FIRM INCENTIVES FOR INVENTION PRIZES WITH MULTIPLE WINNERS

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INTRODUCTION

There is a rich and varied literature on the desirability of using prizes (or awards) as an incentive for research, inventions, and greater work effort. An important early article in this area is by Loury[1979].[1] Some of this research has addressed the possibility of multiple prizes [Glazer and Hassin, 1988; Lazear, 1997; Nalebuff and Stiglitz, 1983; O'Keefe, Viscusi, and Zeckhauser, 1984; Skaperdas and Gan, 1995]. Further, much of the work has focused on compensation schemes for workers [Glazer and Hassin, 1988; Nalebuff and Stiglitz, 1983; O'Keefe, Viscusi, and Zeckhauser, 1984; Rosen, 1986]. Others have embedded the analysis in a rent-seeking probabilistic context [Fullerton and McAfee, 1999; Lazear, 1997; Nti, 1997; Rosen, 1986; Skaperdas and Gan, 1995].[2]

This paper considers the behavior of firms competing for a fixed prize offered by a social planner, which fixed prize can be shared equally. Usually, in the literature, with multiple winners, the total prize amount is not fixed, nor is it equally shared. The model is a hybrid of the traditional tournament model, where the best innovation is rewarded by a specific date, and an innovation race, where the reward is given to the first individual to discover a specific innovation in an unlimited time period.[3] We use the functional form shown in Wright [1983], where the individual probability of success is independent of others' efforts. Further, it is possible that the contest results in no winner. However, because the fixed prize is *shared*, the individual firm's expected benefits from innovation effort are dependent upon other firm's efforts. The social planner can consider that behavior in the determination of that welfare-maximizing optimal prize.

We present three models. First, we present a short-run model where the number of firms is fixed, and is independent of the social planner. In that model, the optimal prize may result in the individual firm's expected profits being less than zero. Next, a long-run model is presented wherein entry and exit of firms occurs until expected profits are zero. Finally, we develop a model where the social planner sets both the amount of the prize and the number of firms. As shown there, the expected profits of the firm are greater than zero.

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A SHORT-RUN MODEL

Posit that in a short-run model there are a fixed number of risk-neutral participants competing for the prize awarded by the social planner. Assume the following notation:

B = societal benefit from the innovation;
T = monetary prize offered by the social planner for the invention;
n = number of firms;
c = research expenditure by the individual firm on the development of the invention; and
$p(c)$ = the probability of development of the invention by a certain time by an individual firm, which is an increasing function of c. This probability function is common for all firms.

In this model it is possible that by that specified time, no firm will have developed the invention, and there will be no winner. The probability of that is given as $1\text{-}(1\text{-}p(c_i))^n$ where $i = 1,...,n$. Thus, this model is a hybrid of the innovation race and the tournament. As discussed in Taylor [1995, pp. 873-4], "a tournament rewards the individual with the best innovation on a specific date, while a race rewards the first individual to discover an innovation of a specific quality." In both of those cases, there is a winner. Here we are specifying that a particular innovation be achieved by a time certain. This model is based upon the functional form specified in Wright [1983, 700-01, equation (14)].[4]

However, a major difference between the model here and in those other articles is that in this model: (1) there can be more than one winner; and (2) if there is more than one winner, those winners share in a specific prize amount, which amount does not change with changes in the number of winners.

If just one firm develops the invention that firm receives all of the prize, while if j firms develop the invention they split the prize equally, such that each firm receives T/j.[5] Thus, from the perspective of firm 1, the probability of it winning the entire prize is $p_1(1\text{-}p_2)(1\text{-}p_3)...(1\text{-}p_n)$ where p_i represents the probability of firm i developing the invention, which is dependent upon its expenditure c_i. Furthermore, by way of example, the probability of firm 1 sharing the prize with firm 2 is $p_1p_2(1\text{-}p_3)(1\text{-}p_4)...(1\text{-}p_n)$.

Firm 1's expected gains from participation in the development of the invention is then p_1V_1T where

$$V_1 = \prod_{j=2}^{n}(1-p_j)+[\sum_{i=2}^{n}p_i\prod_{\substack{j=2\\ j\neq i}}^{n}(1-p_j)]/2!+ \tag{1}$$

$$[\sum_{i=2}^{n}p_i\sum_{\substack{j=2\\ j\neq i}}^{n}p_j\prod_{\substack{k=2\\ k\neq i\\ k\neq j}}^{n}(1-p_k)]/3!+\ldots+\prod_{j=2}^{n}(p_j/n),$$

where Π is the Cartesian product. Note that we assume here that each firm has knowledge of the common probability function, and has knowledge of the amount expended by each of the other firms. Because of the sharing of the prize, T, one firm's expected gains from the contest is dependent upon other firms' efforts. Throughout, we assume that the only benefit that a firm receives from the research is the prize, or its share of the prize. While a firm, then, obtains no first-mover advantage, it does receive a greater return if it is the only one, or one among a few firms, to successfully develop the invention.

Firm 1 seeks to maximize

$$p_1 V_1 T - c_1, \tag{2}$$

where c_1 represents the amount that firm 1 spends on research. Let $p'>0$ and $p''<0$ represent the first and second derivatives, respectively, of p with respect to c. The first-order maximization condition for firm 1 is

$$p_1' V_1 T - 1 = 0 \tag{3a}$$

and the second-order condition is $p_1''<0$. At the Cournot-Nash equilibrium, where it is assumed that all firms are identical, and each firm is doing the best that it can given the expenditures of the other firms, the firms spend identical amounts. It can be shown that $V = [1-(1-p)^n]/np$, where p corresponds to the probability of one firm developing the invention when c, determined from equation (3a) above, is spent, and p and c are the same for all firms $i=1,\ldots,n$.

At that Cournot-Nash equilibrium, equation (3a) can be expressed as

$$p' V T - 1 = 0, \tag{3b}$$

which is the private equilibrium condition. By taking the total differential of equation (3b) we can determine the effect of a change in T on the value of c at the Cournot-Nash equilibrium

$$\left(\left[p'\left(\frac{\partial V}{\partial c}\right) + p''V\right]T\right)dc + (p'V)dT = 0, \tag{4a}$$

or

$$\frac{dc}{dT} = \frac{-p'V}{\left[p'\left(\frac{\partial V}{\partial c}\right) + p''V\right]T}. \tag{4b}$$

Note that

$$\frac{\partial V}{\partial c} = np'[np(1-p)^{n-1} - (1-(1-p)^n)]/[np]^2.$$

The probability of *exactly* one firm developing the invention is $np(1\text{-}p)^{n\text{-}1}$, while the probability of *at least* one firm developing the invention is $1\text{-}(1\text{-}p)^n$. Obviously the probability of *at least* one firm developing the invention is greater than the probability of *exactly* one firm developing the invention, so that

$$np(1-p)^{n-1} - (1-(1-p)^n) < 0.$$

Consequently, $\partial V/\partial c < 0$, and from equation (4b), $dc/dT > 0$. The greater the prize, the greater the amount spent on research by individual firms.

Similarly, by taking the total differential of equation (3b) with respect to c and n we produce

$$\text{(5)} \qquad \frac{dc}{dn} = \frac{-p'(\frac{\partial V}{\partial n})}{[p'(\frac{\partial V}{\partial c}) + p''V]}.$$

We assume that there is no asymmetry in information as between the social planner and the private contest participants for two reasons. First, we are not attempting to make a relative comparison of patents and prizes, as was done in Wright [1983]. Second, in our last model we directly compare a long-run decentralized model with a model where the social planner sets the number of participants. In order to properly compare those two models we do not introduce the possibilities of information asymmetries. The social planner, then, has knowledge of the probability functions, the number of participants, and the amount of expenditures by participants.

The social planner will set the value of T at T^* so as to maximize net societal welfare. As discussed earlier, the probability of at least one firm developing the invention is equal to 1 minus the probability that no firm develops the invention, or $1 - (1\text{-}p)^n$. Hence the social planner wants to maximize

$$\text{(6)} \qquad Z = [1-(1-p)^n]B - nc$$

in order to determine the socially optimal value of Z, Z^*. Throughout we assume that the social planner funds the prize through taxes, and is indifferent as to the level of the prize except to the extent that it can be used to influence social welfare. We also assume that the social planner is not directly influenced by the expenditures of the competitors, except to the extent that those expenditures determine the probability of success. The social planner modifies T, which will then affect c and p, until Z in equation (6) is maximized. *The term T does not figure into any social welfare calculations since it is essentially a one-to-one transfer from taxpayers to inventors. It is simply a tool used to maximize Z.*[6] Consequently, its welfare value nets out to zero.

Thus, we take the derivative of equation (6) with respect to T:

$$n(1-p)^{n-1}p'B\left(\frac{dc}{dT}\right)-n\left(\frac{dc}{dT}\right)=0, \tag{7}$$

where dc/dT is defined from equation (4b). Cancelling out the terms n and dc/dT in equation (7) produces

$$(1-p)^{n-1}p'B-1=0, \tag{8}$$

which is the welfare maximization condition. Given n, B, and the probability function $p(.)$, equation (8) can be solved for c to obtain the societal welfare maximizing value of c, or c^*. The private equilibrium condition, equation (3b), can be combined with the welfare maximization condition, equation (8), to yield

$$(p^*)'V^*T=1=(1-p^*)^{n-1}(p^*)'B, \tag{9a}$$

or

$$T^*=\left(\frac{(1-p^*)^{n-1}np^*}{1-(1-p^*)^n}\right)\mathrm{B}, \tag{9b}$$

the welfare-maximizing prize, where p^*,$(p^*)'$, and V^* are evaluated at c^*. Given c^*, determined from equation (8) above, the social planner can then determine the socially optimal value of T and T^*, from equation (9b).[7]

Even though societal welfare is maximized, Z^*, the welfare maximum, may nevertheless be less than zero, which implies that all social welfare gains, $[1-(1-p^*)^n]B$, are dissipated by the expenditures of the n firms, nc^*. Since $V^*=[1-(1-p^*)^n]/np^*$, Z^* from equation (6) can be expressed as

$$Z^*=np^*V^*B-nc^*. \tag{10}$$

This expression is greater than zero when

$$p^*V^*B-c^*>0. \tag{11}$$

Proposition 1. *The value of the optimal prize, T^*, is always less than the societal benefit of the invention, B.*[8]

As discussed earlier, the probability of exactly one firm developing the invention is $np(1-p)^{n-1}$, while the probability of at least one firm developing the invention is $1-(1-p)^n$. Obviously the probability of at least one firm developing the invention is greater than the probability of exactly one firm developing the invention. Consequently,

$$1-(1-p^*)^n > np^*(1-p^*)^{n-1}.$$

Using this result in equation (9b) it can be easily seen that $T^*<B$;[9] that is, the optimal prize will always be less than the societal benefit of the invention, at the social welfare maximum, Z^*.

Proposition 2. *Expected profits are positive (negative) if the elasticity of probability with respect to research expenditures is less than (greater than) one.*

By substituting the welfare-maximizing prize condition, equation (9b), into the term $p^*V^*T^*$ we obtain

$$p^*V^*T^* = p^*(1-p^*)^{n-1}B. \tag{12}$$

Further, multiplication of the welfare maximization condition, equation (8), by p^* yields

$$p^*(1-p^*)^{n-1}(p^*)'B - p^* = 0,$$

or

$$p^*(1-p^*)^{n-1}B = p^*/(p^*)'. \tag{13}$$

Substitution of equation (13) into equation (12) implies that $p^*V^*T^* = p^*/(p^*)'$ and that expected profits are

$$p^*V^*T^* - c^* = p^*/(p^*)' - c^*. \tag{14}$$

If the indicated elasticity, $(p^*)'c^*/p^*$, is $<(>)1$, equation (14) is $>(<)0$.

If firms are enjoying short-run expected profits ($p^*V^*T^*-c^*>0$), and since $B>T^*$, $Z^*>0$. However, the converse is not necessarily true: if $Z^*>0$, firms may or may not have positive expected profits. In the short-run, firms may have either positive or negative expected profits.[10]

LONG-RUN MODEL

In a long-run model assume that firms enter and exit until expected profits are equal to zero[11] yields

$$pVT - c = 0, \tag{15}$$

which is the long-run firm profits condition.

Proposition 3. *In the long-run, the value of c for each firm is independent of T, B, and n.*

Combining the private equilibrium condition, equation (3b), with the long-run firm profits condition, equation (15), produces

$$\frac{p'_L c_L}{p_L} = 1, \tag{16}$$

where c_L is determined from equation (16) and p_L is evaluated at c_L. Equation (16) implies that in a long-run model, the value of c, or c_L, is *independent* of T, B, and n, and that the elasticity of probability with respect to research expenditures is equal to one. Regardless of the level of the prize, firms will enter and exit such that the ultimate amount of expenditure by an individual firm is unchanged.[12,13] This is analogous to simple models of competition wherein firms will enter and exit as the level of profits (expected benefits here) changed; although the number of firms changes the size of the firm (amount of research expenditure here) does not.

However, the number of firms is dependent on T.[14] As in the short-run model, the social planner maximizes Z with respect to T to yield

$$(1 - (1 - p)^n \ln(1 - p)B - c_L)\frac{dn}{dT} = 0,$$

since $dc_L/dT = 0$. This can be re-expressed as

$$1 - (1 - p)^n \ln(1 - p)B - c_L = 0. \tag{17}$$

Given c_L, determined from equation (16), and B, the socially optimal value of n, n_L, can be determined. In turn those values of c_L and n_L can be utilized in either equations (15) or (16) to determine the socially optimal value of T, T_L.

Proposition 4. *In the long-run model, optimal net societal welfare, Z_L, is always positive.*

Since $B>T^*$ for any value of n, as shown in the short-run model, by equation (15), $p_L V_L B - c_L > 0$, where p_L and V_L are evaluated at c_L and n_L. From equations(10) and (11), societal benefits are always *positive* in this long-run model, and there is no dissipation of benefits as occurs in some rent-seeking models.[15]

DETERMINATION OF THE NUMBER OF FIRMS BY THE SOCIAL PLANNER

In this Section we assume that the social planner determines *both* the prize and the number of firms that can participate. We will also hypothesize that there are always sufficient firms wanting to participate such that the social planner will never be deficient in the desired number of firms. This will occur if expected profits are positive (which is the case here as will be shown later).

The social planner seeks to maximize $[1-(1-p)^n]B$ - nc with respect to n and T. This results in the following two maximization conditions

(18a) $$[-(1-p)^n \ln(1-p)B - c] + n[(1-p)^{n\text{-}1} Bp' - 1]\frac{dc}{dn} = 0,$$

(19) $$(1-p)^{n\text{-}1} Bp' - 1 = 0,$$

where dc/dn is from equation (5),and equation (19) is the welfare-maximizing condition, equation (8). However, the second term on the left-hand side of equation (18a) drops out, because of equation (19), so that equation (18a) becomes

(18b) $$-(1-p)^n \ln(1-p)B - c = 0.$$

Equation (18b) can be re-expressed as

(18c) $$(1-p)^{n-1} = \frac{-c}{(1-p)\ln(1-p)B},$$

which can be substituted into equation (19) to produce

(20) $$\frac{-p'c}{(1-p)\ln(1-p)} - 1 = 0.$$

Equation (20) indicates that, in this model, the socially optimal value of c, c_G, is independent of the socially optimal value of n, n_G, social benefits, B, and the socially optimal prize, T_G.[16] The social planner can maximize welfare by adjusting the number of firms allowed to compete and the prize, but, ultimately, has no control over the "size" (amount of research expenditures) of the firm. This is similar to the result obtained in the long-run model. However, the socially optimal values of firm expenditures, c_L and c_G, are not the same in the two models.

Proposition 5. *The long-run amount of research expenditure by an individual firm, c_L, is less than the socially optimal expenditure amount c_G.*

It is easily shown that for any value of p (where $0<p<1$), $p > -(1-p)\ln(1-p)$, and

(21) $$\frac{1}{p} < \frac{-1}{(1-p)\ln(1-p)}$$

or

(22) $$p'\frac{c}{p} < \frac{-p'c}{(1-p)\ln(1-p)}.$$

All of this implies

(23) $$\frac{p'_G c_G}{p_G} < \frac{-p'_G c_G}{(1-p_G)\ln(1-p_G)} = 1 = \frac{p'_L c_L}{p_L}$$

by equation (20). Note that p_G', p_G, p_L', and p_L are evaluated at c_G and c_L as the case may be.

Note that $d(p/p')/dc = 1 - pp''/(p')^2 > 1$ for $p''<0$. Consequently, $d(p/p')/dc > dc/dc = 1$. For all $c > c_L$, $(p/p')/c > 1 = (p_L/p_L')/c_L$, and for all $c < c_L$, $(p/p')/c < 1 = (p_L/p_L')/c_L$. By Equation (23), $(p_G/p_G')/c_G > (p_L/p_L')/c_L$, so that $c_L < c_G$. The optimal value of c with long-run free entry of firms is less than the optimal value of c when the social planner determines the optimal number of firms.[17]

It would be useful to know whether the total expenditures by firms in the long-run model is greater than the total firm expenditures in the model where the social planner sets the number of firms. As will be shown below, that will be dependent upon the sign of the cross-partial derivative of the expected value of benefits, $[1-(1-p)^n]B$ with respect to c and n.

Let us define $d[1-(1-p)^n]B/dc$ as the marginal effectiveness of the representative firm's research expenditure. Effectiveness is defined in terms of expected value of benefits.

Proposition 6. *The total expenditure in the long-run model, $n_L c_L$, is greater (less) than the total expenditure, $n_G c_G$, in the model where the social planner sets the number of firms, as the marginal effectiveness of the firm's research expenditure decreases (increases) with the number of firms.*

Note that in both models the conditions given by equations (17) and (18b) are equivalent

$$-(1-p)^n \ln(1-p)B - c = 0.$$

We can take the total differential of the above equation, common to both models, and obtain

$$[n(1-p)^{n-1} p' \ln(1-p)B + (1-p)^n p'B/(1-p) - 1]dc + [-\ln^2(1-p)B(1-p)^n]dn. \quad (24)$$

Solving for dn/dc expresses the required relationship between n and c in the two models

$$\frac{dn}{dc} = \frac{-n(1-p)^{n-1} p' \ln(1-p)B - (1-p)^{n-1} p'B + 1}{-\ln^2(1-p)B(1-p)^2}. \quad (25)$$

Also note that in both models

$$c = -(1-p)^n \ln(1-p)B \quad (26)$$

from equations (17) and (18b). Using equation (25) with equation (26) we can determine the change in total expenditures, nc, between the models as c changes

$$\frac{d(nc)}{dc} = n + c\left(\frac{dn}{dc}\right) = -[(1-p)^{n-1} p'B - 1][n + (1/\ln(1-p))]. \quad (27)$$

The right-hand side of equation (27) is less than zero if and only if

$$-[(1-p)^{n-1} p'B - 1][\ln(1-p)][1 + n\ln(1-p)] < 0. \quad (28)$$

The expression $(1\text{-}p)^{n\text{-}1}p'B\text{-}1$ is greater than zero in the range of c_L to c_G by second-order conditions on equation (19) and because $c_L < c_G$ by Proposition 5. Further, $\ln(1\text{-}p)$ is clearly less than zero. It can be easily shown that the expression $1 + n\ln(1\text{-}p)$ is less (greater) than zero as

$$\frac{d^2[1-(1-p)^n B]}{dn\,dc} < (>) 0. \quad (29)$$

The cross-partial derivative term on the left-hand side of the inequality in equation (29) is the rate of change in the marginal effectiveness of the firm's expenditure as the number of firms increases. If that term is negative, then the model of this section results in smaller expenditures than in the long-run model from the previous section.

Proposition 7. *When the social planner sets the number of firms and the prize, firms will always expect positive profits.*

An obvious concern here is whether the firms involved expect to earn positive profits. If not, then this model is not a feasible one for the social planner to undertake. From equation (14) the firm's profits, at optimum, in this case are

$$p_G V_G T_G - c_G \;=\; \frac{p_G}{p_G'} - c_G,$$

where V_G is evaluated at c_G and n_G.

As shown in equation (23), $p_G' c_G / \mathrm{p}_G < 1$, which implies that $p_G VT - c_G = p_G/p_G' - c_G > 0$. Therefore, firms expect to earn positive profits in this model. Furthermore, since $B > T_G$, societal benefits are not dissipated here.

Finally, the net societal welfare is greater than in the short-run and long-run models from the earlier sections, since the social planner has an additional policy instrument, number of firms, not available in those models.

CONCLUSION

This paper considered several multiple winner models with invention prizes, where the social planner determines the social welfare-maximizing prize. In all of the models the prize was always less than the societal benefit from the invention.

The short-run model, with a fixed number of firms, can result in negative expected profits and, conceivably, a negative expected societal benefit, where the welfare gains are totally dissipated by firms' research expenditures. In the long-run model, where firms can enter and exit, it was shown that there is always a positive net welfare gain. Furthermore, the optimal research expenditure by an individual firm is invariant to the amount of the prize. Finally, we considered a model where the social planner set the prize and the number of firms. That model results in smaller total research expenditures than in the long-run model under certain conditions. It also represents the preferred policy for the social planner, since it results in the largest societal welfare (of the three models considered), with positive expected profits for each firm.

NOTES

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1. Other examples are Baik [1993], de Laat [1997], Hughes [1945; 1946], Lee and Wilde [1980] Rogerson [1989], Tirole [1988], and Wright [1983].
2. See Corcoran [1984], Corcoran and Karels [1985], Tullock [1967; 1980], and Wenders [1987] for discussion of the analyses of rent-seeking activity.
3. See Taylor [1995, 873-4]. A variation on the tournament model is the cooperative public goods case, discussed in Romer [1993].
4. As compared with Wright [1983], this paper considers a long-run formulation and a model where the social planner sets the number of competitors.
5. This is similar to the approach taken in Berry [1993].
6. Of course, this is not true in the case of a private sponsor for the contest. That case is not considered in this paper.
7. It is easily shown that $p'' < 0$ is a sufficient condition for second-order conditions to be met. The second derivative of equation (7) with respect to T is

   $$[n(1-p)^{n-1}p'B - n](dc^2/d^2T) + [n(1-p)^{n-1}p'' - n(n-1)(1-p)^{n-2}(p')^2](dc/dT)^2B.$$

   The first term in brackets in the above expression is equal to zero by the welfare-maximizing condition, equation (8). The second term in brackets will be less than zero if $p''< 0$. Hence, if $p'' < 0$, the second-order condition is met.
8. As discussed earlier, the term T is essentially a one-to-one transfer from taxpayers to inventors, and is a tool used to maximize Z. Consequently, it is not necessary that $T^*<Z^*$.
9. This is consistent with the discussion of the "common pool" problem discussed in Dasgupta and Siglitz [1980], and Wright [1983].
10. In the traditional rent-seeking model of Tullock [1980], negative profits occur if the marginal efficiency of expenditure, r, is greater than $n/(n-1)$.
11. The long-run is discussed in Loury [1979] and Taylor [1995], as well as in a rent-seeking context in Corcoran [1984] and Corcoran and Karels [1985].
12. Equation (16) implies that $c = p/p'$. Since $d(p/p')/dc = [(p')^2 - pp'']/(p')^2 > 1$ for all values of c if $p''<0$, that means that $d(p/p')/dc > 1 = dc/dc$ for all values of c. Consequently the function p/p' crosses the 45° line function c only one time, and that there is only one value of c that satisfies $p'c/p = 1$.
13. The primary reason for this is that the probability functional form for the individual firm's probability of success can be separated from the probability of success of the other firms. In contrast, in the Corcoran long-term model, the number of firms is independent of the amount of rents and the amount of rent-seeking expenditures [Corcoran, 1984].
14. The impact of T on n can be determined by taking the total differentials of equations (15) and (16) with respect to dc, dn, and dT, and solving for dn/dT. It can be shown that the result is $dn/dT = V/[T(dV/dn)]$. Further, the value of n may be fractional in this model,
15. In the traditional rent-seeking models, total dissipation always occurs in the long-run (see Corcoran [1984]).
16. As in the prior Section, the optimal value of n may be fractional.
17. Proposition 5 is similar to results obtained in Taylor [1995]. However, in Taylor's analysis a "tournament" model is used, and there is no sharing of the prize among winners.

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