References


Appendix Tables

The following set of 9 Appendix Tables is available on request. These Tables contain the marginal (trend coefficients $a_i$, $b_i$, and mean values $\bar{y}$ (per cent).

**Appendix Table 1** Export Market Shares of Food Items (SITC 0 + 1 + 22 + 4), 1960-1969.

**Appendix Table 2** Export Market Shares of Agricultural Raw Materials (SITC 2 - (22 + 27 + 28), 1960-1969.

**Appendix Table 3** Export Market Shares of Crude Fertilizers, Crude Minerals, Metaliferous Ores and Metal Scrap (SITC 27 + 28), 1960-69.

**Appendix Table 4** Export Market Shares of Fuels and Lubricants (SITC 3), 1960-69.

**Appendix Table 5** Export Market Shares of Chemical Products (SITC 5), 1960-69.

**Appendix Table 6** Export Market Shares of Iron and Steel (SITC 67), 1960-69.

**Appendix Table 7** Export Market Shares of Non-Ferrous Metals (SITC 68), 1960-69.

**Appendix Table 8** Export Market Shares of Manufactured Goods Excluding Iron and Steel and Non-Ferrous Metals (SITC 6 + 8) - (67 + 68).

**Appendix Table 9** Export Market Shares of Machinery and Transport Equipment (SITC 7), 1960-69.

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Real Inputs, Retained Earnings, and Optimizing Behavior of the Financial Intermediary

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The renewed interest in monetary economies of the past two decades has brought forth numerous theoretical models analyzing the behavior of the financial intermediary. Many of these models have dealt specifically with the behavior of commercial banks in both asset and liability markets. Recently, economists have expanded their scope of analysis to include all types of financial firms. For example, several theoretical models have been developed to analyze the actions of the savings and loan association [1, 2, 3, 6, 8, 10]. The major thrust of these analyses concerns the deposit rate setting decision for the firm. The firm is typically considered as a monopolist in deposit markets facing a known demand function for its liabilities. The key decision for the firm is to choose a deposit rate that attracts an optimum level of deposits. The proceeds of the liability issues are then used to purchase assets in the form of mortgages. One major objective of this type of analysis has been to pinpoint key variables to be used in econometric deposit rate equations for the Fed-MIT-Penn model of the economy. The most extensive work done in this area has been performed by Slovin [8]. However, in the majority of the theoretical models, the real input decisions of the firm are ignored. The financial intermediary exists for the purpose of accumulating funds from surplus units in the economy and transferring them to deficit units. Its existence is based upon a profitable process of borrowing and lending funds in a more efficient manner than individuals can do themselves. This fundamental intermediation process necessarily requires the firm to hire real resources in the form of labor and capital. Previous theoretical analyses that have ignored this aspect of behavior give a one-sided analysis of the intermediary. The purpose of this paper is to present a new model of the financial intermediary that integrates the financial and real resource decisions of this type of firm into a single framework. This task is accomplished by relying upon the standard tools of microeconomic analysis.

**Construction of the Model**

**The Objective Function**

The model views the firm operating in a multi-period time horizon with all parameters known with certainty. The objective of the firm is the maximization of the discounted present value of the dividend stream to owners plus the discounted present value of the net worth of the firm at the end of the time horizon. It is assumed that the net worth would be reflected in the future market value of the firm at that point in time.

1 A limited number of models address the concept of transactions costs and the behavior of the financial intermediary [7, 9]. However, none of these explicitly addresses the problem of optimum real input usage in the manner presented in this paper.
particular time. Thus, the firm attempts to maximize the following objective function:

\[
V = \left( \frac{r_f - r_{1r} - r_{1r} - r_C - r_{1r} - r_C}{(1 + r_f)} \right) \frac{E}{(1 + r_f)} \]

where:

- \( V \) = discounted present value
- \( M_f \) = mortgages held at the end of the period
- \( S_t \) = savings deposits outstanding at the end of the period
- \( C_t \) = certificates of deposit outstanding at the end of the period
- \( K_t \) = retained earnings at the end of the period
- \( E \) = value of the firm at the end of the period
- \( r_f \) = interest rate on mortgages held at the end of the period
- \( r_{1r} \) = interest rate on savings deposits outstanding at the end of the period
- \( r_{1r} \) = interest rate on certificates of deposit outstanding at the end of the period
- \( r_{1r} \) = labor services purchased during the period
- \( k_t \) = capital services purchased during the period
- \( r_f \) = wage rate of labor
- \( r_{1r} \) = rental price of capital
- \( r \) = owners' opportunity cost (cost of equity)

Constraints Faced by the Firm

The firm is assumed to face two important types of constraint in each time period. The first of these may be called the "budget constraint," given by:

\[
(M_f - M_{f-1}) - (S_t - S_{t-1}) - (C_t - C_{t-1}) - R_t = 0
\]

This equation states that the firm's planned additions to assets minus its planned additions to liabilities must be equal to its planned additions to retained earnings for each period. Thus, the model explicitly deals with retained earnings as a source of finance in each period. This implies that it is possible for the firm to increase simultaneously its demand for mortgages and reduce its existing supply of liabilities. This formulation of the constraint differs from that of previous studies that restrict the firm to a zero level of wealth accumulation [1, 2]. The model precludes the issuance of new common stock as a source of funds. For simplicity, the budget constraint of the final period of the time horizon is assumed to be different from that of previous periods. It is arbitrarily stipulated that the firm hold the same balance sheet at the end of the second period as it held at the end of the third period. This essentially means that the firm cannot plan beyond its time horizon and thus plans to liquidate at that point in time. This contrasts with the assumption of an infinite time horizon found in many financial models of the firm.

The second major constraint faced by the firm is that of a transactions function. In order to account for the intermediation process, the model assumes that the firm requires a minimal amount of administrative services (transactions) during each period to construct its desired balance sheet. The intermediation requires these transactions to evaluate mortgages and service liabilities. The transactions are assumed to be homogeneous in nature and are given according to the following function:

\[
T_t = \text{Transactions} = T(M_f, S_t, C_t)
\]

All first and second partials of the above function are assumed to be positive. Positive second partials mean that, ceteris paribus, required transactions for increases in assets and liabilities increase at an increasing rate. For convenience purposes cross partials are assumed to be zero.

The administrative transactions required to construct the firm's desired balance sheet are generated via an assumed intermediation production function. Contrary to the typical function of standard theory, this intermediation function generates not a final product for sale but the required transactions for purchasing assets and issuing liabilities. It takes the following form:

\[
T_t = f(o_t, k_t)
\]

This function depicts the maximum amount of transactions in each period that can be generated with given amounts of labor and capital services. In the same manner as the traditional theory of the firm, the function is assumed to be strictly concave. Equations (3) and (4) quite simply represent the real resource behavior ignored in previous models. These constraints can be combined to form the intermediary's real resource constraint:

\[
f(o_t, k_t) = T(M_f, S_t, C_t)
\]

This formulation of the constraint has the advantage of bypassing the problem of defining the "output" of the financial firm. In many previous empirical studies, definition of output in terms of deposits and/or loans has gained much attention (and controversy). Equation (5) simply implies that the output of a financial firm is financial intermediation. It is this constraint along with the form of the budget constraint that differentiates this model from all previous models.

Stable Behavior of the Firm

For expository purposes, it is assumed that the firm has a three period time horizon. By combining equations (1), (2), and (3), the basic problem of the firm is obtained, namely, maximization of the discounted present value of the income stream to owners subject to each period's financial and real resource constraints.

This function can be written in the form of a Lagrangian as:

\[
L = r_f M_f - r_S S_t - C_t + r_{1r} M_{f-1} - r_{1r} S_{t-1} - C_{t-1} + R_t
\]

This is a function of the maximum amount of transactions in each period that can be generated with given amounts of labor and capital services. In the same manner as the traditional theory of the firm, the function is assumed to be strictly concave. Equations (3) and (4) quite simply represent the real resource behavior ignored in previous models. These constraints can be combined to form the intermediary's real resource constraint:

\[
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See the Appendix for the development of the basic equations of the model.
\[
\begin{align*}
\frac{r_t - r_1}{1 + r_1} + \lambda_4 \frac{\Delta T}{\Delta M_1} &= 0 \quad (7) \\
-\frac{r_1}{1 + r_1} + \lambda_5 \frac{\Delta T}{\Delta M_1} &= 0 \quad (8) \\
\frac{r_f}{1 + r_1} + \lambda_6 \frac{\Delta M_1}{\Delta C_1} &= 0 \quad (9) \\
-w_1 - \lambda_7 \frac{\Delta T}{\Delta M_1} &= 0 \quad (10) \\
-p_1 - \lambda_8 \frac{\Delta T}{\Delta M_1} &= 0 \quad (11) \\
T(M_1, S_1, C_1) - f_0 (1, k_1) &= 0 \quad (12)
\end{align*}
\]

It is assumed that a solution for this system of six equations in six unknowns exists. The equations define the demand and supply behavior of the intermediary in asset, liability, labor, and capital markets. By means of some further substitution for the value of \( \lambda_8 \), the following optimal conditions are obtained:

\[(i) \quad \frac{r_t - r_1}{1 + r_1} + \frac{w_1}{\Delta M_1} \frac{\Delta T}{\Delta M_1} = 0
\]

\[(ii) \quad \frac{r_f}{1 + r_1} + \frac{w_1}{\Delta C_1} \frac{\Delta M_1}{\Delta C_1} = 0
\]

\[(iii) \quad \frac{r_t}{1 + r_1} + \frac{w_1}{\Delta C_1} \frac{\Delta M_1}{\Delta C_1} = 0
\]

These variations of the first order conditions can be interpreted in the following manner:

(i) The firm demands mortgage during the current period to the point where the discounted present value of the interest cost of deposits outstanding at the end of the period plus the current marginal labor cost of deposits equals the discounted present value of marginal mortgage revenue minus the marginal labor cost of mortgages.

Similar conditions can be shown to hold for capital services.

Conditions (ii) and (iii) can be stated in perhaps a simpler and more familiar manner. The intermediary desires to supply its optimum amount of liabilities to the point where the gross marginal cost of liabilities is equal to the net marginal revenue of mortgages. These familiar marginal conditions describing the static behavior of the firm point to a distinguishing feature of the model. Whereas conventional models recognize the interest rate as the revenue (cost) of an asset (liability), the new model brings out the important concept of a net yield and a gross cost. In other words, recognition is given to the fact that the intermediary incurs costs in hiring real resources that provide the necessary services for constructing the balance sheet. These conditions may give some insight into real world thrift institutions such as savings and loans increased reliance upon longer term certificates of deposit. These liabilities carry a higher interest rate than passbook accounts. However, the firm may be induced to use certificates if there is a lower marginal transactions cost (including information cost associated with deposit turnover and future interest expense) associated with using this source of funds relative to passbook accounts.

Equations (10) and (11) define the firm's optimal behavior in factor markets. By rewriting these equations and employing a bit of economic intuition, the familiar conditions of competitive hiring are obtained. Substitution of the value of \( \lambda_8 \) given in (7) into (10) gives the following expression for optimal labor service purchases:

\[
w_t = \frac{r_t - r_1}{1 + r_1} \frac{\Delta T}{\Delta M_1}
\]
TABLE 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>M_1</th>
<th>S_1</th>
<th>C_1</th>
<th>( \alpha_1 )</th>
<th>( \beta_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_1 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
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<td>+</td>
<td>+</td>
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<tr>
<td>( \sigma_3 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \sigma_4 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

This form of the system shows the coefficient matrix to be negative definite which is a necessary condition for a maximum. Evaluation of this system of linear equations produces the signs of the derivatives predicting the firm's behavioral reactions to changes in the current period's parameters. The solution of this system is found in Table 1. Analysis of these results shows the intermediary characterized by an upward sloping demand curve for mortgages as a function of the mortgage rate. This, of course, implies a downward sloping demand curve for mortgages as a function of price.

The model predicts that the firm's supply of both liabilities is a negative function of own rates. This result is not obtained in previous models that assume a monopolistic rate setter [1, 2, 3, 6, 8, 10]. By assuming monopoly behavior in deposit markets, these rate setting models are incapable of generating competitive deposit supply functions. The model by Stiller (9), which assumes a competitive firm, does attempt to generate deposit supply curves by assuming increasing marginal cost of issuing deposits and purchasing assets. In addition to the competitive supply of deposits functions, the model indicates that the firm views liabilities as substitutes. As the rate on savings deposits increases, the intermediary issues a larger amount of certificates. The reason for this is that the increase in the deposit rate leads to a decrease of desired savings deposits and hence a decrease in the use of labor and capital. The latter results in a rise in the marginal productivity of inputs and a drop in the corresponding marginal transactions cost of certificates. Thus, the firm finds it profitable to substitute liabilities. The intermediary is characterized by a downward sloping demand curve for labor and capital services. However, it is impossible (without further specification) to determine whether inputs are substitutes or complements. The firm demands a greater amount of mortgages as wage and capital rental rates fall. A decrease in the wage rate reduces the marginal transactions cost of mortgages such that it is profitable to purchase a greater amount of these assets. By similar reasoning, the intermediary will supply a greater amount of liabilities as the price of real resources falls. These predictions represent one of the most important aspects of the model in that they emphasize the interaction of both financial and real resource decisions.

The model predicts some apparently strange results in the firm's demand for mortgages as the interest rates on liabilities change, and in the supply of liabilities as the mortgage rate changes. The results can be easily explained within the framework of the model. The intermediary is hypothesized to desire a greater amount of mortgages as the rate on savings deposits (certificates) increases. The reason is that a decrease in the firm's own rate of supplied liabilities will reduce its demand for assets. This is due to the fact that an increase in the interest cost of liabilities coupled with a constant wage rate of labor and rental price of capital services increases the marginal transactions cost of liabilities. This causes the firm to decrease its supply of liabilities. This reduction in liability supply results in less resource usage such that the marginal products of real inputs rise. The latter implies a drop in the marginal cost of mortgages such that the firm finds it profitable to increase its mortgage purchases. This expansion of assets and a corresponding contraction of liabilities is possible through the retention of a greater amount of earnings. In effect, because the retention of earnings is assumed to involve no administrative transactions, the increase in the rate on liabilities reduces the firm to substitute equity financing for the purchase of a greater amount of mortgages. This type of balance sheet change would not be possible if the model assumed a typical balance sheet constraint that did not allow for the retention of earnings as a source of finance.

The model also predicts that the firm desires a smaller amount of liabilities as the rate on mortgages rises. The reason for this again lies in the change in the marginal productivity of labor and capital. A rise in the mortgage rate induces the firm to purchase more of this asset, requiring the use of more resources at a declining marginal productivity. The decline in productivity increases the marginal transactions cost of liabilities resulting in a decrease in liability issues. The increased mortgage purchases would again be financed by a greater amount of retained earnings. It should be noted that the traditional results of greater liability supply (or an increase in the deposit rate by the monopolist) as the mortgage rate rises can be obtained by eliminating the possibility of earnings retention and assuming that the cost of equity changes by the same amount as the mortgage rate (\( \alpha_2 = \alpha_3 \)). This is essentially the case for a model that constrains the firm's wealth accumulation through business savings to be zero. Hence, Table 1 places both signs under the liabilities as a function of the mortgage rate.

Finally, the intermediary is assumed to make behavioral changes as the cost of owner supplied funds changes. The results of the evaluation in these cases are for the most part ambiguous. The only clear result shows mortgage purchases being negatively related to the cost of equity. This is the same type of result found when the internal rate of return technique of capital budgeting is employed in asset evaluation. As owners find other more profitable investments, the firm would reduce the amount of earnings retention, the purpose of asset expansion. The signs of the derivatives describing liability supply and resource demand are ambiguous because a change in \( \tau_1 \) creates effects upon each of the six endogenous variables. It is assumed that the direct effect of a change in \( \tau_1 \) on the endogenous variable in question outweighs the indirect effects on other variables in the evaluation of the specific derivatives. This gives the results shown in the bottom row of Table 1. The theory predicts that the firm will supply more liabilities as the cost of owner funds rises. This is a common sense result that would apply for the firm using a weighted average cost of capital to evaluate asset purchases. Finally, the intermediary will hire less resources as the cost of equity funds increases.

Conclusion

The model presented in this paper has attempted to demonstrate how standard microeconomic theory can be used to analyze the financial intermediary. The model itself generates a number of static and comparative static propositions describing the decision process of the firm. This is in contrast to rate setting models that limited their scope of analysis to optimum deposit rates and asset purchases [1, 2, 3, 6, 8, 10]. The behavioral predictions stem from two distinguishing characteristics of this approach. The first is the linearity of the firm's budget constraint. All previous models have constrained the firm's asset purchases to equal its liability issues. The model presented above introduces retained
earnings and, implicitly, the dividend decision into the analysis. This inclusion of retained earnings as a source of funds allows the model to generate propositions concerning both asset decisions and the cost of capital usually found in the theory of finance.

The second distinguishing characteristic involves the simultaneous analysis of the financial and real resource decisions made by the intermediary. This is achieved by taking into account the need for real resources in the intermediation process. By taking this approach, the model generates a number of hypotheses concerning the effects of changes in real resource parameters on the decision to purchase assets and issue liabilities. This is perhaps the most important contribution of the model.

A crucial aspect of this type of analysis is the ability to transform the testable hypotheses of the model into an empirically workable framework. Preliminary empirical work by one of the authors has concentrated on the supply of liabilities by the savings and loan associations [5]. Although many of the results were of a mixed nature, the study consistently pointed to the significance of the wage rate proxy in the decision to supply liabilities (in a manner predicted by the model). In addition, empirical work by Greer has shown that a similar wage rate proxy was significant in explaining loan demand by consumer finance companies [4]. Thus, incorporating the real resource decision into the analysis of the financial intermediary may prove to be enlightening in both future theoretical and empirical studies.

References


Appendix

Maximization of the objective function with respect to the endogenous variables leads to the following first-order necessary conditions:

\[
\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0
\]

(13) \[\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0\]

(14) \[\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0\]

These equations show the relationships between the objective function and the endogenous variables under the assumption of the first-order necessary conditions. The value for \(\lambda_2\) given in (15) can be substituted into the appropriate equations. This allows for a system of eight equations where the first period variables are expressed as functions of the first period parameters. This system is:

\[
\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0
\]

(16) \[\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0\]

(17) \[\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0\]

(18) \[\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0\]

(19) \[\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0\]

The value for \(\lambda_2\) given in (15) can be substituted into the appropriate equations. This allows for a system of eight equations where the first period variables are expressed as functions of the first period parameters. This system is:

\[
\frac{\partial L}{\partial \lambda_2} = \frac{r_2}{1 + \lambda_2} - \frac{\lambda_2}{\lambda_3} - \frac{\lambda_2}{\lambda_4} \frac{\partial F}{\partial \lambda_2} = 0
\]
of six equations which constitute the basic system of the model. These equations are:

\[ \frac{\partial L}{\partial \lambda_1} = -\lambda_1 - \lambda_4 \frac{\partial T}{\partial \lambda_1} = 0 \]

\[ \frac{\partial L}{\partial \lambda_4} = (M_1 - M_a) - (S_1 - S_a) - (C_1 - C_a) - R_i = 0 \]

\[ \frac{\partial L}{\partial \lambda_4} = T(M_1, S_1, C_1) - f(n_i, \lambda_i) = 0 \]

\[ \frac{\partial L}{\partial \lambda_1} = -1 - \lambda_1 = 0 \]

The value of \( \lambda_1 \) in (8a) can be substituted into the rest of the system. In addition, since \( R_i \) appears in only one equation, its value is obtained by substituting the optimal values of \( M_1, S_1, C_1, \lambda_1 \) into (6a). Thus, the system of eight equations can be reduced to a system of four equations.

The purpose of this paper is to describe a source of systematic bias in conventional statistical tests of the natural rate of employment hypothesis. The basic problem involves the customary implicit assumption that no significant difference exists between inflation expectations of wage earners and those of employers over time. Econometric models which incorporate this restriction will be shown to be inconsistent with the fundamental principles of the natural rate hypothesis and to provide misleading evidence on its validity.

The study is composed of three sections. First, a distinction is drawn between the short run and long run implications of the natural rate hypothesis, emphasizing the importance of allowing for differences between employer and employee expectations in the short run. Second, evidence is presented indicating that a significant, systematic difference has existed between the average rate of inflation expected by business leaders and the average rate anticipated by American households over the past 10 years. Finally, implications of this difference in expectations for statistical tests of the natural rate hypothesis are discussed.

1. The Natural Rate Hypothesis

For modeling purposes, the labor market will be viewed as essentially competitive or "atomistic" in nature with market participants having limited access to information on current economic conditions. The nominal wage rate is competitively set at the beginning of each time period, over which it remains fixed. Earnings are paid in lump sum amounts at the end of each period. The long run implications of the natural rate hypothesis can be viewed within this framework with the aid of a simple supply and demand illustration. Briefly stated, the hypothesis proposes that a natural or normal rate of employment exists for the economy at any point in time, dependent in part on factors affecting job search and placement, and independent of fully or unanimously anticipated price inflation. The effective supply of and empirical evidence is presented which supports the arguments raised.

These assumptions are consistent with the search theory approach to labor market analysis as presented in Phelps, et al. (1970) and consistent with Tobin's analysis of the tradeoff between inflation and unemployment (1972). When viewed in context of the model presented, attainment of the natural rate of unemployment does not appear to require that forecasts of future inflation become precisely accurate but only that the average rate of inflation expected by suppliers of labor services be equivalent to the average inflation rate anticipated by employers.