Output of the Constrained Revenue Maximizing Firm

C. A. KNOX LOVELL

William J. Baumol (1967) has developed an oligopoly model in which the firm is assumed to seek maximum total revenue, subject to a minimum profit constraint. The model has attracted attention as a potentially fruitful alternative to the traditional unconstrained profit maximization monopoly model, although to date the testable implications of the two models are disappointingly similar.

One well-known implication of Baumol's model asserts that the firm produces a greater rate of output, and reaps a smaller volume of profits, if it is a constrained revenue maximizer (CRM) than if it is a profit maximizer (IMF). This implication has gained wide acceptance, provided the profit constraint takes the form of a minimum dollar value, say $2 million per annum. However, Baumol notes (p. 66) that in practice, few businessmen have such explicit and clear-cut profit goals. And it may be expected that in many cases the firm's minimum profit goal will be approximated better by the requirement that it earn at least, say, 8 per cent on its investment or 20 per cent on its costs or 15 per cent on its dollar sales.

In light of this expectation it seems worthwhile to ascertain whether or not the positive relationship between rates of output under the two alternative behavioral assumptions survives these three re-specifications of the profit constraint.

[diagram of constrained revenue maximization]

1 The profit constraint, whatever its form, is an internally imposed minimum thought necessary to finance future growth and to satisfy stockholders, rather than an externally imposed maximum set by a regulatory agency. The firm is not subject to an external constraint.

[Figure 1]
and nonnegativity constraints on all variables, and the Kuhn-Tucker conditions are necessary and sufficient for a maximum.

Consider first Baumol’s problem of maximizing revenue subject to a profit constraint expressed in dollar terms. Forming the Lagrangian:

\[ G(X, S, \lambda_2) = R(X, S) + \lambda_2 [R(X, S) - C(X)] - S - \Pi_1(X, S) \]  

(1a)

the Kuhn-Tucker conditions are

\[ \frac{\partial G}{\partial X} - \lambda_2 \frac{\partial R}{\partial X} - \lambda_1 C(X) \geq 0, \quad X \geq 0 \]  

(1b)

\[ \frac{\partial G}{\partial S} + \lambda_2 \frac{\partial R}{\partial S} - \lambda_2 \Pi_2 \geq 0, \quad S \geq 0 \]  

(1c)

\[ R(X, S) - C(X) - S - \Pi_1(X, S) = 0; \quad \lambda_2 > 0. \]  

(1d)

Using \( \lambda_2 > 0 \) from (1d) in (1c) gives \( \partial R/\partial S \leq 1 \).

Using \( \lambda_2 > 0 \) in (1b) gives \( \partial R/\partial X = \lambda_2 \Pi_2/\lambda_2 \Pi_2 \) for \( X > 0 \) in the IMO model. Equilibrium rates of sales effort and output are determined by the conditions \( \partial R/\partial S = 1 \) for all \( S > 0 \) and \( \partial R/\partial X = C(X) \) for \( X > 0 \). Hence for this specification of the profit constraint, sales effort and the rate of output are greater under CRM than under IIM. A comparison of the two rates of output appears in Figure 1.

However, if the profit constraint is specified, the relationship between CRM and IIM rates of sales effort and output may be reversed. For example, if the profit constraint is defined as a minimum return on total revenue the Lagrangian becomes

\[ G(X, S, \lambda_2) = R(X, S) + \lambda_2 [R(X, S) - C(X)] - S - \Pi_1(X, S) \]  

(2a)

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\[ R(X, S) - C(X) - S - \Pi_1(X, S) = 0; \quad \lambda_2 > 0. \]  

(1d)

Using \( \lambda_2 > 0 \) from (1d) in (1c) gives \( \partial R/\partial S \leq 1 \) according to \( \Pi_2 \leq 1/\lambda_2 \) for all \( S > 0 \). If the target rate of return on total revenue (I) is set sufficiently high, sales effort may be so restricted as to force marginal revenues above unity. Using this result in (1b) gives \( \partial R/\partial X = \lambda_2 \Pi_2/\lambda_2 \Pi_2 \) for \( X > 0 \) again the target rate of return on total revenue is set sufficiently high, the resulting restriction on sales effort leads to a diminution of demand and a reduction of output below the IMO rate. For Baumol’s result to obtain under this specification of the profit constraint it is necessary that the target rate of return on total revenue be low enough to satisfy \( \Pi_2 < 1/\lambda_2 \). Both possibilities are illustrated in Figure 2, where \( \Pi_2 \) and \( \Pi_2 \) have been selected in such a way as to yield \( \lambda_{CRM} < \lambda_{IMO} < \lambda_{CRM} \).

If the profit constraint is defined as a minimum return on total cost, the Lagrangian becomes

\[ G(X, S, \lambda_2) = R(X, S) + \lambda_2 [R(X, S) - C(X)] - S - \Pi_1(X, S) \]  

(2a)

2In equilibrium \( \lambda_2 \) represents the marginal revenue of relaxing the functional constraint \( \Pi_1(X, S) \), provided there are no effective direct constraints on the adjustment of \( X \) and \( S \). If there is an effective direct constraint on \( X \) and \( S \), this limits their adjustment and tends to reduce the marginal revenue of relaxing the functional constraint. The same analysis holds for the three other forms of the profit constraint.

with Kuhn-Tucker conditions

\[ \frac{\partial G}{\partial X} - \lambda_2 \frac{\partial R}{\partial X} - \lambda_1 C(X) \geq 0, \quad X \leq 0 \]  

(2b)

\[ \frac{\partial G}{\partial S} + \lambda_2 \frac{\partial R}{\partial S} - \lambda_2 \Pi_2 \geq 0, \quad S \leq 0 \]  

(2c)

\[ R(X, S) - C(X) - S - \Pi_1(X, S) = 0; \quad \lambda_2 > 0. \]  

(2d)

Using \( \lambda_2 > 0 \) from (2d) in (2c) gives \( \partial R/\partial S \leq 1 \) according to \( \Pi_2 \leq 1/\lambda_2 \) for all \( S > 0 \). If the target rate of return on total revenue (I) is set sufficiently high, the resulting restriction on sales effort leads to a diminution of demand and a reduction of output below the IMO rate. For Baumol’s result to obtain under this specification of the profit constraint it is necessary that the target rate of return on total revenue be low enough to satisfy \( \Pi_2 < 1/\lambda_2 \). Both possibilities are illustrated in Figure 2, where \( \Pi_2 \) and \( \Pi_2 \) have been selected in such a way as to yield \( \lambda_{CRM} < \lambda_{IMO} < \lambda_{CRM} \).

If the profit constraint is defined as a minimum return on total cost, the Lagrangian becomes

\[ G(X, S, \lambda_2) = R(X, S) + \lambda_2 [R(X, S) - C(X)] - S - \Pi_1(X, S) \]  

(2a)

with Kuhn-Tucker conditions

\[ \frac{\partial G}{\partial X} - \lambda_2 \frac{\partial R}{\partial X} - \lambda_1 C(X) \geq 0, \quad X \leq 0 \]  

(2b)

\[ \frac{\partial G}{\partial S} + \lambda_2 \frac{\partial R}{\partial S} - \lambda_2 \Pi_2 \geq 0, \quad S \leq 0 \]  

(2c)

\[ R(X, S) - C(X) - S - \Pi_1(X, S) = 0; \quad \lambda_2 > 0. \]  

(2d)

Using \( \lambda_2 > 0 \) from (2d) in (2c) gives \( \partial R/\partial S \leq 1 \) according to \( \Pi_2 \leq 1/\lambda_2 \) for all \( S > 0 \). If the target rate of return on total revenue (I) is set sufficiently high, the resulting restriction on sales effort leads to a diminution of demand and a reduction of output. This case is illustrated in Figure 3, where
Baumol's result holds only if the target rate of return is low enough to keep $\delta R/\delta S_1 < 1$ for all $S_1 > 0$.

Finally, if the profit constraint is defined as a minimum return on investment, the Lagrangian becomes

$$G(X, S_1) = R(X, S) + \lambda_4 (R(X, S) - C(X) - S - \Pi_4(X, S)),$$

(4a)

with Kuhn-Tucker conditions

$$\frac{\delta R}{\delta X} + \lambda_4 \frac{\delta R}{\delta X_1} - \lambda_4 C(X) - \lambda_4 \Pi_4 \delta_1 C(X) \leq 0; \quad \lambda_4 > 0.
$$

(4b)

$$\frac{\delta R}{\delta S_1} + \lambda_4 \frac{\delta R}{\delta X_1} - \lambda_4 - \lambda_4 \Pi_4 \delta_2 \geq 0; \quad S_1 \geq 0
$$

(4c)

Using $\lambda_4 > 0$ from (4d) in (4c) gives $\delta R/\delta S_1 \geq 1$ according as $\Pi_4 \geq 1/\Pi_4 \lambda_4$, for all $S_1 > 0$. Using this result in (4b) gives $\delta R/\delta X = (\lambda_4(1 + \theta_1)) C(X)$ according as $\Pi_4 \geq 1/\Pi_4 \lambda_4$. 
1/\beta_1 \lambda_u$ for $X > 0$. In this case the target rate of return on investment (II) may be high enough to force a restriction on sales effort sufficient to cause marginal revenues to exceed unity. However, in contrast to the second and third cases this is neither necessary nor sufficient to cause a reduction in output beneath the IIM rate, which requires $II > 1/\beta_1 \lambda_u$. The possible outcomes of this specification of the profit constraint are illustrated in Figure 4.

II. An Interpretation of the Results

One of the principal implications of Baumol’s model of unregulated constrained revenue maximization asserts that the firm produces a greater rate of output under CRM than under ILM. It has been demonstrated that this proposition lacks general validity, and that the relationship between rates of output under the two hypotheses depends upon the specification of the profit constraint. Only when the profit constraint takes the form of a minimum dollar value does $X_{CRM} > X_{ILM}$. For each of three other specifications of the profit constraint, $X_{CRM} \leq X_{ILM}$ according as the target rate of return exceeds, equals or falls short of some critical value. This result is of some importance since, as Baumol acknowledges, firms are most likely to adopt one of these three types of constraint in practice.

Finally, it should be noted that this modification of Baumol’s analysis has little effect on any further implications of the CRM hypothesis. It does alter the tenor of his implications concerning oligopoly and ideal output. But the response of a CRM firm to changes in overhead costs or lump sum taxes remains unchanged. And the theorems of Bailey and Malone (1970) and of Baumol and Kleinerick (1970) concerning the behavior of the CRM firm under rate of return regulation (the reverse J- effect) remain valid.

References

