External Financing of Hospital Efficiency Projects

DOUGLAS GREGORY*

Several public and private programs designed to stimulate hospital investments in projects which generate cost savings (e.g., mergers, shared services) are currently in the planning stages or extant. The purpose of this paper is to provide policy makers with a tool for planning and to analyze such programs by abstracting and modeling the essential characteristics of the hospital's decision to approach the program. The model may be used to provide planners with predictive information concerning the effect of alternative (program) policies on the hospital's decision to participate in the program.

Hospital costs have been increasing at a rapid rate over the last two decades. Private hospital associations, private and public health care reimbursers, private foundations, and government agencies all have demonstrated substantial interest in curtailing the rate of increase in health care prices. This interest is manifested in the impressive array of cost containment programs sponsored by these groups within the past five or ten years. Several of these programs, designed to stimulate hospital investments in projects which generate cost savings (e.g., mergers, shared services), are currently in the planning stages or already extant. The increasing involvement of government in health service decision-making, combined with the need to control costs, should focus continuing interest on such programs. This paper is specifically concerned with programs which provide financing, or permission to finance, hospital investment projects whose primary aim is to reduce costs (subsequently denoted efficiency projects). The study seeks to provide policy makers with a tool for analyzing the structure of these programs by abstracting and modeling the essential economic characteristics of the hospital's decision to participate in such a program.

Before specifying the hospital decision maker's (d.m.'s) decision problem when applying for external financing of an efficiency project, the impact of such a project on hospital objectives must be understood. In Part I, a hospital objective function is defined following Newhouse (1970) and the impact assessed. In Part II, the hospital's decision to participate in one of these programs is formulated as a competitive bidding problem. In Part III, an important special case of this model is studied, and finally the application of the model is briefly discussed in Part IV.

---

*The author is Assistant Professor of Economics and Management at Oakland University.

Acknowledgement: The author wishes to acknowledge helpful comments by Professors Nancy Schwartz and David Baron.

1See Gregory (1974) Chapter 1, and pp. 44-46, for a partial list of public and private sources. One such program, the Merger Incentive Program, is analyzed in Chapter 6, Gregory (1974). The model developed here could be applied to government programs providing matching funds and certification-of-need programs designed to regulate hospital investments. The hospital applies for permission to finance the investment in the latter.
I. The Impact of an Efficiency Project on Hospital Objectives

The purpose of this section is to develop a theoretical framework within which the hospital’s decision to finance and implement an efficiency project may be viewed. The hospital’s optimal program is postulated, and the impact of the project on this program is studied.

Following Newhouse, assume that the hospital’s objective function is formed over the quantity and quality of care delivered by the institution. Quantitative aspects of hospital product are summarized by a quantity vector with components denoting, among other things, the number of cases treated within each of various case classes. Quality of care analogously is assumed to be characterized by a quality vector with components reflecting the services provided by the hospital, patient-personnel staffing ratios, medical research expenditures, etc. The hospital decision-maker’s preferences over quantity-quality vectors are represented by a strictly quasi-concave utility function \( U(q_n, q_e) \), \( q_n = n \)-dimensional quantity vector, \( q_e = m \)-dimensional quality vector. The hospital’s optimal program is to maximize its utility over quantity-quality vectors subject to the constraint that revenues cover costs:

\[
\max_{q \in H_0} U(q) = U(q^*)
\]

where

\[
q = (q_n, q_e) \in R^{n+m}
\]

\[
H_0 = \{ q \in R^{n+m} : c(q) \leq R(q) \}
\]

\(
C(q) = \text{total cost of treating the patient quantity vector } q_n \text{ with quality } q_e.
\)

\[
R(q) = \text{total revenue generated by the patient vector } q_n \text{ when hospital quality is } q_e.
\]

Next, assume that the hospital implements an efficiency project which generates net total cost savings \( y \). Total cost after project implementation is \( C(q) - y \), and \( y \) is not constrained to be non-negative since negative net cost savings will be a possibility when uncertainty is introduced into the model.³

The feasible set of output opportunities after implementing the project is given by

\[
H_y = \{ q \in R^{n+m} : c(q) - y \leq R(q) \},
\]

and the hospital’s new program is

\[
\max_{q \in H_y} U(q) = u(y)
\]

where \( u(y) \) is the hospital’s indirect utility function defined over the project’s net total cost savings.⁴

³An exogenous shift in total cost (i.e.; \( y \) independent of \( q \)) due to a saving in a fixed cost is a likely outcome in the case of projects involving shared services, since such projects often involve a reduction in duplicate facilities and spreading of remaining fixed inputs over greater utilization rates. The significance of such projects is attested by the fact that 66.6% of the 5,727 hospitals contacted in a recent survey shared from 1 to 73 services at an average of 6.5 services per hospital, and in addition, there was a great amount of interest reported in the survey for initiating further shared service programs. See A. Astolfi and L. Matti, “Survey Profiles for Shared Services,” Hospitals, Journal of the American Hospital Association, September 16, 1972, p. 64.

⁴For a discussion of the technical restrictions of \( H \) necessary to ensure that \( u(y) \) is a well-defined function, see Gregory (1974) pp. 22-23.
The hospital's optimal maximizing its utility over quantity vectors subject to the constraint that its costs:

$$\max_{q \in H_y} U(q) = U(q^*)$$

where $q \in R^{n+m}$.

1. cost of treating the patient with vector $q_n$.

1. revenue generated by the patient when hospital quality is $q_n$.

ne that the hospital implements an project which generates net total cost. Total cost after project implementation is $y$, and $y$ is not constrained to be since negative net cost savings are possible when uncertainty is introduced.

set of output opportunities after the project is given by

$$y \in R^{n+m}: c(q) - y \leq R(q)$$

ital's new program is

$$\max_{q \in H_y} U(q) = u(y)$$

the hospital d.m.'s indirect utility indexed over the project's net total

ous shift in total cost (i.e., $y$ independent of being in a fixed cost is a likely case of projects involving shared care projects, often involves a reduction in fixed costs and spreading of remaining fixed costs utilization rates. The significance of this is attested by the fact that 66.6% of hospitals contacted in a recent survey shared services at an average of 6.5 services per hospital, there was a great amount of data in the survey for initiating further programs. See A. Astolfi and L. Martin, "Hospitals, American Hospital Association, September 4.

2. Assumption of technical restrictions of $H$ ensure that $u(y)$ is a well-defined function (1974) pp. 22-23.

Figure 1 Normal and inferior changes in $q$ after project implementation.

It is easy to show that an efficiency project with positive net cost savings allows the hospital to reach a higher indifference level by expanding its set of output opportunities, i.e., $u(y) > 0$. The new feasible region $H_y$ for $y > 0$ contains $H_y$, so the project's savings allow the hospital to choose quantity-quality vectors which were not previously feasible. Depending on the shape of the d.m.'s utility function, the cost reduction may be employed to increase quality, quality, or both as illustrated in Figure 1 for the special case when $q_n, q_0 \in R^1$. If hospital preferences favor quality over the relevant range, then all of the cost savings might find their way into quality enhancing equipment or the hiring of more competent personnel. This appears to have happened in the recent merger of eight Phoenix hospitals. Figure 1 depicts the normal case as well as the case where one of the attributes is "better" over the relevant range of values, i.e., either $q_n$ or $q_0$ decreases after project implementation. Such inferior effects might be exhibited in the presence of substantial indivisibilities of factors purchased with project savings.

Uncertainty is introduced into the analysis through the hospital's application for funding of the prospective project. In particular, there are initially two possible actions—to submit or not to submit application for funding of the project. Associated with the first action are two possible outcomes, the application is accepted or rejected. Label the net cost saving implications of these two outcomes $y_1$ and $y_2$ respectively. Associated with $y_1$ and $y_2$ are two points $q^1$ and $q^2$ such that $\max_{H_{q^i}} U(q^i) = U(q^i)$, $i = 1, 2$. Now the probability distribution over $y_1$ and $y_2$, which gives the acceptance and rejection probabilities, applies to the points $q^1$ and $q^2$. If an application for funding is submitted, then the feasible region shifts to $H_{y_1}$ or $H_{y_2}$, and the associated output points are $q^1$ and $q^2$, respectively. Thus, a probability distribution over cost savings induces a distribution over quantity-quality vectors of hospital care. It is assumed accordingly that the d.m. can order probability distributions over a cost vector $y(q)$ of hospital care. This is analogous to Arrow's and Pratt's three-stage process. Recognizing that ultimate project savings are related to the extent of planning and search for cost saving opportunities, MIP planners require extensive pre-merger information from program applicants. The program's funding decision then is made on the basis of that information. Finally, post-merger information is required to determine whether forecasted cost savings are actually being generated. See Gregory and Rapoport for an analysis of this program based on the results of the model developed here.
assumption that an entrepreneur’s risk preferences are conditioned by the level of his assets. The difference is that hospital assets may result from exogenous contributions, endowment, grants, and subsidies, whereas a firm’s assets are generated internally from profits. The d.m.’s preferences over \( q \) are represented as follows:

\[
U(q_0, q_1, w) = U(q, w)
\]

\(= \text{d.m.'s von Neumann-Morgenstern utility function over } q, \text{ parameterized by } w.\)

\(w = \text{hospital’s assets.}\)

Expected utility associated with application for external financing of the project is derived from the indirect utility function as follows:

\[
E_U(y, w) = P(r = y_1) u(y_1, w) + P(r = y_2) u(y_2, w)
\]

(1)

where \( y_1 \) is the net savings when the application is accepted; \( y_2 \) is net savings (or loss) when rejected; \( u(y_1, w) = \max_{u, U} U(q, w) \) and \( u(y_2, w) \) is defined analogously.

Conditions under which monotonicity, concavity, and decreasing risk aversion are inherited by \( u(y, w) \) from \( U(q, w) \) are given in Gregory, Chapter 2. We will make use of these properties of the indirect utility function in subsequent analysis.

### II. The Program Participation Decision

In this section, the hospital’s decision to participate in the program is represented more explicitly using the framework developed in the last section. The implications of this formulation of the problem are then explored.

Recall that the d.m. must apply to some external agency for funding of a hospital efficiency project, and the hospital’s application may be rejected. The probability of rejection depends in part upon the project’s potential cost savings. The d.m. is accordingly motivated to plan the prospective project at some cost to the hospital in order to improve the project’s acceptance probability. The planning investment could be measured by the cost of collecting information with respect to (1) the search for efficiency investments, and (2) the organizational and material aspects of implementing the project.

The following notation and assumptions are used in subsequent analysis:

- \( I = \text{planning or information costs including costs of meeting the informational requirements of the funding agency.} \)
- \( x(I) = \text{gross total cost savings of the project. Furthermore, } x'(I) > 0, x''(I) < 0. \)
- \( \hat{x} = \text{proposed gross total cost savings of the project. } \hat{x} \text{ is the amount of cost saving which the d.m. informs the external source that the project will generate.} \)
- \( w = \text{hospital assets} \)
- \( L = \text{size of the loan for which application is made. The loan is repaid at the end of the period.} \)
- \( \alpha(x - x(I)) = \text{penalty cost/reimbursement, if } \alpha \text{ maximum belief of cost savings.} \)
- \( \alpha > 0 \text{, for mis-stating the cost savings. It is a function of the difference and actual cost } \)

\[ \begin{align*}
\tilde{y} &= \text{net total cost savings (random)} \\
\tilde{y}_1 &= x(I) - L - \alpha(\hat{x} - x(I)) \\
\tilde{y}_2 &= -L, \text{ if } p_1 \end{align*} \]

If the application for finance is accepted, the hospital wins, and the project is implemented, savings of \( x(I) \) are generated at a rate of \( (\alpha(x - x(I))). \) If the application is rejected, the hospital loses its investment. If cash flows are valued at a discount rate of \( \pi \)
EXTERNAL FINANCING OF HOSPITAL EFFICIENCY PROJECTS

The d.m. is accordingly motivated to implement a project at some cost in order to improve the project’s potential and the agency’s prospects. The planning investment is measured by the cost of collecting information with respect to (1) the search for investments, and (2) the organizational and informational aspects of implementing the decision to invest. The planning investment, $I$, is equal to the sum of the costs of meeting the informational requirements of the funding agency. The proposed gross total cost savings of the project, $\bar{x}$, is the amount of cost saving which the d.m. forecasts will be achieved. This is the amount of cost saving which the d.m. forecasts will be achieved. Therefore, $x(I) > 0$, $x(I) < 0$.

$\gamma = \text{proposed gross total cost savings of the project}$. $\bar{x}$ is the amount of cost saving which the d.m. forecasts will be achieved. This is the amount of cost saving which the d.m. forecasts will be achieved. Therefore, $x(I) > 0$, $x(I) < 0$.

$\psi = \text{hospital assets}$

$\gamma = \text{size of the loan for which application is made}$. The loan is repaid at the end of the period. It is helpful to assume that the loan size requested is simply equal to the fixed cost of implementing the project. Thus, $L$ also denotes the implementation cost of the project.

$\delta = \text{payback fraction applied to the loan}$. That is, a loan of size $L$ entails a commitment to repay $kL$ at the end of the period. It is set by the funding agency and is parametric to the hospital (e.g., $k = 0$ for a grant and $k = 1 + r$ for a loan with full market payback, $r = \text{private interest rate}$).

$F(\bar{x}; L) = \text{probability that a project with proposed cost savings of at least } \bar{x} \text{ is accepted when a loan of } L \text{ is requested.}$ Assume that $\frac{\partial F}{\partial L} < 0$, $F(\bar{x}; L) \rightarrow 1$ as $\bar{x} \rightarrow c$, where $c$ represents the maximum believable statement of cost savings. Clearly, there is some such number since the external funder would not believe that arbitrarily large levels of cost savings could be attained. For $\bar{x} > c$, $F(\bar{x}; L)$ would not necessarily be one since the financial situation would not believe such a proposal.

$\alpha(\bar{x} - x(I)) = \text{penalty, cost/earnings function imposed by government agency for mis-stating the project’s cost savings.}$ It is a linear function of the difference between stated and actual cost savings.

$\gamma = \text{net total cost savings of project}$. The application for financing is approved, the project is implemented, and gross cost savings of $x(I)$ are generated at a cost of $I + kL + \alpha(\bar{x} - x(I))$. If the application for financing is rejected, the hospital loses its planning investment. All cash flows are valued at the beginning of the period. The expected utility of the gamble ($\bar{U}$) associated with applying for external funding is:

$$
\bar{U}(\bar{y}, w) = F(\bar{x}, I) u(y_1, w) + (1 - F(\bar{x}; L)) u(y_2, w).
$$

The values of the decision variables $I$ and $x$ which maximize the above expected utility are denoted $I^* = x^0$. If

$$
\max_{x(I), \gamma} \bar{U}(\bar{y}, w) > u(0, w)
$$

then the optimal planning investment, $I^*$, is made and a proposal for financing the efficiency project (with specified savings $\bar{x}$) is submitted to the external agency. If $\max_{x(I), \gamma} \bar{U}(\bar{y}, w) < u(0, w)$, then the optimal planning investment is zero and no proposal is submitted. Hence, optimal planning is

$$
I^* = \begin{cases} 
I^*, & \text{if }\max_{x(I), \gamma} \bar{U}(\bar{y}, w) > u(0, w) \\
0, & \text{otherwise}
\end{cases}
$$

The necessary and sufficient condition for proposal submission involves the usual certainty equivalent for inequality (2). The hospital d.m. chooses to apply for financing of a cost reducing investment if the project's risk adjusted expected net benefits (savings) with program financing are positive, i.e.,

$$
F(\bar{x}; L) [x(I) - kL - \alpha(\bar{x} - x(I))] - I - \pi > 0,
$$

where $\pi = \text{risk premium for the gamble}$. In certain instances, third party payers (e.g., Blue Cross, Medicaid) may reduce their reimbursement for patients to absorb some portion of the gross cost savings generated by the project, $(1 - \beta) x(I)$. If this occurs, the expected net benefits of the project would be

$$
\bar{U}(\bar{y}, w) = u(\delta \bar{y}; I - kL - \alpha(\bar{x} - x(I)) + \beta x(I)), w).
$$

The incentive for hospitals to undertake such projects accordingly would be diminished, and economic projects might not be financed unless the payback fraction were sufficiently low, perhaps...
even negative, to compensate the d.m. for possible losses due to the revised reimbursement. This may partially explain the incidence of grant programs which subsidize costs of certain efficiency projects.

Another reason for subsidizing implementation costs with a low payback fraction may be to compensate program participants for certain intangible costs inherent in the type of investment being funded. For example, hospital mergers and major shared service ventures may entail costs due to loss of local autonomy. In addition, organizational inertia or x-efficiency may be conceptualized as a cost to planning and implementing efficiency projects. Such costs could be incorporated formally into the model by subtracting some positive constant from the project's net benefits. We neglect intangible costs and revised reimbursements (assuming $\beta = 1$) with the understanding that they could be incorporated easily into the analysis.

Next, an obvious result is stated with important implications to the distribution of the program's funds. It specifies the impact of risk aversion on the d.m.'s decision to approach the program.

**Lemma 1:** Consider two utility functions $u^1(y, w)$ and $u^2(y, w)$ such that $r_{u^i}(w) \geq r_{u^j}(w_i)$ for every $w$ and $i$, where

$$r_{u^i}(w) = \frac{\partial u^i(y, w)}{\partial y_i}, i = 1, 2.$$ Then $E u^1(\tilde{y}, w) - u^1(0, w) \leq E u^2(\tilde{y}, w) - u^2(0, w).$ (Let $u^i_0(y, w) = \frac{\partial u^i(y, w)}{\partial y^i}$.)

**Proof:** Scale $u^2(0, w)$ so that $u^2_0(0, w) = u^1_0(0, w)$, since $u^1$ and $u^2$ are von Neumann-Morgenstern utility functions, the underlying preference orderings are invariant under this change in scale. The following implication is due to Pratt, equation 21, p. 129: $r_{u^i}(w) > r_{u^j}(w_i)$ for every $w$ implies

$$\frac{u^1(s, w) - u^1(t, w)}{u^1_0(t, w)} < \frac{u^2(s, w) - u^2(t, w)}{u^2_0(t, w)}, s > t.$$ Let $\pi_i$ be defined by $E u^{i}(\tilde{y}, w) = u^i(E \tilde{y} - \pi_i, w)$,

$i = 1, 2$, assumes $E \tilde{y} > \pi_1$, and denote $s = E \tilde{y} - \pi_1$ and $t = 0$. It follows from the inequality that

$$u^1(E \tilde{y} - \pi_1, w) - u^1(0, w)$$

$$< u^2(E \tilde{y} - \pi_1, w) - u^2(0, w),$$

since $u^1_0(0, w) = u^2_0(0, w)$ by choice of scale. Pratt (p. 128) shows in Theorem 1 that $r_{u^i}(w) > r_{u^j}(w)$ implies $\pi_1 > \pi_2$, so

$$u^1(E \tilde{y} - \pi_1, w) - u^1(0, w)$$

$$< u^2(E \tilde{y} - \pi_2, w) - u^2(0, w).$$

Combining the two preceding inequalities

$$E u^1(\tilde{y}, w) - u^1(0, w) < E u^2(\tilde{y}, w) - u^2(0, w),$$

for $E \tilde{y} > \pi_1$. In equation (22), Pratt shows that $r_{u^i}(w) > r_{u^j}(w)$ implies

$$\frac{u^1(s, w) - u^1(t, w)}{u^1_0(t, w)} > \frac{u^2(s, w) - u^2(t, w)}{u^2_0(t, w)}$$

for $s > t$. Letting $s = 0$ and $t = E \tilde{y} - \pi_1$, an analogous proof can be used to show that the same result holds for $E \tilde{y} < \pi_1$.

$r_{u^i}(w)$ is the familiar Arrow-Pratt risk index, and the lemma states that the less risk averse a d.m. is, the more likely he is to seek financing of an efficiency program, i.e., the more likely it is that inequality (2) holds. It follows easily from this that a d.m. whose risk preferences exhibit decreasing risk aversion (i.e., $r_{u^i}(w)$ decreases in $w$) will be more likely to seek program financing when hospital assets are great. The policy implication of this is that programs providing funds for efficiency projects may find that their applicants tend to be the wealthiest hospitals if decreasing risk aversion hypothesis regarding hospitals, and accordingly there may be distribution of program funds between wealthier hospitals. We will return to this point later.

Next, we turn to the d.m.'s problem with respect to the payment $I$ and forecasted costs $C$. We are interested in the case where financing is submitted (otherwise assume that

$$\max_{L, \tilde{x}} E u^*(\tilde{x}) > u(L),$$

where $w$ is suppressed for notation until later. We can reasonably assume that a finite interior (positive) maximization problem exists.

For $\alpha = 0$ and no positive specifications cost savings is incurred in $\tilde{x}$ since

$$E u^*(\tilde{x}) = F(\tilde{x}; L) \{u(y^1) - u(y^2)\},$$

for $\tilde{x} > \tilde{x}$ following from (4) and independent of $x$ for $\alpha = 0$. The notion of cost savings $x^*$ is acco-

ced to maximum believable savings. Imposition of the penalty d.m., to evaluate the trade-off by acceptance and penalty $\alpha$ $\upsilon$ for now that $\alpha > 0$ and $x^*$ in case in which this assumption is described at the end of this section.

Letting $y^i$ denote $y^i$ evaluate maximum $(y^i, x^*)$ and $y^2 = -T$, the definition for a maximum with respect

$$\frac{u^i(y^1)}{u^i(y^2)} - \frac{1 - F(x^*)}{F(x^*; \tilde{x})},$$

and for $\tilde{x}$ it is

$$\frac{u^i(y^1)}{u^i(y^2)} - \frac{F(x^*)}{F(x^*; \tilde{x})},$$

where $F(x^*; I) = 0$, $u(x^1)/u(x^2)$, $F(x^*; I) = $ density function associated.
The following implication is trivial, equation 21, p. 129: 

\[ r_w \rightarrow (w) > \]

every \( w \) implies

\[ t^1(t, w) \leq \frac{u^2(s, w) - u^2(t, w)}{u^1(t, w)} \], \( s > t \).

Defined by \( E u^1(\tilde{z}, w) = u^1(E \tilde{z} - \pi_1, w) \), assumes \( E \tilde{z} > \pi_1 \), and denote \( s = E \tilde{z} - \pi_1 \). It follows from the inequality

\[ w > u^1(0, w) > u^2(E \tilde{z} - \pi_1, w) - u^2(0, w), \]

\[ w = u^2(0, w) \text{ by choice of scale.} \]

128 shows in Theorem 1 that \( r(w) \) implies \( \pi_1 > \pi_1 \), so

\[ w > u^2(0, w) \]

\[ u^1(t, w) \leq \frac{u^2(s, w) - u^2(t, w)}{u^2(s, w) - u^2(0, w)}, \]

the two preceding inequalities

\[ u^1(0, w) < \frac{u^2(E \tilde{z} - \pi_1, w) - u^2(0, w)}{E \tilde{z} - \pi_1} \]

(22). Pratt shows that \( r_w \rightarrow (w) > \]

\[ -u^1(t, w) \geq \frac{u^2(s, w) - u^2(t, w)}{s}, \] \( w \)

Letting \( s = 0 \) and \( t = E \tilde{z} - \pi_1 \), an roof can be used to show that the holds for \( E \tilde{z} < \pi_1 \).

The familiar Arrow-Pratt risk index, ma states that the less risk averse a more likely he is to seek financing acy project, i.e., the more likely it utility (2) holds. It follows easily that a d.m. whose risk preferences easing risk aversion (i.e., \( r_w(w) \) de-)

will be more likely to seek pro-
ging when hospital assets are great.

Implication of thi this that programs nds for efficiency projects may find applicants tend to be the wealthier

hospitals if decreasing risk aversion is an accurate hypothesis regarding hospital risk references, and accordingly there may be a bias in the distribution of program funds favoring wealthy hospitals. We will return to this possibility later.

Next, we turn to the d.m.'s optimization problem with respect to the planning investment, \( I \), and forecasted cost savings, \( \tilde{x} \). Since we are interested in the case when a proposal for financing is submitted (otherwise \( I^\alpha = 0 \), assume that

\[ \max_{I, \tilde{x}} E u(\tilde{y}) > u(0), \] \( \tilde{y} \)

where \( w \) is suppressed for notational simplicity until later. We can reasonably post the existence of a finite interior (positive) solution to the maximization problem except for one important case. If \( \alpha = 0 \) and no penalty to over-

specifying cost savings is incurred, then \( E u(\tilde{y}) \)

is increasing in \( \tilde{x} \) since

\[ E u(\tilde{y}) = F(\tilde{x}; L) [u(y_1) - u(y_2)] + u(y_2), \]

\[ y_1 > y_2 \] following from (4) and (3), and \( y_1 \) independent of \( \tilde{x} \) for \( \alpha = 0 \). The optimal state-

ment of cost savings \( \tilde{x} \) is accordingly \( \tilde{x} = c \), c = maximum believable statement of cost saving. Imposition of the penalty cost causes the d.m. to evaluate the trade-off between probability of acceptance and penalty costs. It is assumed for now that \( \alpha > 0 \) and \( \tilde{x} < c \). A special case in which this assumption is not required is described at the end of this section.

Letting \( y^\alpha \) denote \( y_1 \) evaluated at the optimum \( (I^\alpha, \tilde{x}^\alpha) \) and \( y^* = -I^\alpha \), the first order condition for a maximum with respect to \( I \)

\[ u'(y^\alpha) \frac{dy^\alpha}{dI} - 1 - F(\tilde{x}^\alpha; L) = 0, \] \( \tilde{y} \)

and for \( \tilde{x} \) it is

\[ u'(y^\alpha) \frac{dx^\alpha}{d\tilde{x}} - u(y^\alpha) \frac{dF(\tilde{x}^\alpha; L)}{d\tilde{x}} = 0, \]

where \( F(\tilde{x}^\alpha; L) > 0 \), \( du(y)/dy = u'(y) \), and \( f(\tilde{x}^\alpha; L) = \) density function associated with \( F \).

The first term in (5) represents the ratio of the marginal utility of net gain from incremental increase in \( I \) to the marginal utility of loss due to the increment in \( I \). The second term in (5) represents the rejection odds associated with proposal submission (e.g., say \( F = 1/4 \) and \( (1 - F) = 3/4 \), then \( 1 - F/F = 3/1 \), the rejection odds). It is easy to show that the first term is non-increasing in \( I \) near the optimum \( (I^\alpha, \tilde{x}^\alpha) \) when \( u(\cdot) \)

is concave in cost savings (i.e., the d.m. is risk averse). Hence, the optimal investment in information is arrived at by increasing \( I \) (and thereby driving down the utility term) until the ratio of the marginal utilities of net gain to loss from the last increment in \( I \) just equals the rejection odds. Notice also that the optimal planning investment is increased to the point where the project's marginal net benefit with program financing is zero \( (dy^\alpha/d\tilde{x} = 0) \) if \( F = 1 \), i.e., there is no uncertainty regarding acceptance by the program.

The first term in (6) is the proportionate change in utility due to an incremental increase in \( \tilde{x} \) and it is non-decreasing in \( \tilde{x} \) at \( \tilde{x}^\alpha \). The second term is the proportionate increase in the acceptance probability due to a small increase in \( \tilde{x} \). Kamien and Schwartz define \( f(\tilde{x}; L)/F(\tilde{x}; L) \) as the acceptance rate and they assume that the acceptance rate is decreasing in \( \tilde{x} \). They reason that the "proportionate rate of increase in the acceptance probability with a given bid (proposals of cost savings here) increment is larger when the bid (and its probability of acceptance) is small than when the bid is already large." If the acceptance rate is decreasing in \( \tilde{x} \), then the optimal decision rule for \( \tilde{x} \) is to increase the stated level of a project's cost savings (thereby increasing the utility term and decreasing the acceptance rate) until the proportionate loss in utility due to increased penalty cost just equals the acceptance rate.

Turning to the comparative static effects of alternative program structures, we concentrate

\[ ^{10} \text{M. Kamien and N. Schwartz, (1970), p. 22.} \]
on the implications of payback policy. This is undoubtedly the most important program instrument, and it can serve as the basis for a supply-demand analysis of program policy decisions (see Part IV). Let us consider then the impact of changes in \( k \) on the equilibrium values of \( I \) and \( \dot{x} \). Letting \( I \), \( x \), and \( k \) vary, totally differentiating the first order conditions (5) and (6), and solving for \( dl/dk \) and \( dx/dk \) using Cramer’s rule yields the following equations:

\[
\frac{dl}{dk} = \begin{bmatrix}
\frac{\partial^2 Eu^*}{\partial k} & \frac{\partial^2 Eu^*}{\partial \dot{x}}^2
\end{bmatrix} \begin{bmatrix}
1
\end{bmatrix}
\]

\[
\frac{dx}{dk} = \begin{bmatrix}
\frac{\partial^2 Eu^*}{\partial k} & \frac{\partial^2 Eu^*}{\partial \dot{x}}
\end{bmatrix} \begin{bmatrix}
1
\end{bmatrix}
\]

where \( Eu^* = \max_{(x,t)} Eu(t) \) and \( H \) denotes the positive (for a local maximum) Hessian Determinant of \( Eu^* \) with respect to \( I \) and \( \dot{x} \).

If the acceptance rate, \( f/J \), decreases, then it can be shown that \( \frac{\partial^2 Eu^*}{\partial \dot{x}^2} < 0 \) (see Gregory, p. 75). The sign of \( \frac{\partial^2 Eu^*}{\partial \dot{x}^2} \) can also be determined:

\[
\frac{\partial^2 Eu^*}{\partial \dot{x}^2} = f(u'(y_1^*) \dot{y}_1^*/\partial \dot{x} + u'(y_2^*)
\]

\[
-Fu'\dot{x}/\partial \dot{x} > 0,
\]

\( F = F(x^*;L), f = f(x^*;L) \) = density function associated with \( F \). Positivity follows from \( u() \) concave in cost savings and \( \dot{y}_1^*/\partial \dot{x} > 0 \) by (5).

Finally, \( Eu^* \) is concave in \( I \) near the optimum.

\[
\frac{\partial^2 Eu^*}{\partial I^2} = \frac{1}{F} \left[ u''(y_1^*) \frac{\partial y_1^*}{\partial I} + u'(y_1^*) \frac{\partial^2 y_1^*}{\partial I^2} \right] + (1 - F) u''(y_2^*) < 0
\]

since \( \frac{\partial^2 y_1^*}{\partial I^2} = x''(I^*)(1 + \alpha) < 0 \) by assumption.

Now, substituting the cross-partial derivatives with respect to \( k \) into (5) and (6), we have

\[
\frac{dl}{dk} = \frac{L}{H} \left[ Fu''(y_1^*) \frac{\partial y_1^*}{\partial I} (\frac{\partial^2 Eu^*}{\partial \dot{x}^2}) \right]
\]

\[
- \left( Fu'(y_1^*) - Fu''(y_1^*) \alpha (\frac{\partial^2 Eu^*}{\partial I \partial \dot{x}}) \right)
\]

\[
\frac{dx}{dk} = \frac{L}{H} \left[ (-Fu''(y_1^*) \frac{\partial y_1^*}{\partial I}) (\frac{\partial^2 Eu^*}{\partial \dot{x}^2}) \right]
\]

\[
+ \left( Fu'(y_1^*) - Fu''(y_1^*) \alpha (\frac{\partial^2 Eu^*}{\partial I^2}) \right)
\]

Unfortunately, the signs for \( dl/dk \) and \( dx/dk \) are ambiguous, since the first term in both (7) and (8) is positive while the second is negative.

An unambiguous effect obtains in the special case when the d.m. is risk neutral, i.e., has linear utility. Since \( u''(\cdot) = 0 \) in this case, \( dl/dk < 0 \) and \( dx/dk < 0 \). Thus, an increase in the payback fraction imposed on borrowed funds causes the expected value maximizer to decrease his planning investment as well as his statement of the project’s cost savings. As the terms of the borrowing are improved for the d.m., he devotes more resources to planning a project and proposes more in cost savings with a corresponding increase in the acceptance probability.

Certain results can also be obtained in special cases when the utility function is concave. It can be shown, for example, that the risk averse d.m. will definitely not respond to an increase in \( k \) by decreasing \( I \) and increasing \( \dot{x} \) simultaneously, i.e., it can’t be that both \( dl/dk > 0 \) and \( dx/dk > 0 \) (see the appendix for proof of this).

Both of these actions would be detrimental to the project’s net benefits given proposal acceptance since lower net cost savings are achieved at the reduced \( I \) under \( \dot{y}_1^*/\partial \dot{x} > 0 \) and penalty costs are greater when \( x(\dot{I}) \) decreases and \( \dot{x} \) increases. The only situation in which the optimal proposal of cost savings increases following an increase in \( k \) is when optimal planning is also increased. In that case, an increase in \( x \) improves the acceptance probability, any increase in penalty cost is more than offset by the simultaneous increase in \( x(\dot{I}) \).

In the limiting case when the d.m. with program participation is low (i.e., \( F = 1 \)), one would expect the risk averse d.m. to have in the same fashion as the risk aversionco. facing a change in the payback. Thus, since \( \dot{y}_1^*/\partial \dot{x} = 0 \) when \( F = 1 \), it follows that \( dl/dk < 0 \) and \( dx/dk < 0 \) (8). The following proposition summarizes the preceding results.

**Proposition 2:** (i) A risk neutral d.m. will not react to an increase in the payback fraction by decreasing his planning investment. (ii) A risk neutral d.m. will not react to an increase in the payback fraction by decreasing his planning investment. (iii) If \( F(x^*;L) = 1 \), then optimal cost savings are both d.m. in the acceptance probability.

### III. Univariate Mode

Next, an important special case of a preceding model is presented. Pro type described here often require concerning the project before the decision is made (see footnote 6), quasitance to financing can severely affect the project's opportunity to overspend on cost savings, and therefore it is study the optimal planning in the proposed savings coincide with \( x = x(I) \). Expected utility in

\[
Eu(y, w) = F(x(I);L) u(x(I) - l)
\]

\[
+ \left[ 1 - F(x(I);L) \right] U
\]

\[
= F(x(I);L) \left[ u(y, w) + u(y, w) \right]
\]

where \( y = x(I) - l - kL \), and \( w \) is typically represented as an argument of...
\( \dot{x} \) improves the acceptance probability and any increase in penalty cost is offset by the simultaneous increase in \( x(t) \).

In the limiting case when the risk associated with program participation is low (\( F \) “close” to 1), one would expect the risk averse d.m. to behave in the same fashion as the risk neutral d.m. following a change in the payback fraction. Since \( \dot{y}_t = 0 \) when \( F = 1 \), it does indeed follow that \( df/dk < 0 \) and \( dx/dk < 0 \) in (7) and (8). The following proposition summarizes the preceding results.

**Proposition 2:** (i) A risk neutral d.m. adjusts to an increase in the payback fraction by decreasing both his planning investment and his statement of the project’s cost savings, i.e., \( df/dk < 0 \) and \( dx/dk < 0 \). (ii) A risk averse d.m. will never react to an increase in \( k \) by simultaneously reducing his planning investment and increasing his statement of cost savings, i.e., not both \( df/dk < 0 \) and \( dx/dk > 0 \). (iii) If \( F(x^*; L) = 1 \), then optimal planning and stated cost savings are both decreasing in \( k \) for the risk averse d.m.

### III. Univariate Model

Next, an important special case of the preceding model is presented. Programs of the type described here often require information concerning the project before the financing decision is made (see footnote 6). Such a prerequisite to financing can severely limit the participant’s opportunity to overstate a project’s cost savings, and therefore it is important to study the optimal planning investment when proposed savings coincide with actual savings, i.e., \( x = x(t) \). Expected utility in this case is:

\[
Eu(y, w) = F(x(t); L)u[x(t) - I - kL, w] + \left[ 1 - F(x(t); L) \right] U[-I, w] = F(x(t); L)u(y_2, w) - u(y_2, w) + u(y_1, w)
\]

where \( y_3 = x(t) - I - kL \), and \( w \) is again explicitly represented as an argument of \( u \).

The first order condition for a maximum of \( Eu(y, w) \) is:

\[
\frac{dEu^*}{dy} = f(x(t^*); L)u(y_2, w) - u(y_2, w) + \frac{F(x(t^*); L)}{1 - F(x(t^*); L)} u_1(y_2, w) x'(t^* - 1) - \frac{1 - F(x(t^*); L)}{F(x(t^*); L)} u_1(y_2, w) = 0
\]

where \( u^* \) is now understood to be optimal expected utility evaluated at \( t^* \). The three effects of increases in \( f \) on expected utility can be seen in (9). The first term evaluates the increased acceptance probability which results from an increase in planning and the project’s cost savings. The second term represents the expected marginal utility of an increase in \( I \) given proposal acceptance, and the third term represents the same given proposal rejection.

It is clear from (9) that the d.m. may invest in planning up to a point \( t^* \) where \( x'(t^*) - 1 < 0 \). If \( x'(t^*) < 1 \), then the first term in (9) must be sufficiently positive to offset the negative second and third terms. Hence, the d.m. must perceive large gains in acceptance probability due to incremental \( I \) if he is to make uneconomic investments in information \( x'(t^*) < 1 \). Such an investment is uneconomic because the marginal cost savings generated by the last unit of search investment is less than the marginal cost. If the acceptance rate doesn’t decrease rapidly, then it is possible that the d.m. will be led to make such uneconomic investments in information, and this possibility has policy implications to external sources whose goal is to promote efficiency.\(^\text{11}\)

\[^{11}\text{In order to examine this possibility more closely, rewrite (9) as follows:}\]

\[
x'(t^*) - 1 = \frac{1 - F(x(t^*); L) u_1(y_2^*, w)}{F(x(t^*); L)} u_1(y_2^*, w) + \frac{F(x(t^*); L) u_1(y_2, w) - u(y_2, w)}{F(x(t^*); L)} x'(t^*)
\]
Next, we consider the impact of risk aversion on the optimal planning investment for the normal case which excludes the possibility of uneconomic planning.

**Proposition 3:** Let \( EU^*(T) = \max I \) \( EU^*(\tilde{T}, w) \), \( i = 1, 2 \); and denote the optimal values of \( I \) as \( I_1 \) and \( I_2 \), respectively. If \( x'(I_1) > 1 \), then for \( r_u(w) \) as defined in Lemma 1

\[
r_u(w) > r_u^*(w) \quad \text{for every } y \text{ and } w, \text{ implies } \quad I_1 < I_2
\]

**Proof:** The first order condition for a maximum of \( EU^*(\tilde{T}, w) \) at \( I_1 \) can be written as follows:

\[
\frac{u^1(y_1^2, w) - u^2(y_1^2, w)}{u_1^2(y_1^2, w)} \left( f(x(I_1); L)x'(I_1) \right) + \frac{u_1^1(y_1^3, w) - u_1^2(y_1^3, w)}{u_1^2(y_1^3, w)} \left( F(x(I_1); L)(x'(I_1) - 1) \right)
\]

where

\[
y_1^3 = x(I_1) - I_1 = kL, y_1^2 = -I_1, y_1^3 > y_1^2, i = 1, 2.
\]

Pratt (p. 128) shows that \( r_u^*(w) > r_u^*(y, w) \) implies

\[
\frac{u^1(y_1^3, w) - u^2(y_1^2, w)}{u_1^2(y_1^3, w)} < \frac{u_1^2(y_1^3, w) - u_1^2(y_1^2, w)}{u_1^2(y_1^3, w)}
\]

and

\[
u^1(y_1^3, w)/u^1(y_1^3, w) < u^2(y_1^3, w)/u^2(y_1^3, w)
\]

so long as \( y_1^3 > y_1^2 \). It follows from this and (11) that

\[
\frac{u^2(y_1^3, w) - u^2(y_1^2, w)}{u_1^2(y_1^3, w)} \left[ f(x(I_1); L)x'(I_1) \right]
\]

\[
+ \frac{u_1^2(y_1^3, w)}{u_1^2(y_1^3, w)} \left[ F(x(I_1); L)(x'(I_1) - 1) \right]
\]

\[
- [1 - F(x(I_1); L)] > 0
\]

since \( r_u^*(w) > r_u^*(w) \) and \( x'(I_1) - 1 > 0 \). But this is simply a positive multiple of the first order condition for \( EU^*(\tilde{T}, w) \) evaluated at \( I_1 \). That is, \( \frac{\partial EU^2}{\partial I} (\tilde{T}, w) |_{I = I_1} > 0 \).

The second order concavity condition holds at a regular maximum, and I must be increased in order to drive the above inequality to zero. Therefore, \( I_2 > I_1 \).

Therefore, the more risk averse the d.m. is in the normal case the less he invests in project planning. Alternatively, an increase in risk aversion causes the d.m. to reduce planning which in turn implies lower cost savings (since \( x'(I_1) > 1 \) by assumption) and lower acceptance probability. Thus, an increase in risk aversion causes the d.m. to substitute a reduction in the planning costs (due to rejection of the program) for lower cost savings and acceptance probability.

It is now possible to specify the impact of a change in \( w \) on optimal \( I \).

**Corollary 4:** Let \( I^* \) maximize \( EU(\tilde{T}, w) \) as before, and let \( x'(I^*) = 1 \). If the increasing risk aversive, then optimal increasing in \( w \).

**Proof:** Let \( u_1(y, w) = u(y, w) \) and \( u(y, w + \Delta) \), \( \Delta > 0 \), in proposition decreasingly risk aversive, then \( ru(w) \) and \( I^* < I^* \) where \( I^* \) maximizes \( EU^*(\tilde{T}) \).

If hospital decision makers are risk averse, one would expect their greater assets (and financial stress better prepared for implementing a project than poorer hospitals). strengthens the policy implication concerning the possible bias in pro due to decreasing risk aversion amo now appears that, ceteris paribus hospitals (1) may be more likely the program with a proposal, and (onstrate greater cost savings of planning investments. If wealth serve a wealthier clientele, then at tributional bias could result with implications to federal programs.

Next we examine the effect of change in the payback faction search. Implicitly differentiating the condition (9) with respect to \( \lambda \) yields

\[
\frac{dI}{d\lambda} = \left[ \frac{\beta^2 EU^2}{\partial I} \right]
\]

\[
\left[ f(x(I^*); L)u_1(y_2^3, w)x'(I^*) \right]
\]

\[
- F(x(I^*); L)u_1(y_2^3, w)x'(I^*)
\]

The sign of (12) is determined by bracketed term at a regular maximum

\[
\frac{dI}{d\lambda} \leq 0 \Rightarrow \frac{1}{u_1(y_2^3, w)x'(I^*) - 1} \frac{u_1(y_3^3, w)}{x'(I^*)}
\]

\[
= r_u(y_3^3, w) \left[ \frac{x'(I^*) - 1}{x'(I^*)} \right] \leq \frac{f(x(I^*))}{F(x(I^*))}
\]

Hence, optimal search is decreases
implies
\[ -\frac{u^1(y_{I^*}^1, w)}{u^1(y_{I^*}^1, w)} < \frac{u^2(y_{I^*}^2, w) - u^2(y_{I^*}^1, w)}{u^1(y_{I^*}^1, w)} \]
\[ y_{I^*}^1 > y_{I^*}^2. \]
It follows from this and
\[ -\frac{u^2(y_{I^*}^2, w)}{\frac{1}{2}} [f(x(I_1^*); L)x'(I_1^*)] \]
\[ \frac{\partial}{\partial x'} \left( y_{I^*}^2 \right) = \frac{\partial}{\partial x'} \left( y_{I^*}^1 \right) - 1 \]
\[ \frac{\partial}{\partial x'} \left( y_{I^*}^1 \right) - 1 > 0. \]
But a positive multiple of the first order concavity condition holds maximum, and I must be increased to drive the above inequality to zero.
\[ y_{I^*}^2 > y_{I^*}^1. \]
the more risk averse the d.m. is in the case the less he invests in project.
Alternatively, an increase in risk averse the d.m. to reduce planning which implies lower cost savings (since assumption) and lower acceptance.
Thus, an increase in risk averse is a reduction in the planning costs (due to rejection on lower cost savings and probability.

possible to specify the impact of a on optimal \( I \).

Let \( I^* \) maximize \( Eu^2(y, w) \) as before, and let \( x'(I^*) > 1. \) If the d.m. is decreasingly risk averse, then optimal search is increasing in \( w \).

\[ \text{Proof: Let } u_1(y, w) = u(y, w) \text{ and } u_2(y, w) = u(y, w + \Delta), \Delta > 0, \text{ in proposition 2. If } u \text{ is decreasingly risk averse, then } r_\Delta(w) > r_\Delta(w + \Delta) \text{ and } I^* < I^\Delta \text{ where } I^\Delta \text{ maximizes } Eu(y, w + \Delta). \]

If hospital decision makers are decreasingly risk averse, one would expect hospitals with greater assets (and financial strength) to be better prepared for implementing the efficiency project than poorer hospitals. This result strengthens the policy implications of Lemma 1 concerning the possible bias in program funding due to decreasing risk aversion among d.m.'s. It now appears that, ceteris paribus, wealthier hospitals (1) may be more likely to approach the program with a proposal, and (2) may demonstrate greater cost savings due to greater planning investments. If wealthier hospitals serve a wealthier clientele, then an income distributional bias could result with important implications to federal programs.

Next we examine the effect of a parametric change in the payback fraction on optimal search. Implicitly differentiating the first order condition (9) with respect to \( k \) yields:

\[ \frac{df}{dk} = \left[ L \frac{\partial^2 Eu^*}{\partial x'^2} \right] \]
\[ \quad \cdot [f(x(I^*); L)u_1(y_{I^*}^2, w)x'(I^*) - F(x(I^*); L)u_{I^*}(y_{I^*}^2, w)(x'(I^*) - 1)] \]
\[ \text{(12)} \]

The sign of (12) is determined by the second bracketed term at a regular maximum:

\[ \frac{df}{dk} \leq 0 \quad \text{as } \frac{u_{I^*}(y_{I^*}^2, w)x'(I^*) - 1}{u_1(y_{I^*}^2, w)x'(I^*)} \]
\[ \leq \frac{f(x(I^*); L)}{F(x(I^*); L)}. \]

Hence, optimal search is decreasing in \( k \) when the risk index, weighted by the ratio of marginal net benefit (from incremental \( I \)) to marginal gross benefit, is less than the acceptance rate. If \( r\Delta(w) \leq 0 \), then \( df/dk \leq 0 \); but \( r\Delta(w) = 0 \) implies that the d.m. is risk preferring, a possibility which has been excluded from this analysis. If the d.m. is risk neutral or slightly risk averse, then \( df/dk > 0 \). This is consistent with proposition 2. The same result obtains when \( x'(I^*) \) is close to one. On the other hand, extreme risk aversion and \( x'(I^*) - 1 \) large positive imply that \( df/dk \leq 0 \). The very risk averse d.m. increases his planning investment in order to increase his chances of acceptance and to regain some of the lost benefits due to higher \( k \).

This interpretation follows from the fact that \( x'(I^*) > 1 \) for the risk averse d.m. who responds to an increase in \( k \) by increasing \( I \) (i.e., \( df/dk > 0 \)). Hence, increasing \( I \) leads to greater net cost savings (\( x'(I^*) > 1 \)) if the project is funded, and the very risk averse d.m. responds to an increase in \( k \) by attempting to improve the worsened terms of the gamble.

One last question is posed for the univariate model. A program policy-maker can exercise some influence over the form of the distribution function \( F \) assessed by the d.m. He does this through policy statements and informal communication with hospital administrators. One might ask then, “what effect does an exogenous increase (or decrease) in \( F \) have on optimal planning?” The next proposition provides a qualified answer when \( F \) is positively translated by \( \Delta > 0 \). We will require the following additional notation:

Define \( Eu^\Delta = F(x(I) + \Delta; L) [u(y_{I^*} - u(y_{I^*})] \). \( Eu^\Delta \) is simply \( Eu(y, w) \) with \( F \) positively translated by the increment \( \Delta > 0 \).

Proposition 6: Let \( 0 \leq \partial^2 F/\partial x^2 (x(I); L) \) Let \( I^* \) maximize \( Eu^2(y, w) \) and let \( I^\Delta \) maximize \( Eu^\Delta \). Then \( I^\Delta > I^* \) as \( \Delta > 0 \).
Proof: let \( F = F(x(I^*); L) \), \( F^\Delta = F(x(I^*) + \Delta; L) \), \( f = f(x(I^*); L) \), \( \max_y Eu(y; w) = Eu^* \), \( f^\Delta = f(x(I^*) + \Delta; L) \), and \( \Delta > 0 \)

\[
\frac{\partial Eu^*}{\partial I} = f[u(y^*_x, w) - u(y^*_x, w)]x(I^*) \\
+ F[u_1(y^*_x, w)(x(I^*) - 1) \\
+ u_1(y^*_x, w)] \\
+ u_1(y^*_x, w) \\
< F^\Delta[u(y^*_x, w) - u(y^*_x, w)]x(I^*) \\
+ F^\Delta[u_1(y^*_x, w)(x(I^*) - 1) \\
+ u_1(y^*_x, w)] - u_1(y^*_x, w) = \frac{\partial E u^\Delta}{\partial I} \bigg|_{I=I^*} 
\]  

(14)

since \( F^\Delta > F \), \( f^\Delta > f \) by assumption that \( \partial^2 F/\partial x^2 \geq 0 \), and \( [u_1(y^*_x, w)(x(I^*) - 1) + u_1(y^*_x, w)] > 0 \). The inequality follows from the fact that \( x(I^*) > 0 \), since \( x(I^*) < 0 \) implies that a slight decrease in \( I \) would increase the project's benefits (gross cost savings and acceptance probability) and decrease its cost. Expected utility would consequently be increased, so such an \( I \) could never be optimal. It follows that

\[
[u_1(y^*_x, w)(x(I^*) - 1) + u_1(y^*_x, w)] \\
\geq [u_1(y^*_x, w) - u_1(y^*_x, w)] > 0 
\]

since \( y^*_x > y^*_x \) and \( u \) is concave in \( y \).

Now it follows that

\[
\frac{\partial E u^\Delta}{\partial I} \bigg|_{I=I^*} > 0.
\]

If the second order concavity condition is assumed to hold, then \( I \) must be increased in order to drive the above inequality to zero.

\( \therefore f^\Delta > f^* \)

The fact that optimal search is increasing in positive exogenous translations of \( F \) holds without restriction of \( u \) except that it be concave in \( y \). The inequality \( 0 \leq \partial^2 F(x; 1)/\partial x^2 \) requires \( F \) to be convex in \( I \) but includes the important case in which the d.m. assesses an information- less prior (uniform distribution) over his acceptance odds, and it does not exclude the possibility of decreasing acceptance rate. The proposition holds with the opposite sign when \( \Delta < 0 \).

The comparative static analysis of the last two sections generally demonstrated that parametric changes diminishing the expected utility of financing the efficiency project (e.g., increase in \( k \), negative translation in \( F \), increase in risk aversion) cause the risk neutral, or mildly risk averse, d.m. to reduce his planning investment. If the d.m. is "very risk averse" in the sense of inequality (13), however, he may respond to an increase in the payback fraction by increasing \( I \) and trying to recapture some of the lost benefits due to higher \( k \).

IV. Conclusion and Applications

The model developed here provides a basis for program policy analysis. A theory of demand for program funding can be constructed from the notion of the payback fraction as the price paid by participating health care units for funds obtained from the program. Since the d.m.'s expected utility is decreasing in \( k \), it becomes more likely that a given d.m. chooses to apply for funding as \( k \) decreases; and if there are sufficient prospective applicants, demand for program funding may be characterized by a continuous downward sloping curve.

The program's supply curve is positively sloped to reflect the fact that increases in \( k \) make more effective funds available for projects; that is, money repaid to the program can be used to fund additional projects in subsequent periods, so the present value of the long-run effective supply of funds is increasing in the payback fraction.

Figure 2 depicts one possible set of supply-demand relationships in which \( D \) represents the demand curve and \( S \) the supply curve. Observe that \( kAB \) denotes the actual amount of funding that the program can support, since no more

funds can be distributed than \( \overline{\ell} \). Now, various policy decisions can be made in terms of their effects on the program's incremental funding curve, \( k^\Delta \).

A change which tends to reduce expected utility from program final may shift the curve downward. For expansion of extensive pre-implementa- tion requirements constrains the planning investment and may affect utility from program participation. Communication of these requirements constrains the planning investment and may affect utility from program participation. Communication of these requirements is necessary to ensure that the expected utility from program participation is maximized.

Finally, the supportable funds are mapped into an aggregate cost which depicts the various levels of program cost savings generated by various levels of \( k \). The effect of policy...
uniform distribution) over his acceptances, and it does not exclude the possibility of decreasing acceptance rate. The question holds with the opposite sign when the comparative static analysis of the last proposition demonstrated that an increase in risk averse risk reduces the expected utility of the efficiency project (e.g., increased reliance on $F$), thus increasing the risk neutral, or mildly risk averse, planning investment to reduce his planning investment, is “very risk averse” in the sense of the formula, however, he may respond to the payback fraction by increasing $g$ to recapture some of the lost benefit over $k$.

**Conclusion and Applications**

This developed here provides a basis for policy analysis. A theory of demand for funding can be constructed from the payback fraction as the price of health care units for funds from the program. Since the d.m.'s expected utility is decreasing in $k$, it becomes less attractive to a given d.m. as $k$ decreases; and if there are susceptible applicants, demand for funding may be characterized by a downward-sloping curve. The program's supply curve is positively related to the fact that increases in $k$ reduce effective funds available for projects, and the money repaid to the program can fund additional projects in subfields, so the present value of the longer supply of funds is increasing in the action.

**Diagram**

![Diagram of Aggregate Demand (L) and Supply for MIP Funds](image)

**Appendix**

In this appendix, a proof is offered of the following hypothesis of proposition 2:

**Lemma:** If $u$ is concave in $y$, then not both $dI/dk < 0$ and $dI/dk 
eq 0$.

**Proof:** Let $u$ be concave and assume both the inequalities hold. We will derive a contradiction:

$$
\frac{dI}{dk} = \frac{1}{H} \left[ \left( -\frac{\partial^2 E_u}{\partial k \partial I} \right) \left( \frac{\partial^2 E_u}{\partial k^2} \right) \right. \\
- \left. \frac{\partial^2 E_u}{\partial k \partial y} \right|_{\partial I} \frac{\partial^2 E_u}{\partial I \partial x} \right] < 0
$$

which implies

$$
\frac{\partial^2 E_u}{\partial k \partial x} \frac{\partial^2 E_u}{\partial k \partial I} > \frac{\partial^2 E_u}{\partial k \partial y} \frac{\partial^2 E_u}{\partial I \partial x} \quad (i)
$$

since $\frac{\partial^2 E_u}{\partial k \partial I} \frac{\partial I}{\partial k} = -E_u(y) \frac{\partial y}{\partial I} L > 0$ and $\frac{\partial^2 E_u}{\partial I \partial x} \frac{\partial x}{\partial I} > 0$ as shown in Part II. Further,

$$
\frac{dI}{dk} = \frac{1}{H} \left[ \left( -\frac{\partial^2 E_u}{\partial k \partial I} \right) \left( \frac{\partial^2 E_u}{\partial k^2} \right) \right. \\
+ \left. \frac{\partial^2 E_u}{\partial k \partial y} \right|_{\partial I} \frac{\partial^2 E_u}{\partial I \partial x} \right] > 0
$$

which implies

$$
\frac{\partial^2 E_u}{\partial k \partial x} \frac{\partial^2 E_u}{\partial k \partial I} > \frac{\partial^2 E_u}{\partial k \partial y} \frac{\partial^2 E_u}{\partial I \partial x} \quad (ii)
$$

Combining (i) and (ii)

$$
\frac{\partial^2 E_u}{\partial k \partial x} > \frac{\partial^2 E_u}{\partial I \partial x} \quad \frac{\partial^2 E_u}{\partial k \partial I} \quad \frac{\partial^2 E_u}{\partial I \partial x} \quad \frac{\partial^2 E_u}{\partial I \partial I} \quad \frac{\partial^2 E_u}{\partial I \partial I}
$$

or

\[ \left( \frac{\partial^2 E_u}{\partial k \partial x} \right) \left( \frac{\partial^2 E_u}{\partial I \partial x} \right) < \left( \frac{\partial^2 E_u}{\partial I \partial I} \right)^2 \]

12 For an analysis which proceeds along these lines see Gregory and Rappaport (1974).
which contradicts the assumption that \( H > 0 \)
(for a regular maximum). Therefore, not both
\[ \frac{df}{dk} < 0 \text{ and } \frac{d^2f}{dk^2} > 0. \]

References


