COORDINATION, FRAGILITY, HIGH-POWERED MONEY, AND THE LIQUIDITY TRAP:

A "TOBINESQUE" PARABLE

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INTRODUCTION

Recently there has been renewed interest in production complementarity and coordination failure in Macroeconomics. Indeed, Evans, Honkapohja, and Romer [1998, 506] identify production complementarity as the only plausible mechanism for generating macroeconomic coordination failure. Colander [1996] and Cooper [1999] provide excellent overviews and Romer [1996, 294-299] an excellent brief summary. The particular concern of this coordination approach is with models of multiple Pareto ranked equilibria, where production complementarity induces the multiplicity of equilibria. A "coordination failure" is the realization of such a Pareto dominated equilibrium. The realization of such an equilibrium is "bad" in an unambiguous sense; that is, the economy is, in this circumstance, unambiguously malfunctioning. Thus the existence of such coordination failures provides a reasonable explanation of how macroeconomic problems can come into existence, even as all agents are rational.

If production complementarity is the mechanism for generating macroeconomic coordination failure generally, then it should be the mechanism for generating a prominent feature of macroeconomic coordination failure, monetary fragility. This paper draws the link between production complementarity, coordination failure and monetary fragility, by exploiting standard, archetypal production complementarities. Thus the paper describes a simple and stylized environment in which the two basic production complementarities identified by Cooper [1999, 41-45] generate macroeconomic coordination failure and monetary fragility.

The first of these basic production complementarities is between the suppliers of inputs to a final product fabricator. This is the archetypal "input game," with an "internal constant returns to scale production process," in Cooper's terms. In this input game a fabricator needs to coordinate her suppliers of complementary inputs. It is the interaction between fabricator and suppliers, in this input coordination game, which induces a role for money. Further, the threat of coordination failure induces a demand for high-powered money.

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Secondly, there is a general production complementarity between the fabricators themselves, "external returns" in Cooper's terms. That is, in this second basic production complementarity "the returns to scale are created by the effects of other agents outside an internal constant returns to scale production process [Cooper, 1999, 43]." Demand for high-powered money interacts with this second production complementarity to trigger general recession, and thereby generate a liquidity trap. The second general production complementarity is both the propagation mechanism for recession, and the source of a true liquidity *trap*. It is further worth noting that this liquidity trap parable involves "liquidity" perhaps more nearly in a store of value sense of Tobin [1958], than in a, perhaps more monetarist, sense of "liquidity services" *per se*.

PRODUCTION COMPLEMENTARITIES, COORDINATION, AND FRAGILITY

Production complementarity induces fragility. There is a basic standard parable of production complementarity. In this parable, there are N > 1 individuals whose inputs e_i , "effort," jointly determine output. In particular, the payoff to effort for each individual *i* is a $[\min(e_1,...,e_N)] - be_i$, where a > b > 0 and $e_i \in [0,1]$. a $[\min(e_1,...,e_N)]$ is the utility gain to the individual from the joint output, and $-be_i$ is the utility loss to the individual from her (own) effort. The individuals choose their effort levels independently. This game has a continuum of Pareto ranked Nash equilibria, where e_i are equal for all *i*.¹ This is illustrated in Figure 1, where the bold line represents individual *i*'s payoff, when the minimum effort of the other individuals is \underline{e}_i .



FIGURE 1

A fortiori, such multiple equilibria are a concern in the slightly more complicated version of this parable developed in this paper. One approach game theory takes to multiple equilibria is to select particularly salient equilibria. Specifically, where they exist, Pareto dominant and secure equilibria are selected [Cooper, 1999]. The above basic parable of decentralized production has both. Namely, $e_i = 1$, for all i, is Pareto dominant, indeed Pareto optimal. On the other hand, $e_i=0$, for all i, is the secure equilibrium. That is, $e_i = 0$ maximizes the player's payoff, given that she believes that the others will choose the strategy that minimizes her payoff, namely zero effort on their part. With others choosing zero effort, individual i gets a return of -b on her own

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effort. "Fragile" is taken to mean that the economy has both Pareto dominant and secure equilibria, like this basic parable of decentralized production. The slightly more complicated version of this basic parable, developed in this paper, is fragile as well.

The economic environment in this extended parable is still simple. There are two production technologies in the model. There is a primitive production technology that does not involve production complementarities, and an advanced production technology that does. Recession is modeled as the general use of the primitive production technology.

First consider the primitive, complementarity-free, production. This primitive production technology is "Robinson Crusoe." There are NJ individual producers endowed with leisure, in the amount L each. These individuals have identical utility functions, which are smooth, increasing and strictly quasi-concave, defined on leisure, C_1 , and a single consumption commodity, C_2 . They can produce the consumption commodity, on their own, and in isolation, with leisure. The rate of exchange of leisure for commodity is a fixed 1:1. Assume for the moment that this is the only production technology available. Then there is a unique optimal amount of input of leisure into this primitive production technology, I^* . That is, $I^* = \operatorname{argmax}_I U(L - I, I)$, where U is the common utility function. This is illustrated in Figure 2.





Security is now found in foregoing efficient, advanced, specialized, joint production, in favor of this primitive production; that is, in an inefficient, but secure (from coordination failure) self-sufficiency. By using primitive production, in isolation, individuals can guarantee themselves a rate of return of one, at any production level. Thus, the addition of primitive production, to the basic parable of production complementarity, yields a more natural, appealing version of a secure equilibrium than that of the "zero effort level" equilibrium. The general flight to this security is a recession.

The advanced production technology has the two production complementarities. The first production complementarity involves the use of multiple inputs. These inputs take the form of component parts. The consumption commodity is produced from leisure in a two stage process. First, component parts are produced with leisure, and then, second, the component parts are assembled into the consumption commodity. The NJ individuals endowed with leisure are also input producers. There are J inputs, component parts, and the NJ input producers are preassigned to the inputs, N

preassigned to each input. Thus the input producers are specialized, which creates the need for coordination. In the first stage of production, input producers produce their preassigned input from leisure, at the fixed rate of exchange of 1:1. Their choice variable is how much leisure to convert into input. Each can use either primitive or advanced production, but not both.

In the second stage of advanced production, fabricators assemble the J inputs into the consumption commodity. In addition to the NJ input producers, there are N fabricators. These fabricators are identical, have no endowment, and consume only the consumption commodity. Each fabricator is capable of handling the input of just one input producer of each of the J inputs. The input producers are also preassigned to fabricators. Other than their preassignment to input and to fabricator, the input producers are identical. Fabricators are zero cost. Their assembly of inputs yields $xJ\min[I_1,...,I_j]$ units of commodity from $(I_1,...,I_j)$ units of the J inputs. Hence, if a fabricator has equal amounts of each input, then each unit of leisure ultimately yields x units of commodity. If a fabricator has unequal amounts of inputs, output of commodity is determined by the least amount of any input, and any excess amounts of inputs are just costlessly discarded as waste. Thus the rate of return x is achieved only up to the minimum amount of any input, beyond that point individual inputs have a rate of return of zero; hence the need for coordination. This second stage is the perfect complements, component parts assembly, aspect of the advanced production technology; where component parts come in infinitesimal increments. This advanced production technology is just a goods version of the standard basic parable of production complementarity. This basic production complementarity is discussed, for example, by Cooper [1999, viii-xiii, 2-5, 41-46, 60], Van Huyck, Battalio and Beil [1990] and Bryant [1983].

Fragility, as defined above, involves Pareto dominant and secure equilibria. Primitive production provides security, as there the rate of return of one is sure, in production in isolation, without coordination. It remains to characterize the Pareto dominant, advanced production, equilibrium, in this slightly more complicated version of the basic parable of production complementarity. Assume that (almost) all of the rents from the advanced production technology go to the fabricators, as only the fabricators have the ability to assemble component parts.² Specifically, suppose that the fabricator makes a take it or leave it offer to her input suppliers, an offer which is indifferent to (or epsilon better than) their producing in the primitive production technology, on their own. That is, fabricators ask their suppliers to supply a fixed amount of input for a fixed amount of payout, amounts which yield the suppliers a utility of $U(L - I^*, I^*)$. If $x \ge 1$, the fabricator's problem is:

$$\begin{aligned} & \text{Max} \; [xH-V] \\ & H,V \\ & \text{subject to: } U(L-H,V) \geq U(L-I^*,I^*) \;, \end{aligned}$$

where *H* is the amount of each input produced, and the hours worked by each input supplier, and *V* is the amount of consumption good paid to each input supplier by the fabricator. Let the solution values be $H = H^*$ and $V = V^*$. The fabricator's problem implies $[U_1(L - H^*, V^*)/U_2(L - H^*, V^*)] = x$ and $U(L - H^*, V^*) = U(L - I^*, I^*)$. Thus the

fabricators are perfectly discriminating monopsonists. Under the above assumptions on tastes, there is for each value of x a unique optimal amount of input production, $I^*(x)$, solving the above fabricator's problem, $I^*(x) = H^*$.³ Moreover, for $x \ge 1$, $I^*(x)$ is a continuous increasing function of x, bounded below by I^* and above by L. On the other hand, if x < 1, only the primitive production technology is used. Notice also that, for x > 1, there are Pareto dominant, advanced production, and secure, primitive production, equilibria separately for the respective groups of input producers, preassigned to each fabricator.

This fabricator's profit maximization problem is illustrated in Figure 3, where $C_1 = L - H$ and $C_2 = V$ (compare with Figure 2).



FIGURE 3

Importantly, the advanced production technology also involves a second, general, production complementarity. Recession is modeled as the general use of the primitive production technology. Importantly, then, introduction of this second, general, production complementarity motivates the general choice of the security of primitive production, which characterizes recession, and generates a liquidity trap. This general production complementarity is between the final product fabricators themselves. Jointly the fabricators exhibit increasing returns. Specifically, the rate of return x is a function, X, of the total amount of input successfully assembled by all of the fabricators to set. X is a known, continuous, positive, and strictly increasing function defined on [0, NJL]. As usual, N is assumed to be large, and the value of X little affected by the input assembled by a single fabricator. Consequently, the fabricators and input producers ignore their own negligible effect upon x (and the use of a fixed x in the earlier analysis remains valid). This is, then, exactly the second basic production complementarity discussed by Cooper [1999, 41-43, 55-60], for example.

This general production complementarity motivates the general recession and liquidity trap. It is assumed that, for small amounts of inputs assembled, x < 1. That is, at low rates of aggregate production, the advanced technology yields a rate of return less than the primitive technology. Specifically, X(JL) < 1. Thus a single fabricator alone producing cannot compete with primitive production. This assumption that X(JL) < 1 motivates the general recession and liquidity trap. It is further assumed that for large amounts of input assembled x > 1, specifically $X(NJI^*) > 1$. That is, if all input producers use the primitive production technology, they input leisure at a rate that justifies using the advanced production technology instead (if they coordinate on

doing so). This second assumption implies that there is at least one equilibrium rate of input, in the advanced production technology, that yields a rate of return greater than the primitive production technology.

Proposition 1: There is at least one equilibrium with a rate of return greater than one.

Proof: By assumption $X(NJI^*(1)) = X(NJI^*) > 1$. Consider $x^m > X(NJL)$. Then $X(NJI^*(x^m)) \le X(NJL) < x^m$. Thus at $x = 1 X(NJI^*(x)) - x > 0$, and at $x = x^m X(NJI^*(x)) - x < 0$. As the function $X(NJI^*(x)) - x$ is continuous, by the intermediate value theorem, there exists a rate of return x^e , $x^m > x^e > 1$, such that $X(NJI^*(x^e)) - x^e = 0$. (This proof is illustrated in Figure 4.)



Thus, with one or more equilibria, in advanced production, with a rate of return greater than one, the equilibrium with the highest rate of return is the Pareto dominant equilibrium.⁴

RECESSION

Recession is modeled as the general use of the primitive production technology. There are Pareto dominant, advanced production, and secure, primitive production, equilibria separately for the respective groups of input producers, preassigned to each fabricator. Thus a recession could be the result of the general choice of primitive production by the separate groups of input producers. This general choice can simply be interpreted as being the result of waves of animal spirits, self-fulfilling prophecies or sunspots [Romer, 1996, 296]. However, in addition, the second general production technology provides a propagation mechanism for recession. That is, because of the general production complementarity, separate coordination failures have spillover effects.

This propagation mechanism for recession is a simple one. Given the general production complementarity, even limited input coordination failure triggers recession. That is, a sufficient number of separate failures to coordinate inputs, in interaction

with the general production complementarity, triggers general use of the primitive production technology. Refer to fabricators, whose respective group of suppliers, separately, choose security as "disrupted fabricators," and to the others as "undisrupted fabricators." Indeed, suppose that the number of disrupted fabricators is known before undisrupted fabricators coordinate input suppliers. Say that $N_1 < N$ fabricators are undisrupted and can coordinate inputs. If N_1 is sufficiently small, there is general recession. Indeed, the proof of Proposition 1 has an immediate corollary.

Corollary 1: If N_1 satisfies $X(N_1JI^*(x)) - x < 0$, for all $x \ge 1$, then there is no equilibrium in which N_1 fabricators produce.

As X is increasing, there also is no equilibrium in which fewer than N_1 fabricators produce. A simple sufficient condition for the condition in Corollary 1, namely that $X(N_1JI^*(x)) - x < 0$, for all $x_1 \ge 1$, is that $X(N_1JL) < 1$ (which already holds by assumption for $N_1 = 1$). Summarizing, then, if $(N - N_1)$ disrupted fabricators cannot produce, because their inputs cannot be coordinated, then the rate of return in the advanced production technology, x, is less than one, even if all the remaining fabricators produce. The fabricators all realize this, none produce, and there is general recession. Limited input coordination failure has triggered general recession.

Corollary 1 exhibits a direct relation between technological productivity and recession. An economy with a less productive advanced technology, a sufficiently lower rate of return function X, has a lower "trigger point," $(N - N_1)$, for recession. This further suggests that an economy hit by a negative technology shock would also be a prime candidate for a coordination failure induced recession.

EXTENSIONS OF THE SIMPLE PARABLE: HIGH-POWERED MONEY AND RECESSION

The above parable can be interpreted as a very simple model of money. It is assumed above that fabricators provide their suppliers with a payout in the consumption commodity. However, the advanced production technology involves a two stage process. Input is produced in the first stage of production, while the commodity is not fabricated until the second stage. This two stage production process, then, immediately induces a demand for claims of some sort with which fabricators can pay suppliers. While the structure of this coordination parable is simple, consideration of such claims may, nonetheless, provide insight into the demand for money, and, in particular, into a coordination-based demand for high-powered money by disrupted fabricators. In the first instance, suppose that the undisrupted fabricators are simply paying suppliers, in the first stage, in promises to consumption commodity in the second stage; promises which might be viewed as a rudimentary "inside money," that is, claims backed by promises of future payment. However, under this money interpretation of the parable, disrupted fabricators are those whose promises of consumption commodity are worthless. That is, unfortunately, inside money, a fabricator's promise to consumption commodity, is worthless to a supplier, if the fabricator's other suppliers do not accept it as well. The fabricator cannot meet her promises of consumption commodity without getting all of the inputs. Thus primitive production is indeed secure,

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it maximizes the supplier's payoff given that that supplier believes that the fabricator's other suppliers also will not accept the inside money. Suppliers of disrupted fabricators want security, which inside money does not provide. This raises the question of whether, with another form of claim available, disrupted fabricators could still produce. Indeed, high-powered money can serve this function. That is, suppliers, seeking security, would accept payment, up front, in a high-powered money, if available.

To allow for a simple form of high-powered money, gold is introduced to the parable. Following McAndrews and Roberds [1999], "gold" is treated as already existing consumption commodity. Parenthetically, if, instead, one modeled a fiat high-powered money, then it would be necessary to consider the possibility of the inside monies, the fabricators' promises to consumption commodity, themselves becoming fiat. That is, fiat money models, with multiple types of paper available, have multiple equilibria, with the various papers serving as "monies."⁵ While doubtless of theoretical interest, this feature of fiat money models does not seem to be of great practical interest. In any case, this, conceivably interesting, but tangential, issue is avoided by simply assuming a "gold" high-powered money, which is consumption commodity.

Disrupted fabricators can use high-powered money to coordinate inputs. Indeed, by paying in currency, "cash on the barrel head," a disrupted fabricator insures input production. This coordination-based demand for a high-powered money is a simple notion. Assume fabricators are now endowed with gold. Further, disrupted fabricators pay their input suppliers with notes, 100 percent backed by that gold. Input suppliers, paid in such high-powered notes, do not have to concern themselves with the behavior of the other input suppliers. They are paid in full in any case, as they can simply consume the backing gold directly. Indeed, then, these high-powered notes provide the desired security. Hence input producers, paid in such high-powered money, do, indeed, produce input. In short, high-powered money facilitates coordination.

Naturally, if there is a large enough number of disrupted fabricators, and small enough endowments of gold, general recession still results.⁶ Hence high demand for high-powered money triggers general recession.

EXTENSIONS OF THE SIMPLE PARABLE: LIQUIDITY TRAP AND DEPRESSION

One might even speculate that this simple parable provides insight into the Great Depression. In particular, the model exhibits a liquidity trap.

There is a view that during the Great Depression the economy was awash in liquidity, hence the notion of the liquidity trap. With the interpretation of this parable, as described above, this may actually have been more of a high-powered money trap, than a "liquidity trap," *per se*, at all. With a large number of disrupted fabricators, their demand for high-powered money is high, up until the point of general recession. However, in the general recession, fabricators are not producing at all, and are not using high-powered money. That is, if enough disrupted fabricators cannot produce, because they do not have access to high-powered money, then it is not efficient for any fabricators to produce, and there is general recession, with no fabricator demand for high-powered money. Thus a high-powered money shortage triggers general recession, resulting, in turn, in a high-powered money glut. Hence the, seemingly paradoxical, twin perils of high-powered money shortage and high-powered money glut are reconciled.

The consistency with the liquidity trap goes deeper, however. The general production complementarity may generate a true liquidity trap. While triggered by a shortage of high-powered money, a general recession might take on a life of its own. Imagine, in a repeated version of the parable, that, first, a general recession is triggered by high demand for high-powered money. Because of this experience of recession, fabricators and input producers might then subsequently become "focused" on the secure equilibrium. That is, with them all now anticipating a low rate of return, a low x, advanced production is unattractive relative to primitive production. With no fabricators using advanced production, its rate of return x, indeed, is low. That is, given the general production complementarity, the low rate of return is now self-fulfilling prophesy. A general recession, initially triggered by demand for high-powered money, is sustained on its own momentum, as pessimism now pervades the economy.

This possibility is particularly intriguing. There is now a true high-powered money *trap*. Once fabricators and input producers are all focused on the secure equilibrium, introducing more gold does not move the economy out of recession. The problem now is not an inadequate supply of high-powered money, but the general pessimism, the general anticipation of bad times. A fabricator, even if offering payment in high-powered money, still only anticipates an unacceptable rate of return of x < 1. Consequently, at this point, enhancements in high-powered money supply are, indeed, a matter of "pushing on a string." Not only is the economy awash in high-powered money, there is a true high-powered money trap. Now no amount of additional high-powered money helps.

In this liquidity trap parable, at least, the critical attribute of high-powered money is its assured store of value. That is, quite purposefully, in this parable, the highpowered notes are not assumed to have any advantage, over inside money, in terms of portability, divisibility, recognizability, validation, or legal or recording costs; matters involving "liquidity services." Indeed, the high-powered notes do not have any advantage in terms of information or uncertainty either, at least in equilibrium. In the Pareto dominant equilibrium the respective inside money is known to be of value, while in the secure equilibrium it is known to be valueless. It is, rather, just this valuelessness of inside money, in the secure equilibrium, itself, which is the problem. If you will, the high-powered money is acting as an assured store of value. So, with this interpretation, and for this purpose, it is its assured store of value, not "liquidity services" per se, which is the crucial attribute of high-powered money. This is, then, perhaps, closer to Tobin's [1958] approach to liquidity preference, than to a, perhaps more monetarist, "liquidity services" interpretation. Moreover, note that, perhaps, indeed, this "Tobinesque" interpretation of liquidity trap should not be rejected out of hand. Indeed, suppose one takes the apocryphal Great Depression stories of currency buried, in tin cans, in the back yard, seriously. It is not at all clear that "liquidity services," per se, are the attraction of a buried currency!

In short, the Great Depression may have had a monetary trigger, but a real source of persistence.

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NOTES

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- 1. The literature treats both continuous and discrete versions [Van Huyck, Battalio and Beil, 1990; Romer 1996, 294-299; Cooper, 1999, viii-xiii, 2-5, 41-46, 60], where this is variously referred to as the "min rule," the "stag hunt" and "Bryant's game."
- 2. The particular allocation of rents is not crucial, but this allocation of the rents to the fabricators facilitates discussion.
- 3. Hence $I^*(1) = I^*$.
- 4. As analyzed by Cooper [1999, 19-25], there may be multiple equilibria in the advanced production technology. Note also that this Pareto dominant equilibrium is not Pareto optimal, as it is in the standard basic parable of production complementarity itself.
- 5. One would further have to address the fact that the holding of a truly pure fiat money, itself, is not secure (albeit involving a less "risky" game [Cooper, 1999] than the min rule).
- 6. Indeed, this simple structure can induce a rudimentary fractional reserve system, in which, in effect, undisrupted fabricators provide their gold backed notes to disrupted fabricators. Even with this sharing of high-powered money, if aggregate gold supply is small enough, general recession remains possible.

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