an aggregate Cobb-Douglas production function is mistaken. Causation runs the other way and the apparent success of aggregate Cobb-Douglas production functions is due to the relative constancy of labor's share" [1971, 306].

To see what occurs empirically when one estimates a production function, first we estimate the income accounting identity under the assumption that factor shares are constant. This corresponds to the estimation of equations (2) and (3), in growth rates and levels, respectively. Estimation of these forms can be interpreted simply as a test for whether factor shares are constant. If indeed they are, these expressions should work empirically, (i.e., will yield very high fits, parameters very close to the average factor shares, and because they are identities, they will have to yield [almost] identical results). Results shown in equations A (growth rates) and B (levels) in Table 2 are clear. The four parameters estimated are very close to the average factor share, (i.e., $\gamma_1 \cong \bar{a}$; $\gamma_2 \cong 1-\bar{a}$; $\gamma_3 \cong \bar{a}$; $\gamma_4 \cong 1-\bar{a}$). (The average labor share, its standard deviation, as well as maximum and minimum values are shown in the table.) The only way to simultaneously explain the proximity of the parameters to the factor shares, the extremely high t-statistics and fits, and the fact that it does not matter whether we use growth rates or levels, is by arguing that what has been estimated is an identity. This analysis confirms that factor shares are (sufficiently) constant, and thus verifies the first hypothesis for this data set. We conclude that because factor shares are (sufficiently) constant, a Cobb-Douglas "production function" should work properly with this data set.

To continue with the argument, we now fit the Cobb-Douglas form with the linear trend. As discussed above, this expression follows from the previous identity (under the assumption that factor shares are constant) by imposing a second assumption, namely, that wage and profit rates grow at constant rates. Estimation results are shown in equations C (growth rates) and D (levels) in Table 2. The results are striking

TABLE 2
Value Added Accounting Identity
(Time Series) Equation 2

A. (Growth rates) $q_t = \gamma_1\varphi_{wt} + \gamma_2\varphi_{rt} + \gamma_3 l_t + \gamma_4 k_t$

γ_1	γ_2	γ_3	γ_4	$\bar{a}$; σ	R^2	D.W.
0.579 (230.94)	0.418 (270.98)	0.582 (247.76)	0.417 (190.08)	0.576; 0.012	0.9999	1.56

Max.(a)=0.594; Min.(a)=0.555

B. (Levels) $\ln Q_t = c + \gamma_1 \ln w_t + \gamma_2 \ln r_t + \gamma_3 \ln L_t + \gamma_3 \ln K_t$

Constant	γ_1	γ_2	γ_3	γ_4	R^2	D.W.
−5.94 (−334.28)	0.574 (454.00)	0.423 (306.45)	0.578 (228.53)	0.422 (212.33)	1.00	1.24

C. (Growth rates) $q_t = c + \gamma_3 l_t + \gamma_4 k_t$

Constant	γ_3	γ_4	R^2	D.W.
0.056 (3.26)	1.15 (6.65)	−1.35 (−2.59)	0.63	1.15

D. (Levels) $\ln Q_t = c + \varphi t + \gamma_3 \ln L_t + \gamma_4 \ln K_t$

Constant	φ	γ_3	γ_4	R^2	D.W.
4.99 (0.98)	0.024 (1.25)	0.287 (0.85)	−0.029 (−0.059)	0.982	0.35

t-statistics in parentheses. Data for the U.S. private sector for 1960-89 (OECD's Analytical Database). $\bar{a}$ is the average labor share; σ is the standard deviation of the labor share; Max. and Min. are maximum and minimum values of the labor share, respectively.

in that now the estimates diverge significantly from the factor shares (these are what anyone would call very poor regression results). Why do these forms fail to perform well? In the light of this analysis, the answer is obvious: because the second assumption (i.e., that the weighted average of the growth rates of the wage and profit rates is a constant) is incorrect for this data set. We need to search for an alternative path for the weighted average.

THE AGGREGATE PRODUCTION FUNCTION WITH INFRASTRUCTURE

To see how this argument relates to the case at hand (i.e., the production function with infrastructure expenditures), we proceed as follows. Continue assuming that in this economy factor shares are constant. (We have already tested this assumption and concluded that it is empirically valid.) Alternatively, we have seen that wages and profit rates do not grow at constant rates (this is the incorrect assumption that makes the regression with a linear trend yield such poor results). Rather, suppose

that in this economy the growth rates of the wage and profit rates follow (i.e., they are well tracked by) the paths $\varphi_{wt} = \delta_1 g_t$ ($w_t = G_t^{\delta_1}$ in levels) and $\varphi_{rt} = \delta_2 g_t$($r_t = G_t^{\delta_2}$ in levels), where g_t denotes the growth rate of public infrastructure expenditures (G_t is the level), and δ_1 and δ_2 are constants. These assumptions need not have theoretical content in the sense that we are not trying to model the wage and profit rates (see discussion below). They are simply empirical assumptions, the same as those above when we hypothesized that these growth rates were constant, which can be corroborated or refuted (and remember that the aggregate production function continues, in all likelihood, without existing). Under these circumstances, φ_t in equation (2) becomes

$$(4)\qquad \varphi_t = a_t\varphi_{wt} + (1-a_t)\varphi_{rt} = a\varphi_{wt} + (1-a)\varphi_{rt} = a\delta_1 g_t + (1-a)\delta_2 g_t = \delta g_t$$

where $\delta = a\delta_1 + (1-a)\delta_2$.

Equation (2) then becomes:

$$(5)\qquad q_t = a\varphi_w + (1-a_t)\varphi_r + a\ell_t + (1-a)k_t = \delta g_t + a\ell_t + (1-a)k_t.$$

If we now integrate equation (5), we obtain:

$$(6)\qquad Q_t = A_0 G_t^{\delta} L_t^{a} K_t^{1-a}.$$

What is equation (6)? Given the derivation above, it is the income accounting identity for output of the private sector, equation (1), rewritten under the assumptions of constant factor shares and the paths of the wage and profit rates assumed in this section, namely, $w_t = G_t^{\delta_1}$ and $r_t = G_t^{\delta_2}$. Suppose one estimated equation (6) unrestricted with the parameters of infrastructure investment, labor and capital, denoted λ, α, β, respectively and assumed the hypotheses made about the data to derive equation (6) were correct. OLS estimation will inexorably yield $\lambda = \delta = a\delta_1 + (1-a)\delta_2$, $\alpha = a$, $\beta = 1-a$, and a suspicious perfect fit. However, as before, the elasticities equal the factor shares simply because of the accounting identity. Likewise, the finding $\delta > 0$ cannot be interpreted as evidence of increasing returns. We have simply estimated an accounting identity. On the other hand, if these assumptions were incorrect for the data set in question, but one nevertheless estimated equation (6), one will not find a perfect fit, and the estimates will diverge form those derived above. Negative estimates are perfectly possible (see Table 1).[6] From an econometric point of view, if G does not track the weighted average of the wage and profit rates in equation (2) well, including it in the regression will lead to a mix of bias of the labor and capital coefficients due to omitted variables (for missing a proper account of the weighted average in the production function), and to inefficiency (for introducing a potentially irrelevant variable, G, in the regression).

An important question is why wage and profit rates are connected to infrastructure. It could then be argued that if such relations exist (i.e., if verified econometrically), then infrastructure *must play a role* (in a causal sense) in production. That is, suppose we estimate equation (6) unrestricted and the coefficient of G appears to be positive, statistically significant, and with a "reasonable" size. One then might argue that the reason is that the aggregate-production (technological) relationship must exist. Such an argument is an example of the so-called *fallacy of affirming the consequent*.[7] If the aggregate production function cannot be derived theoretically, estimation of equation (6) does not prove its existence, and thus one cannot claim that the estimated coefficients are the technological parameters [Blackorby and Schworm, 1984, 647]. The assumptions about the wage and profit rates have to be interpreted in the spirit of Fisher. That is, suppose there is no well-behaved aggregate production function, and yet an equation like (6) works econometrically. This can occur only because equation (6) is simply tracking the identity equation (1). Reversing the argument and concluding that the production function must exist because an equation like (6) appears to yield "sensible" econometric results is a fallacy.

Munnell [1992] and Gramlich [1994] discussed in their surveys three possible econometric problems that could affect the results: (1) spuriouness and common trends, (2) missing variables, and (3) endogeneity of the regressors. None of these issues can be considered a problem. If $a_t = a$, $w_t = G_t^{\delta_1}$, and $r_t = G_t^{\delta_2}$ are the approximately correct paths, then it is clear that an equation like (6) should work (R^2 will be very high, and the three estimated parameters will take on the theoretical values derived here), and thus those three problems do not apply (even in case where the regression shows a low Durbin-Watson, and tests for cointegration indicate that the variables are indeed cointegrated, which is possible in the context of an identity). Naturally, if we are not estimating the identity exactly (because, for example, factor shares vary slightly, or because we are not tracking the weighted average of the factor prices with the paths $w_t = G_t^{\delta_1}$, and $r_t = G_t^{\delta_2}$ well, and we nevertheless fit a Cobb-Douglas), the regression may improve from an econometric point of view by running it in first differences or using cointegration methods, if the variables happen to contain unit roots;[8] or by including extra variables in the regression (e.g., a time trend or a measure or capacity utilization) to track the path of the wage and profit rates better; or by building a simultaneous equation model. But all these issues are secondary. What has likely happened in most empirical applications discussed in the literature is that the path of the weighted average of the wage and profit rates has been incorrectly tracked. All we have to do is search for the correct one, which certainly exists. This will lead to another functional form, (i.e., other production function), which is ultimately another way of writing the income accounting identity in a way that is consistent with the data set.

If factor shares are not constant, we know that the Cobb-Douglas will not work, and we need a different path. This is not a problem, for once we identify their path, we will plug it into equation (2) and upon integration we will obtain another form that will look like another production function, such as the translog, which can be seen as a more complex approximation to the income identity. For the translog to work, factor shares will have to vary significantly. Let us derive this form from the identity (We continue working under Fisher's conclusions (i.e., there is no well-behaved APF). Suppose in this economy factor shares are not constant; in fact they show some trend, which can be modeled as:

(7) $$a_t = \alpha_2 + 2\alpha_5 \ln L_t + \alpha_7 \ln K_t + \alpha_9 \ln G_t$$

(8) $$(1-a)_t = \alpha_1 + 2\alpha_4 \ln K_t + \alpha_7 \ln L_t + \alpha_8 \ln G_t$$

Likewise, suppose the weighted average of the growth rates of the wage and profit rates follows the path:

(9) $$\varphi_t = \lambda + (\alpha_3 + 2\alpha_6 \ln G_t + \alpha_8 \ln K_t + \alpha_9 \ln L_t) g_t$$

Substituting now these three paths into the identity equation (2) yields:

(10) $$q_t = \lambda + (\alpha_1 + 2\alpha_4 \ln K_t + \alpha_7 \ln L_t + \alpha_8 \ln G_t) k_t + (\alpha_2 + 2\alpha_5 \ln L_t + \alpha_7 \ln K_t + \alpha_9 \ln G_t) \ell_t + (\alpha_3 + 2\alpha_6 \ln G_t + \alpha_8 \ln K_t + \alpha_9 \ln L_t) g_t$$

and integrating we obtain:

(11) $$\ln Q_t = \alpha_0 + \alpha_1 \ln K_t + \alpha_2 \ln L_t + \alpha_3 \ln G_t + \alpha_4 (\ln K_t)^2 + \alpha_5 (\ln L_t)^2 + \alpha_6 (\ln G_t)^2 + \alpha_7 (\ln K_t \ln L_t) + \alpha_8 (\ln K_t \ln G_t) + \alpha_9 (\ln L_t \ln G_t) + \lambda t.$$

As before, given the derivation, equation (11) must be interpreted as the income accounting identity rewritten under the paths in equations (7), (8), (9). The difficulty is that it *looks like* a translog production function. This implies that equation (11), when fitted econometrically, will work if and only if the data follow the paths given by equations (7)-(8)-(9).[9]

EMPIRICAL EVIDENCE

Recall that to derive equation (6) two assumptions about the data were imposed. First, factor shares are constant. The second refers to the precise path of the wage and profit rates. The first assumption was tested and confirmed in Table 2. We now proceed to test the second assumption, (i.e., the paths of the wage and profit rates are $\varphi_{wt} = \delta_1 g_t$ (or $w_t = G_t^{\delta_1}$ and $\varphi_{rt} = c_2 g_t$(or $r_t = G_t^{\delta_2}$)), conditional on the constancy of the factor shares. To carry out this second test, we substitute the paths of the wage and profit rates in terms of the infrastructure variable. This amounts to an unrestricted fitting of equation (5) (growth rates), or equation (6) (levels). If these paths are correct, we should expect the coefficient of infrastructure expenditures to be equal to $[a\delta_1 + (1-a)\delta_2]$ and the parameters of capital and labor should continue to be equal to the factor shares. In fact, this was done in Table 1, where we obtained rather poor results.

Further results are provided in Table 3. It shows a total of four regressions, the first two in growth rates and the other two in levels, with the narrow and broad definitions of infrastructure, and including a time trend and a measure of utilization capacity, as done by Aschauer [1989a]. Results are similar to those reported in the literature. The difference is that now we know what caused them. All the "extra" variables do is provide a proxy for the weighted average of the growth rates of the wage and profit rates. If the approximation is good, the regression results will be "good"; and if, not, they will be "bad."

TABLE 3

Cobb-Douglas Production Function with Infrastructure (Equations 5 and 6) Augmented with Time Trend and Capacity Utilization

A. **(Equation 5. Narrow measure)** $q_t = \delta\, g1_t + \alpha \ell_t + \beta k_t + \tau t + \pi cu_t$

Constant	δ	α	β	τ	π	R^2	D.W.
0.007	0.81	0.15	−0.23	0.00042	0.38	0.92	2.42
(0.36)	(2.32)	(0.62)	(−0.55)	(0.88)	(5.28)		

B. **(Equation 5. Broad measure)** $q_t = \delta\, g2_t + \alpha \ell_t + \beta k_t + \tau t + \pi cu_t$

Constant	δ	α	β	τ	π	R^2	D.W.
0.026	0.79	0.22	−0.53	0.000014	0.37	0.91	2.24
(1.48)	(1.48)	(0.85)	(−1.03)	(−0.03)	(4.32)		

C. **(Equation 6. Narrow measure)** $\ln Q_t = \delta \ln G1_t + \alpha \ln L_t + \beta \ln K_t + \tau t + \pi cu_t$

Constant	δ	α	β	τ	π	R^2	D.W.
3.45	0.66	0.51	−0.86	0.033	0.31	0.999	1.19
(3.26)	(7.66)	(2.89)	(−3.15)	(6.26)	(4.76)		

D. **(Equation 6. Broad measure)** $\ln Q_t = \delta \ln G2_t + \alpha \ln L_t + \beta \ln K_t + \tau t + \pi cu_t$

Constant	δ	α	β	τ	π	R^2	D.W.
3.26	0.97	0.37	−1.07	0.037	0.37	0.998	0.83
(2.41)	(5.17)	(1.64)	(−2.45)	(4.74)	(4.50)		

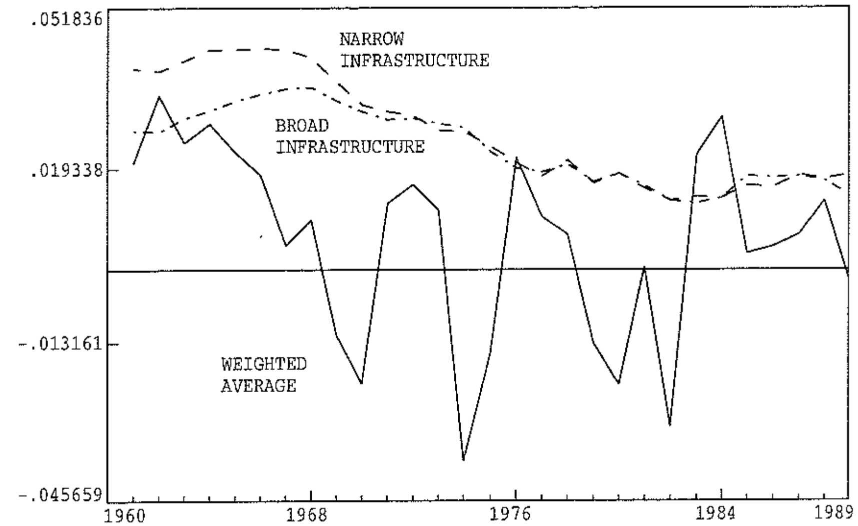
t-tests in parenthesis. Data for the U.S. private sector for 1960-89 [Ford and Poret, 1991].

The problem, from a graphical point of view, is the following. Figure 1 shows the weighted average of the growth rate of the wage and profit rates as well as the growth rates of the narrow and broad definitions of infrastructure. The latter two display much less variation than the weighted average. This explains why we obtained such poor results when we fitted the Cobb-Douglas with infrastructure in Table 1. While factor shares are constant, and thus we should expect a Cobb-Douglas to work, the approximation that infrastructure expenditures provides to the weighted average of the wage and profit rates is very poor (and the same argument applies to the inclusion of the time trend), and this biases the estimates.

It has been shown that neither a linear time trend nor public expenditures proxies the evolution of the weighted average correctly. Remember that ultimately we are approximating an identity, and that we have seen that factor shares are sufficiently constant in this data set so that a Cobb-Douglas form should work. All we need, as argued above, is a mathematical function that approximates the weighted average. Can this be done? It is simply a matter of trial and error. We have found a function for the regression in levels that yields the following results:

FIGURE 1

Weighted Average of the Growth Rates of the Wage and Profit Rates Growth Rates of Broad and Narrow Measures of Infrastructure



$$\ln VA_t = \underset{(1.85)}{0.0072\, A(t)} + \underset{(2.87)}{0.566 \ln L_t} + \underset{(1.38)}{0.301 \ln K_t}$$

with $R^2 = 0.982$ (t-statistics in parentheses). If we compare this regression with equations C-D in Table 1, or equation D in Table 2, we will conclude that this one is substantially better. The most remarkable features are the proximity of the estimated parameters to the factor shares, and the fact that the negative sign of the capital stock has disappeared. The interesting question is: what is $A(t)$? $A(t) = Constant + T - \mathrm{Sin}(T^2) + \mathrm{Sin}(T^4)$, where T is a time trend.[10] Although the approximation is not perfect yet, it proves the point.[11] And the message behind this exercise must be clear: all we have done is to approximate the income accounting identity equation (1).[12]

SOME EXTENSIONS: ALGEBRAIC TRANSFORMATIONS

Here we extend the discussion to two other frameworks that have been used, but which ultimately are also transformations of the income accounting identity and thus do not provide a solution, because they suffer from the same problem of interpretation. The first is the estimation of the equation for total factor productivity growth, and the second is the estimation of the first-order condition.

Aschauer [1989a] and Ford and Poret [1991] used a Cobb-Douglas function with infrastructure expressed in growth rates, assumed competitive product and factor markets, and added together the total contribution of the private sector inputs labor and capital, each weighted by its share in output. That is,

(12) $$q_t = \delta g_t + \theta[a_t \ell_t + (1-a_t)k_t] = \delta g_t + \theta PSI_t$$

where $[a_t \ell_t + (1-a_t)k_t] = PSI_t$ represents the total contribution of private sector inputs. The parameter δ is the elasticity of output growth with respect to infrastructure, and θ is the elasticity of output with respect to the private sector input bundle. $\theta = 1$ implies constant returns in private inputs. The previous expression can be transformed into:

(13) $$TFP_t = \delta g_t + (\theta - 1)[a_t \ell_t + (1-a_t)k_t] = \delta g_t + (\theta - 1) PSI_t = \delta g_t + \psi PSI_t$$

where $TFP_t = q_t - [a_t \ell_t + (1-a_t)k_t]$ is the rate of growth of total factor productivity, and $(\theta-1) = \psi$ (constant returns to scale imply now $\psi = 0$). This is the relationship Aschauer, and Ford and Poret used. However, note that the identity equation (2) can be transformed as follows:

(14) $$q_t = a_t \varphi_{wt} + (1-a_t)\varphi_{rt} + \mu[a_t \ell_t + (1-a_t)k_t] = a_t \varphi_{wt} + (1-a_t)\varphi_{rt} + \mu PSI_t, \text{ or}$$

(15) $$TFP_t = a_t \varphi_{wt} + (1-a_t)\varphi_{rt} + (\mu - 1)[a_t \ell_t + (1-a_t)k_t]$$
$$= \rho[a_t \varphi_{wt} + (1-a_t)\varphi_{rt}] + (\mu-1) PSI_t = \rho RES_t + (\mu-1)PSI_t = \rho RES_t + \tau PSI_t$$

where $RES_t = [a_t \varphi_{wt} + (1-a_t)\varphi_{rt}]$, and $(\mu\text{-}1) = \tau$. We can now compare equations (13) and (15). Because the latter is an identity, and given its derivation, OLS estimation will surely yield $\rho = 1$, $\tau = 0$, and a perfect fit. We can use this fact as our reference point. Why? Because if g_t in equation (13) tracks exactly the path of RES_t in equation (15), then equation (13) will yield a perfect fit. And if that is the case, it will also be true that the estimated parameters will be $\delta = 1$ and $\psi = 0$. The latter (i.e., $\psi = 0$), however, cannot be interpreted, as constant returns to scale. On the other hand, if the approximation is not good enough, the fit will be less than unity, and the lower the correlation between g_t and RES_t, the farther the point estimates δ and ψ will be from the reference values $\delta = \rho = 1$ and $\psi = \tau = 0$. Evidence is provided in Table 4.

The first two regressions in the table are equation (13). The first one includes the narrow measure of infrastructure (g1), and the second one includes the broad measure (g2). It can be appreciated that in both cases the fit has decreased substantially with respect the potential $R^2 = 1$. We also note that the estimates are relatively far from the reference values $\delta = 1$ and $\psi = 0$, and both equations imply unbelievably high putative "increasing returns" in both private sector inputs (θ is around 1.4) and overall inputs $((\delta + \theta)$ is close to 2).

Aschauer [1989a, Table 1, Panel B, equation 1.7] and Ford and Poret [1991] argued that the above regressions do not take into account cyclical variations, and for this reason they included the rate of capacity utilization (cu) as a regressor.[13] The results including this variable, shown in the lower half of Table 4, seem to indicate a substantial improvement with respect to the previous two regressions, at least in terms of fit. It is easy to understand why this occurs. The regression given by equa-

TABLE 4
Total Factor Productivity Growth and the Accounting Identity Equation 13

A. (Narrow measure) $TFP_t = c + \delta\, g1_t + \psi[a_t \ell_t + (1-a_t)k_t]$

Constant	δ	ψ	R^2	D.W.
−0.02	0.584	0.406	0.168	1.28
(−1.71)	(1.89)	(1.44)		

Implied $\theta = 1+\psi = 1.40$ (5.00); Implied $(\delta+\theta) = 1.99$ (4.58)

B. (Broad measure) $TFP_t = c + \delta\, g2_t + \psi[a_t \ell_t + (1-a_t)k_t]$

Constant	δ	ψ	R^2	D.W.
−0.012	0.357	0.359	0.074	1.18
(−0.85)	(0.74)	(1.21)		

Implied $\theta = 1+\psi = 1.36$ (4.60); Implied $(\delta+\theta) = 1.72$ (3.07)

C. (Narrow measure) $TFP_t = c + \delta\, g1_t + \psi[a_t \ell_t + (1-a_t)k_t] + \pi cu_t$

Constant	δ	ψ	π	R^2	D.W.
0.016	0.451	−0.899	0.412	0.85	2.27
(2.64)	(3.41)	(−5.31)	(10.89)		

Implied $\theta = 1+\psi = 0.10$ (0.59); Implied $(\delta+\theta) = 0.55$ (2.43)

D. (Broad measure) $TFP_t = c + \delta g2_t + \psi[a_t \ell_t + (1-a_t)k_t] + \pi cu_t$

Constant	δ	ψ	π	R^2	D.W.
0.017	0.579	−1.01	0.435	0.84	2.09
(2.62)	(2.84)	(−5.70)	(10.92)		

Implied $\theta = 1+\psi = -0.012$ (-0.07); Implied $(\delta+\theta) = 0.56$(2.19)

t-tests in parenthesis. Data for the U.S. private sector for 1960-89 [Ford and Poret 1991].

tion (13) "misspecifies" the weighted average of the growth rates of the wage and profit rates in two combined ways. First, by omitting the weighted average from the regression, and second, by including a series of (potentially) irrelevant variables (infrastructure and capacity utilization). The size and sign of PSI depends on the combined effect of these two problems (although we know that the first problem is far more serious econometrically).[14] Note that the growth rate of the profit rate (φ_{rt}) is highly procyclical while the growth rate of the wage rate (φ_{wt}) is not (these are well known facts). This indicates that variations in the weighted average are mainly driven by variations in the growth rate of the profit rate (factor shares are constant so that they do not affect variations in the weighted average). When the growth rate of the profit rate is omitted from the regression (strictly speaking, it is misspecified), the regression deteriorates. However, if one now includes a procyclical variable such as the rate of capacity utilization, the regression will again more closely approximate the identity. Hence the increase in fit [Felipe, 2001; Felipe and McCombie, 2001].

The regression results are, nevertheless, far from optimal. Note that the estimates of ψ have changed drastically. They are around -1, implying (private sector) returns to scale of 0. This, in turn, implies that private sector inputs have little effect on output at the margin. And overall returns to scale $(\delta + \theta)$ are slightly over 0.5. Is this sensible if what had been estimated were a "production function"? When Ford and Poret estimated this equation for 11 OECD countries they could not draw any coherent inference (estimates differed widely from country to country), leading them to conclude that "the regression results suggest that the numerical estimates of the effect on infrastructure productivity are not robust enough to support a policy recommendation of a sharp acceleration of infrastructure investment" [1991, 74]. It should be clear why this is indeed the case.

A further alternative to the frameworks discussed above is to estimate the first-order condition(s). The optimization condition is $w = dQ/dL$ for labor, and $r = dQ/dK$ for capital. In the case of the Cobb-Douglas these expressions reduce to $w = a(Q/L)$ and $r = (1-a)(Q/K)$. But what would estimating these regressions yield? As we are going to show, if factor shares are sufficiently constant so that the Cobb-Douglas is the correct form, estimating the above expressions (unrestricted estimation as $w = \gamma_1(Q/L)$ and $r = \gamma_2(Q/L)$ must yield $\gamma_1 = a$ (the labor share) and $\gamma_2 = (1-a)$ (the capital share). To see why this is the case, consider the definition (an identity) of the labor share, (i.e., $a_t = (w_tL_t/Q_t)$). This can be rewritten as $w_t = a(Q_t/L_t)$. Suppose, as above, that in this economy the labor share is (sufficiently) constant, (i.e., $a_t = a$) so that $w_t = a(Q_t/L_t)$. This last expression, *an identity*, is indistinguishable from the labor marginal productivity condition. Because it is an identity, its estimate says nothing about the technology, producers' behavior, or market conditions (i.e., profit maximization and competitive markets). The parameter estimated will be the labor share (a similar argument holds for the profit rate, yielding $r_t = (1-a)(Q_t/K_t)$). For the case under consideration, OLS regression of $w_t = \gamma_1(Q_t/L_t)$ yields $\gamma_1 = 0.577$ (t-statistic = 282.23), statistically equal to the average labor share (a). OLS regression of $r_t = \gamma_2(Q_t/L_t)$ yields $\gamma_2 = 0.424$ (t-statistic = 186.96), statistically equal to the average capital share $(1-a)$. It must be stressed that these results cannot be interpreted as evidence of profit maximization and competitive markets. They only confirm that factor shares are (sufficiently) constant. Summing up: the marginal productivity conditions, the same as the aggregate production function, are not testable forms because they cannot be statistically rejected.[15]

Aschauer [1988] used an extension of this framework to test the importance of public sector capital accumulation on the rate of return to private capital. However, in the light of the arguments in the previous paragraph, it should be obvious that such a framework has also a problematic interpretation. How did Aschauer proceed? He substituted the production function $Q_t = A_0G_t^{\delta}L_t^{\alpha}K_t^{\beta}$ back into the numerator of the first-order condition for capital, and imposed the restriction that the coefficients of public expenditures, labor, and capital, add up to unity $(\alpha + \beta + \delta = 1)$. This yields $r_t = \beta(L_t/K_t)^{\alpha}(G_t/K_t)^{\delta}$, an expression which, under the arguments put forward in the paper, does not have a structural interpretation unless it can be shown that the aggregate production function exists.

CROSS-SECTIONAL DATA

Not surprisingly the argument remains for cross-sectional data. It is interesting, however, to see how the relationship between the identity and the production function arises in this case.[16] For a cross section, the labor share can be written as $a_i = (w_iL_i/Q_i)$; and similarly the capital share as $1-a_i = (r_iK_i/Q_i)$ (where i denotes the units of the cross section). For a low dispersion in factor shares, the approximation $\bar{a} \cong (\bar{w}\,\bar{L}/\bar{Q})$, where a bar denotes the average value of the variable, holds. Then the following also holds:[17]

$$(16)\qquad (\bar{a}_i/\bar{a}) \cong (w_i/\bar{w})(L_i/\bar{L})/(Q_i/\bar{Q})$$

and a similar expression follows for the capital share $(1-a_i)$:

$$(17)\qquad (1-\bar{a}_i)/(1-\bar{a}) \cong (r_i/\bar{r})(K_i/\bar{K})/(Q_i/\bar{Q}).$$

For small deviations of a variable X_i from its mean $\bar{X}$, it follows that $\ln(X_i/\bar{X}) \cong (X_i/X)-1$. Thus, taking logs in equations (16)-(17) and using this approximation we can write:

$$(18)\qquad \ln(w_i/\bar{w}) + \ln(L_i/\bar{L}) - \ln(Q_i/\bar{Q}) \cong (a_i/\bar{a}) - 1$$

$$(19)\qquad \ln(r_i/\bar{r}) + \ln(K_i/\bar{K}) - \ln(Q_i/\bar{Q}) \cong [(1-a_i)/(1-\bar{a})] - 1$$

Multiplying equations (18) and (19) by $\bar{a}$ and $(\bar{a}-1)$, respectively, adding them, and rearranging the result yields:

$$(20)\ \ln Q_i \cong B + \bar{a}\ln w_i + (1-\bar{a})\ln r_i + \bar{a}\ln L_i + (1-\bar{a})\ln K_i = B + A(i) + \bar{a}\ln L_i + (1-\bar{a})\ln K_i$$

where $B = (\ln\bar{Q} - \bar{a}\ln\bar{w} - (1-\bar{a})\ln\bar{r} - \bar{a}\ln\bar{L} - (1-\bar{a})\ln\bar{K})$ is a constant. Notice the similarity between equation (20) and a Cobb-Douglas production function. The latter with infrastructure expenditures for a cross-section is:

$$(21)\qquad Q_i = C\,G_i^{\delta}L_i^{\alpha}K_i^{\beta}$$

where C is the constant term. But if one estimates equation (21) unrestricted (in logarithms) for a cross-section of industries, regions or countries, and the variable $\ln G_i$ correctly approximates the term $A(i) = \bar{a}\ln w_i + (1-\bar{a})\ln r_i$ in equation (20), it follows that the estimated coefficients will be very close to the factor shares, and the fit will be very high (potentially unity). From an econometric point of view, if the variation in w_i and r_i is picked up by the variation in G_i (i.e., if infrastructure expenditures pick up the paths of the wage and profit rates, $\ln w_i = \delta_1\ln G_i$ and $\ln r_i = \delta_2\ln G_i$) then the regression will work, and will yield the estimates $\alpha = \bar{a}$, $\beta = \bar{b}$, and $\delta = \bar{a}c_1 + (1-\bar{a})c_2$, which might be interpreted incorrectly as "increasing returns." It must be stressed that the assumption of low variability in the factor shares above is, as in the time-series case, not the key question. What the argument says is that if the dispersion in factor shares is low (for whichever reason), then an equation like (20) will yield very good results, whether an aggregate production function exists or not. And the opposite: equation (20) will never work if the dispersion in factor shares is relative large.

Suggestive empirical evidence is shown in Table 5. First we estimate the identity equation (20) for a sample of 12 OECD countries. The proximity of the estimates to the average shares (i.e., $\gamma_1 \cong \bar{a}$; $\gamma_2 \cong 1-\bar{a}$; $\gamma_3 \cong \bar{a}$; $\gamma_4 \cong 1-\bar{a}$) indicates that indeed factor shares are sufficiently constant for this sample of 12 advanced countries. Hence once should expect a Cobb-Douglas production function to work. The second regression is equation (21). This regression yields what any economist would label as excellent results. The estimates of labor and capital continue being in the neighborhood of the factor shares, adding up to 0.964, statistically not different from unity. And adding infrastructure, the three parameters add up to 1.097, statistically different from unity, thus implying slight increasing returns. As discussed above, these results follow simply because the variable $\ln G_i$ proxies well $[\bar{a}\ln w_i + (1-\bar{a})\ln r_i)]$. Indeed for this data set, the correlation between $\ln G_i$ and $(0.645\ln w_i + 0.355\ln r_i)$ is 0.81. As argued above, if the aggregate production function does not exist, this high correlation cannot be interpreted in a behavioral sense, and the elasticities obtained are determined by the accounting identity.

We can now shed light on Munnell's [1990b, Table 5] results, and what she refers to as "sensible" estimates. The coefficients of capital and labor are almost identical with their shares in total income, they are highly significant, the fit is above 0.99, and her results show slight increasing returns. What explains all this simultaneously is that her equation is tracking the value-added identity impeccably well. Why? Because factor shares for the forty-eight states are almost constant (thus a Cobb-Douglas form must work), and public capital appears to be highly correlated with the weighted average of the factor prices. But the conclusion is, as in the case of time series, that this estimate of δ cannot be taken to be a measure of the productivity of infrastructure.

THE PRODUCTIVITY PUZZLE

As indicated in the Introduction, Aschuer's work took place in the context of the debate over the productivity decline of the 1970s, after economists observed a sharp decline in the rate of total factor productivity growth. The analysis in this paper allows some hypotheses that have been unexplored. From equation (2) it follows that:

$$\varphi_t = q_t = -a_t \ell_t - (1-a_t)k_t = a_t \varphi_{wt} + (1-a_t)\varphi_{rt} \tag{22}$$

Recall that equation (22) has been derived without any reference to a production function. Thus, the so-called Solow residual (TFP growth) is simply our weighted average of the growth rates of the wage and profit rates (nothing to do with the *dual* interpretation of TFP growth!). And we must insist: without reference to a production function, such an expression cannot be interpreted as a measure of productivity.[18] It is simply a measure of distributional changes. We have seen that factor shares vary

TABLE 5
Value Added Accounting Identity and Cobb-Douglas Regression Cross-Section Data, 1988

Equation 20. $\ln VA_i \cong B+\gamma_1\ln w_i+\gamma_2\ln r_i+\gamma_3\ln L_i+\gamma_4\ln K_i$

B	γ_1	γ_2	γ_3	γ_4	R^2
−7.96	0.610	0.352	0.638	0.367	0.9999
(−48.38)	(36.88)	(28.50)	(49.02)	(27.49)	
		$\bar{a}$ =0.645; σ=0.063			

Equation 21. $\ln VA_i = c + \alpha\ln L_i + \beta\ln K_i + \delta\ln G_i$

Constant	α	β	δ	R^2
−3.13	0.521	0.443	0.133	0.995
(−5.11)	(2.79)	(2.15)	(1.60)	

G in these equations represents the narrow definition of infrastructure.
Source: OECD database. Countries: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Sweden, U.K., U.S.A. $\bar{a}$ is the average labor share, and σ is the standard deviation of the labor share.

little, so variations in TFP growth are (mostly) associated with variations in the growth rate of the wage rate, and variations in the growth rate of the profit rate. Figure 2 shows again the weighted average of the growth rates of the wage and profit rates φ_t (i.e., the TFP rate) together with the growth rates of the wage and profit rates, φ_{wt} and φ_{rt}, respectively. One interesting aspect is that the profit rate varies substantially more than the wage rate (i.e., the profit rate is markedly procyclical while the wage rate is only mildly procyclical). This is what economists, essentially, observe when they calculate Solow's residual.

Figure 3, on the other hand, graphs the profit rate in levels (r_t). It shows a downward trend after achieving a maximum of almost 25 percent in 1965/66. Summing up: if economists want to continue analyzing and exploiting the information contained in the Solow residual, perhaps it would be more illuminating to study the evolution and determinants of wage and profit rates (without relating them to an aggregate production function). To this purpose, perhaps a framework that directly links growth and distribution, such as those of Kaldor [1956] and Pasinetti [1962; 1974], who also used the income accounting identity as their starting point, would be more enlightening.[19] This would also help us understand better the productivity miracle of the 1990s, which most likely is associated with a recovery in profit rates.

CONCLUSIONS

This paper has reassessed the conundrum surrounding the measurement of the impact of public infrastructure spending in the light of the theoretical literature on aggregation (i.e., if most likely the aggregate production function probably cannot be derived theoretically, how can one interpret the empirical estimates?), and has of-

FIGURE 2
Growth Rates of TFP, Wage Rate, and Profit Rate

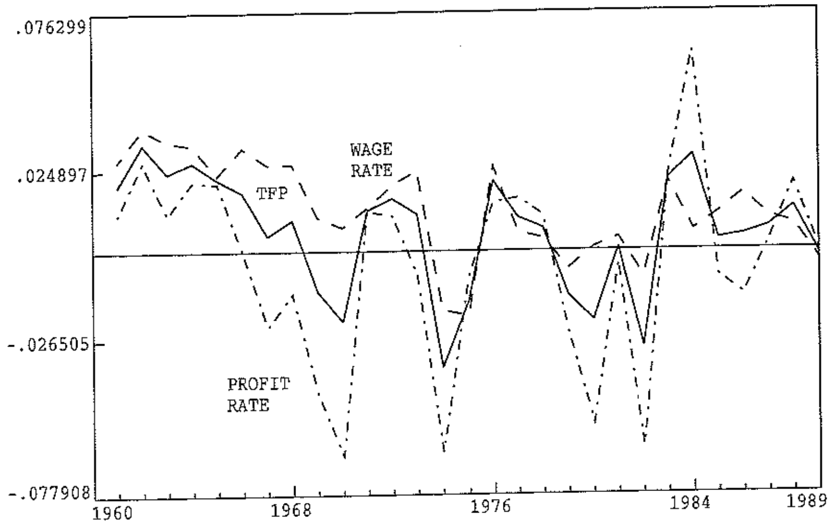
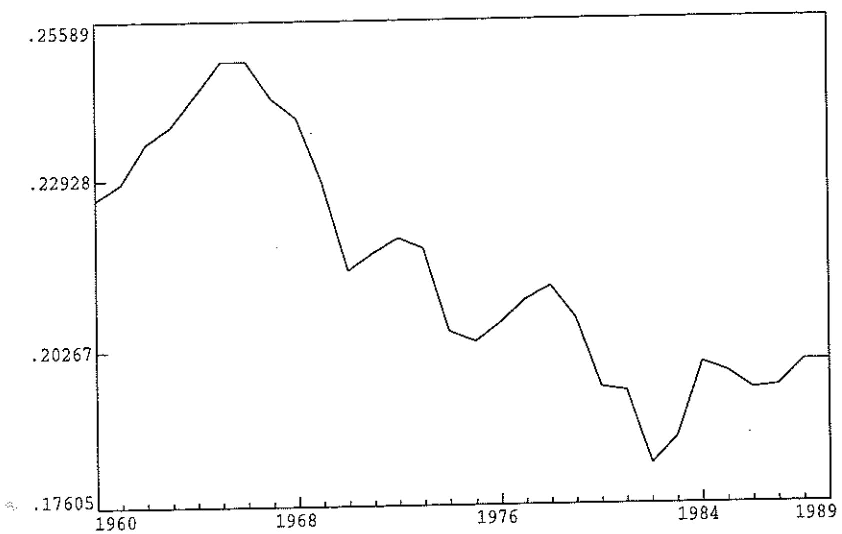


FIGURE 3
Profit Rate of the U.S. Private Sector



fered a parsimonious explanation for the variety of results obtained. The paper has shown that the coefficient of the measurement of infrastructure spending in an aggregate production function cannot be taken to provide unambiguously a measure of the productivity of infrastructure. The coefficient estimated in a Cobb-Douglas "production function" regression with labor, capital and infrastructure expenditures, and assuming a correct specification, is a weighted average of the wage rate, profit rate, and public expenditures, where the weights are the factor shares in income. There is no reason why this should be interpreted as the productivity of public capital, given that this result has been obtained by rewriting the income accounting identity that relates value added to the wage bill plus profits as a form that *looks like* an aggregate production function. The conclusion is, therefore, that one cannot use an "aggregate production function" directly to measure the productivity of infrastructure, and thus this type of exercises bear no policy implications. To measure the productivity of public infrastructure one needs to use data at the project level.[20] This is much more difficult to do due to the (general) lack of available data. But it cannot be an excuse for continuing to use a framework that is intrinsically useless.

NOTES

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1. The theoretical question under study was to determine the causes underlying the decline in productivity growth since the 1970s. Aschauer advanced as a possible reason the sharp deceleration of public investment.
2. Fisher's complete work on aggregation appeared a few years ago in an edited volume [Fisher, 1993].
3. For example, Fisher [1965] analyzed the case where each firm produces a single output with a single labor type, but two capital goods, (i.e., $Y(v) = f^v(K_1,K_2,L)$). Aggregation across firms over one type of capital (e.g., K_1) requires very stringent conditions related to the so-called Leontief theorem. And Fisher [1983] showed that the simultaneous existence of a full and a partial capital aggregate (e.g., plant) implies the existence of a complementary partial capital aggregate (e.g., equipment), and that the two partial capital aggregates are perfect substitutes. Blackorby and Schworm [1984] is an extension of Fisher [1983]. They present an alternative formulation of the problem in which one can have both a full and a partial capital aggregate without the restrictive substitution implications derived by Fisher. They show that there need be only one partial aggregate and that if there are two partial aggregates, they need not be perfect substitutes. The conditions nevertheless remain very restrictive.
4. The general argument follows Simon [1979], Shaikh [1974; 1980], and McCombie [1987]. Inexplicably, it has been largely ignored.
5. The measure of output used here, Gross Value Added at factor cost, excludes indirect business taxes. It corresponds to the correct measure of output used to estimate a production function.
6. Certainly spuriousness might induce a higher fit [Aaron, 1990]. However, Felipe and Holz [2001], using a Monte Carlo simulation, showed that spuriousness plays a minor role. Most of the high R^2 is accounted for by the accounting identity.
7. The structure of this fallacy is as follows: "If p, then q. Therefore p."
8. See, for example, Uimonen [1993] and Otto and Voss [1996].
9. Berndt and Hansson [1992] used a translog cost function. The form they used can be seen as an approximation to the expression $Q_t = A_0 w_t^a r_t^{1-a} L_t^a K_t^{1-a}$ (discussed in above) for L/Q.

10. Certainly nothing in neoclassical economics says that "technical progress" has to be approximated through a linear time trend.
11. The correlation between ($0.567\ln w$) + ($0.424^*\ln r$) and $A(t)$ is 0.55 (0.576 and 0.424 are the average labor and capital shares, respectively).
12. Aaron [1990, 55], to cast doubt on Aschauer's [1989a] results, introduced the exchange rate dollar/yen in the regression explaining output, for no good theoretical reason, and simply on the basis that the series had a pattern similar to that of public sector investment. The theoretical reason behind his findings (and correct analysis!) is the one provided here.
13. Aschauer [1989] also included a time trend, which in our case is insignificant.
14. Omission of a relevant variable leads to biased estimates; while inclusion of an irrelevant variable affects the efficiency of the least-squares estimator, but does not bias the slope parameter estimates of any of the variables. The bias of the parameter of PSI with respect to its theoretical value of zero (in the identity), as in the case of the first two regressions, is the result of the combination of the covariances of the omitted term (i.e., $a_t\,\phi_{wt} + (1-a_t)\phi_{rt}$) with the included terms (i.e., $g1_t$, or $g2_t$, and cu_t).
15. On top of this, there is the problem raised during the Cambridge Capital controversies [Harcourt, 1972], different from the problems discussed here. Because we are working with an aggregate production function, the profit rate is no longer determined by a purely physically defined marginal product of capital. Under these circumstances, the marginal product of capital is the *change in the value of output with respect to the change in the value of capital*. This implies that distribution depends not only on independent physical magnitudes but also on prices which, in turn, depend on distribution.
16. See Da Silva Costa et al. [1987]; Aschauer [1990a]; Munnell [1990b] for cross section regressions for the individual states of the United States.
17. This derivation follows the arguments in Cramer [1969] and McCombie [1987].
18. Hulten [2000, 11-12] argues that there is a "close link" between the income accounting identity and the production function, and that one must assume constant returns to scale to estimate the profit rate residually, as in, for example, Jorgenson and Griliches [1967], or here. This is not true. The income identity equation (1) stands by itself without the production function, which, as argued in the text, most likely cannot even be derived theoretically.
19. It is very interesting, perhaps even ironic, that the decline in the profit rate is a basic tenet of Marxian economics. On the decline of the profit rate (and the importance of this variable) see, among others, Bowles et al. [1986], Clark [1984], Nordhaus [1974], Shaikh [1993], Wolff [1986], and Zarnowitz [1999]. The fact, for example, that in real business cycle models technological shocks are measured through the Solow residual does not undermine the general criticism put forward here. True, variations in profit rates are part of the so-called Solow residual; but in real business cycle models the shocks are part of an "imaginary" aggregate production function (which, according to Prescott [1998], is a well-tested theory). I do not think real business cycle theorists are aware of the direct connection between the residual and profit rates and how the latter are the link between the supply shocks and the effects on the real economy.
20. Kocherlakota and Yi [1996] proposed a test in the context of time series analysis to distinguish between endogenous and exogenous growth models. The test was based on whether temporary innovations in government policies can affect the per capita level of GNP. Some of the variables considered were different types of infrastructure expenditures. Their test did not use an aggregate production function explicitly, although it is implicit, and thus their test is not immune to the argument presented in this paper. Kunihisa and Kaiyama [1998] despite referring to "highway construction" still used aggregate data (i.e., data in value terms), and thus their analysis suffers from the same problem.

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