The Dubious Case for Decreasing Costs

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Many textbook authors complete their discussions of long-run supply schedules by recognizing the possibility of "decreasing cost industries." Noting the incompatibility of internal economies and competition, authors frequently attribute decreasing costs to external economies that arise from input prices that decline as industry output expands. Pushing the explanation back to the behavior of input prices naturally raises the question of the cause of decreasing costs for input suppliers. External economies in the output industry cannot be ascribed to external economies in the input industry since this involves infinite regress, and the existence of internal economies in the input industry implies monopoly. Thus, the absence of alternative explanations in many expositions suggests that authors picture inputs as being supplied by monopolists who experience internal economies and sell larger quantities at lower prices. In this article it is argued that a general case for decreasing costs cannot be based on this circumstance.

The argument against attributing decreasing costs in an output industry to internal economies experienced by a monopolistic input supplier consists, first, of a demonstration in Section I that there is no output supply curve if inputs are bought from a monopolist and that this circumstance, therefore, cannot provide the basis for the negatively sloped long-run supply curve of a decreasing cost industry. Second, since this demonstration may appear to dispose of a substantive question by an appeal to a definition, a less rigorous interpretation of decreasing costs is considered in the following section. Here it is shown that, aside from the fact there is no output supply curve when inputs are produced monopolistically, the existence of decreasing costs in the input industry is not sufficient to ensure that a competitive output industry can produce a larger output at a lower price. In fact, the quantity of a competitively produced commodity may *decline* when the demand for it increases if that commodity is produced with a monopolistically supplied input, even if the input is produced at declining marginal cost. Conclusions appear in the final section.

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I. Monopoly Input Pricing and Competitive Supply

In the following model of a competitive industry that buys an input from a monopolist, we make the following assumptions:

(a) the demand for product X is represented by

$$p_X = f(x, \lambda) \qquad (1)$$

where x is the quantity of product X, λ is a shift parameter, the partial derivative f_x is negative, and the partial derivative f_λ is positive;

(b) X is produced using two inputs, A and B. The X industry is competitive and is the sole purchaser of input A. All firms producing X

are identical. Furthermore, for each such firm there is a unique level of output, independent of industry output, at which average cost is minimum;[1]

(c) input A is produced by a monopolist; and

(d) input B is supplied to the X industry competitively and at constant long-run supply price.

The assumptions in (b) imply that

$$x = H(a, b) \tag{2}$$

is the X industry's production function where x is quantity produced, and a and b are the amounts of the inputs A and B used by the entire X industry. Furthermore, since average cost is minimum at the same level of output for all firms, a change in output demand that results in, say, a doubling of the industry's output will result in a doubling of the number of firms in the industry and, therefore, of a and b. Therefore H is homogeneous of degree one.[2] Finally, since linear homogeneity implies that the isoquants corresponding to H are strictly convex within the relevant ranges of input usage, partial derivatives have the following signs:

$$H_a, H_b, H_{ab} > 0,$$

and

$$H_{aa}, H_{bb} < 0.$$

Long-run competitive equilibrium for the X industry implies that profit maximization will yield zero long-run profits across the industry, i.e.,

$$xp_X = p_A a + p_B b \tag{3}$$

and

$$p_A / p_B = H_a / H_b. \tag{4}$$

Assumption (d) above implies that

$$p_B = \bar{p}_B. \tag{5}$$

Equations (1)-(5) can be used to derive the X industry's demand for A, which is the demand faced by the monopolistic supplier of A since the X industry is the sole purchaser of A. Profit maximization by the monopolist requires that

$$R(a, p_B, \lambda) = C(a), \tag{6}$$

where R is the function describing the monopolist's marginal revenue curve and C is his marginal cost function. We require that the second-order condition for profit maximization be fulfilled in equilibrium, i.e.,

$$R_a - C_a < 0.$$

The resulting system of six equations can be solved for x, a, b, p_X, p_A, and p_B, yielding the long-run equilibrium values. An increase in the parameter λ will result in new equilibrium values of x and p_X. Furthermore, if the monopolistic supplier of input A experiences decreasing costs (if $C_a < 0$), it is possible that a higher value of x will be associated with a lower $p_X (dp_X / d\lambda < 0)$. It is not correct, however, to interpret the new equilibrium pair as lying on a declining X industry long-run supply curve. This is so because there *is no X industry supply curve* in this model. That is, the system of equations (1)-(6) allows us to determine the quantity of X that will be supplied for any given output demand *function* (1), but not for any given output *price*, as required by the definition of supply. A demand function for X is necessary for solving the system because a demand curve for A, which is necessary for deriving (6), cannot be derived without knowing the whole demand function for X. Knowing just the price of X is not sufficient. Thus, although internal economies experienced by a monopolistic input producer may well result in a fall in output price being associated with a rise in output quantity as demand increases, this cannot be interpreted as a negatively sloped output supply curve.

1 When the expenditure elasticity of an input is equal to one, the minimum average cost output remains unchanged in response to changes in the price of that input [3, 6]. The definition of expenditure elasticity implies that its value will equal one for all inputs in the case of a homogeneous production function regardless of the degree of homogeneity. Therefore, a sufficient condition for our assumption to be true is that each firm's production function be homogeneous of any degree.

2 Assuming conditions that imply a linearly homogeneous production function for a perfectly competitive industry is not uncommon in the literature. [1; 4, p. 84; 8; 9, pp. 209–12].

II. Perverse Response of Output to Changes in Demand

As noted above, it is possible for an increase in the demand for X to result in a decline in the price of X. Aside from the fact that this possibility cannot be interpreted as declining supply price, it is not even correct that decreasing marginal costs in the input supplying industry are a sufficient condition to ensure that an increase in the demand for X will produce a decline in the price of X. It depends, of course, on whether marginal costs in the A industry decrease sharply enough to offset the increased difference between price and marginal cost that may result from a demand increase. If marginal costs do not decrease sufficiently, then an increase in demand for X will produce a rise in both the monopolistic price of A and the price of X in spite of A's declining marginal costs.

It is also possible, but less obvious, that an increase in the demand for X may result in a *decline* in the quality of X produced and an *increase* in its price even if industry A experiences decreasing marginal costs. We use the model developed in the previous section to demonstrate this possibility.

Substituting equations (1), (4), and (5) into equations (2), (3), and (6) yields the following three equations in the three variables x, a, and b:

$$xf(x, \lambda) - a\bar{p}_B H_a / H_b - \bar{p}_B b = 0 \tag{7}$$

$$x - H(a, b) = 0 \tag{8}$$

$$R(a, \bar{p}_B, \lambda) - C(a) = 0 \tag{9}$$

Taking the total differential of each of these equations then produces:

$$(f + xf_x)dx - \left[\frac{H_a \bar{p}_B}{H_b} + \frac{H_{aa}\bar{p}_B a}{H_b} - \frac{H_a H_{ba} \bar{p}_B a}{H_b^2}\right] da - \left[\frac{H_{ab}\bar{p}_B a}{H_b} - \frac{H_a H_{bb} \bar{p}_B a}{H_b^2} + \bar{p}_b\right] db = -xf_\lambda d\lambda \tag{10}$$

$$dx - H_a da - H_b db = 0 \tag{11}$$

$$R_a da - C_a da = -R_\lambda d\lambda \tag{12}$$

In matrix form, this system of differential equations can be written as follows:

$$\begin{bmatrix} f + xf_x & \frac{\bar{p}_B}{H_b^2}(H_a H_{ba} a - H_{aa} H_b a - H_a H_b) & \bar{p}_B \left(\frac{H_a H_{bb} a}{H_b^2} - \frac{H_{ab} a}{H_b} - 1\right) \\ 1 & -H_a & -H_b \\ 0 & R_a - C_a & \end{bmatrix} \begin{bmatrix} dx \\ da \\ db \end{bmatrix} = \begin{bmatrix} -xf_\lambda d\lambda \\ 0 \\ -R_\lambda d\lambda \end{bmatrix} \tag{13}$$

Let Δ represent the determinant of the matrix of coefficients and let z_{ij} be the element in the ith row and jth column of this matrix. Then,

$$\Delta = (R_a - C_a)(z_{13} + H_b [f + xf_x]). \tag{14}$$

Note that the second-order condition for profit maximization by the monopolist producing A requires that $(R_a - C_a)$ be negative. Furthermore, since $\bar{p}_B / H_b = f$ and $H_{ab} = -bH_{bb}/a$,

$$z_{13} = \bar{p}_B (H_a H_{bb} a / H_b^2 - H_{ab} a / H_b - 1) = fH_{bb} x / H_b - p_B. \tag{15}$$

Therefore,

$$z_{13} + H_b (f + xf_x) = x(fH_{bb}/H_b + H_b f_x), \tag{16}$$

which is always negative. Hence, Δ must always be positive.

Solving (13) for dx and da using Cramer's rule yields:

$$\frac{dx}{d\lambda} = \frac{R_\lambda(z_{12}\mathbf{H}_b - z_{13}H_a) - H_b x f_\lambda(R_a - C_a)}{(R_a - C_a)\, x(fH_{bb}/H_b + H_b f_x)} \quad (17)$$

and

$$\frac{da}{d\lambda} = -\frac{R_\lambda}{R_a - C_a} \quad (18)$$

Clearly $da/d\lambda$ will be negative only when (but always when) R_λ is negative, i.e., only when the marginal revenue curve for A shifts in the opposite direction from the demand curve for X. That this is possible follows from the fact that marginal revenue curves do not necessarily shift in the same direction as their demand curves.[3]

The cases for which $dx/d\lambda$ is negative are more restricted but nevertheless possible within the assumptions of our model. First of all,

$$z_{12}H_b - z_{13}H_a = -H_{bb}x^2 f^2/\bar{p}_B a \quad (19)$$

which is always positive. Furthermore,

$$H_b x f_\lambda(R_a - C_a) < 0 \quad (20)$$

Therefore, $dx/d\lambda$ will be negative only when

$$R_\lambda < \frac{H_b x f_\lambda(R_a - C_a)}{z_{12}H_b - z_{13}H_a} < 0. \quad (21)$$

Using (19) and the fact that the elasticity of substitution between A and B can be expressed as[4]

$$\sigma = H_a H_b / H_{ab} x, \quad (22)$$

this condition reduces to:

$$R_\lambda < \frac{f_\lambda}{f}\, b\, \frac{H_b}{H_a}\, \sigma(R_a - C_a) < 0, \quad (23)$$

which is equivalent to requiring that

$$\frac{da}{d\lambda} < -\frac{f_\lambda}{f}\, b\, \frac{H_b}{H_a}\, \sigma < 0, \quad (24)$$

i.e., that $da/d\lambda$ not only be negative, but also be sufficiently large in absolute value that the profit level of each X-producing firm will fall below zero (since p_A is simultaneously rising). The decline in the quantity of A relative to the increase in the price of X that is sufficient to produce this reduction in profits can be found by multiplying (24) by $-f/f_\lambda a$ to obtain

$$-\frac{da/a}{f_\lambda d\lambda/f} > \frac{bH_b}{aH_a}\sigma > 0. \quad (25)$$

Thus, given a positive change in λ, X will fall if the ratio of the resulting (negative) percentage change in A to the percentage change in p_X is greater than the product of the relative shares of the inputs and the elasticity of substitution.[5]

III. Conclusions

Many text authors attribute negatively sloped long-run industry supply curves to input prices that decline as industry usage increases. It may be true that a competitive industry can buy larger quantities of an input at lower prices if the input is produced under decreasing cost conditions by a monopolist. Section I shows, however, that there is no long-run industry supply curve in this case and that the case, therefore, fails to provide an explanation for a negatively sloped industry supply curve.

Defining a decreasing cost industry as one that has a negatively sloped long-run supply curve seems consistent with the usual definitions of constant and increasing cost industries. Some authors, however, appear to give the decreasing cost label to a circumstance in which one equilibrium has a lower price and higher quantity than another, even if the two combinations do not lie on a correctly defined supply curve. Even under this less rigorous definition, an explanation for decreasing costs that relies on inputs being produced under decreasing costs by a monopolist encounters difficulties. It has been shown in Section II that a competitive industry may respond perversely to an increase in demand even if it buys inputs from a decreasing cost monopolist.

An explanation of decreasing costs that is frequently employed by text authors has been found wanting. In the Viner [10] classification, this leaves external economies that are technological rather than pecuniary as a possible explanation. Yet technological economies that can be attributed solely to the increased output of one industry are probably not sufficiently common to provide a basis for a general case of decreasing costs. In fact, authors who rely on this explanation almost invariably slip into examples of what Marshall called "those external economies which result from the general progress of industrial environment" [5, p. 365]. Thus, instead of straining to produce the economic curiosity of decreasing costs in the static context that is proper to microeconomics, authors should limit the list of possibilities to increasing and constant cost industries while saving explanations of historically declining costs for books on development, where the phenomenon can be correctly attributed to events that occur with the passage of time rather than with increases in output.

[3]Consider a monopolist facing the demand curve $p_X = f(x, \lambda)$. Then $MR = f + xf_x$ and

$$\frac{\partial MR}{\partial\lambda} = f_\lambda + xf_{x\lambda}.$$

Letting $\eta = f/xf_x$ be the price elasticity of demand, the preceding expression will be positive only when

$$\frac{f_\lambda}{f} > \left(-\frac{1}{\eta}\right)\frac{f_{x\lambda}}{f_x},$$

or, multiply by $d\lambda$, when

$$\frac{f_\lambda d\lambda}{f} > \left(-\frac{1}{\eta}\right)\frac{f_{x\lambda}d\lambda}{f_x},$$

i.e., when the percentage vertical shift is greater than the product of $(1/\eta)$ and the percentage change in slope. Thus, if the demand curve becomes sufficiently steeper at the appropriate quantity as it shifts up, marginal revenue at that quantity will fall. Boulding mentions the disappearance of a substitute good as an example of when this might happen [2, pp. 459-60]. Note that if f is a linear function of x both before and after the shift (which means that $f_{x\lambda} = 0$), then $\partial MR/\partial\lambda = f_\lambda$ at every quantity and marginal revenue will always shift in the same direction as price.

[4]See Henderson and Quandt, p. 86, for the proof that this is true for all linear homogeneous production functions.

[5]Note that if H is such that there are no possibilities for substitution between A and B, then $\sigma = 0$, and $dx/d\lambda$ will be negative whenever $da/d\lambda$ is negative.

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