Factor Transferability: The Efficiency of Single Product and Multiproduct Firms

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Introduction

To this point the literature on economies and diseconomies involved in multiproduct production has been relatively sparse. Generally, differences in efficiency between a multiproduct firm and a group of single product firms have been attributed to externalities in joint, as opposed to alternative, production (Carlson [3], Domo [4], Hicks [7] and Mauer and Naylor [9]). Basomol [2] and Panzar and Willig [10] investigate the restrictions that these externalities impose on the joint cost and the joint production functions. Weldon [13] catalogs potential economies and diseconomies that arise from differences in the way factors may be utilized by the multiproduct firm.

In this paper factor transferability is shown to provide a measure of adaptability and hence, a cost savings to the multiproduct firm not available to a group of single product firms. This adaptability can be expected to be of benefit to firms having a wide variety of behavioral objectives. One practical consequence is that factor transferability is a type of "synergy" which would encourage mergers and to some degree the addition of new product lines. Further, one could expect the adaptability provided to alter the multiproduct firm's investment behavior in the long run.

A Benchmark

Consider a simple multiproduct firm characterized by a capital stock which, in the short-run, is fixed in both amount and array. The firm uses a fixed and m variable factors to produce n outputs. If the firm is assumed to have n independent production processes, then the firm's cost of producing a given bundle of outputs, $X = (x_1, \ldots, x_n)$, is:

$$C^*_F = \min_{F} C_F = \sum_{j=1}^{n} \sum_{k=1}^{m} w_{jk}j_{jk} + F $$

is a somewhat more general behavioral trait. In particular, not only are profit-maximizing firms cost efficient, but so are others, for example the constrained revenue maximizing firm (Basomol [2]). Consequently, the efficiency provided by factor transfer would benefit any firm which as part of its behavioral objective could be expected to cost minimize. Thus, at optimum it is possible to show that any profit maximizing firm would be more profitable and that a constrained revenue maximizing firm would have a higher revenue as a result of the ability to transfer factors.

*The capital stock is fixed over the short-run horizon by assuming that the transaction cost of shifting these inputs either by rental or purchase is so insurmountably large that it becomes impractical if not impossible. (c.f. Klein, Crawford and Alchian [8]).
such that
1. \( x_r - f(x, z_r) \leq 0 \) \quad \forall r = 1, \ldots, n
2. \( y \geq 0 \) \quad \forall r = 1, \ldots, n;

where:
\( y = (y_1, \ldots, y_m) \)
\( z_r = (z_{r1}, \ldots, z_{rn}) \)
\( w_r \) = factor price of the \( r \)th variable factor.
\( y_r \) = the amount of the \( r \)th variable factor used to produce the \( r \)th product.
\( F = \) the amount of the fixed costs.
\( z_r^* \) = the initial (and here the final) amount of the \( r \)th type of capital good used to produce the \( r \)th product.
\( F_r^* \) = the initial amount of the \( r \)th type of capital good suited to produce the \( r \)th product, which is idle.
\( x_r^* \) = the (minimum) amount of the \( r \)th product which the firm is constrained to produce.
\( f_r(x) \) = production function of the \( r \)th product
\( \bar{y}_r = (y_1, \ldots, y_m) \)
\( \bar{z}_r = (z_{r1}, \ldots, z_{rn}) \)

Note that for simplicity all markets for variable factors are competitive.

Next, consider a single product firm producing the \( r \)th output. The firm's cost of producing a quantity \( x_r \) is:
\[
C_r^* - \text{Min} \frac{C_r}{y} \times \sum_{j=1}^{m} w_j y_j + F_r.
\]

such that
1. \( x_r - f(x, z_r^*) \leq 0 \)
2. \( y_r \geq 0 \)

The above program is representative of the short-run problem facing any single product firm (c.f. Samuelson [12]). With no externalities the costs incurred by a group of \( n \) single product firms in producing the bundle \( (x_1, \ldots, x_n) \) is \( \sum_{r=1}^{n} C_r^* \).

The intent here is to compare the efficiency of the single multiproduct firm with that of a group of single product firms in the production of an arbitrarily given vector \( x \). In this comparison the following are assumed:
1. The endowment of fixed factors is the same for both the simple multiproduct firm and the group of single product firms.  
2. Consequently, the fixed costs of the simple multiproduct firm equal the sum of the fixed costs of the single product firms, i.e., \( F = \sum_r F_r \).
3. Production technology is the same in each case. That is, the form of the production function of any output is the same whether that output is produced by a single product firm or by a multiproduct firm.

Under these assumptions the program of the simple multiproduct firm is separable so that \( \sum_r C_r^* = C_r^* \).

Factor Transferability^1

In the short-run firms pass factors for which no external market exists. However, Clemens [4], and Phelps [11] note that while it is not possible to buy or sell these fixed factors, multi-product firms, in particular, have the ability to transfer these factors among different activities. Thus, by changing molds a given pressing machine can produce a wide variety of products. What is to be shown is that a multiproduct firm with such an ability (hereafter called the adaptive multiproduct firm) is in general a more efficient producer than either the simple multiproduct firm or a group of \( n \) single product firms.

The fixed factors of the adaptive multiproduct firm are partitioned into \( p \) groups each containing \( m \) factors. The firm may transfer at a cost between any two factors in the same group. Each group may be thought of as a type of capital good although no assumption is made about the homogeneity of units within the group. Further within each group there are a pair of factors, one associated with each vector. Accordingly define:
\( u_r \) = that position associated with the \( r \)th type of capital good employed in the \( r \)th sector, \( \forall r = 1, \ldots, n \); \( u_r^* \) = that position associated with the \( r \)th type of capital good suited to produce the \( r \)th output, but which is idle \( \forall r = 1, \ldots, n \).

The firm chooses amounts which it wishes to switch between points in the same group. Hence, define \( \Delta(u_r - u_r^*) \) as the amount of factor at \( u_r \) transferred to \( u_r^* \). Note that this amount is measured in units of the source factor and represents input into the switching process. The switches are actually accomplished external to the multiproduct firm by a switching firm (or firms). Because no homogeneity assumption is made one must allow that switching takes place between distinct, disparate factors. Therefore, the output of the switching process must be measured in units of the destination factor. The correspondence \( g \) is defined to relate the inputs and outputs of the switching process. Alternatively, one may regard the \( g \) functions as a type of production correspondence through which factors of one type are diverted as inputs. Transfer, then, results in the "production" of another factor. While the possibility of joint switching processes exist, for simplicity the assumption here is that all switching processes are independent. Thus, \( g(d(u_r - u_r^*)) \) denotes the number of units at \( u_r^* \) transferred from \( u_r \). There are \( p \cdot 2(m-1) \) \( g \) functions, one for each possible channel of switching, and consequently the \( g \) functions are indexed by their arguments. The amounts transferred between points serve to alter the array of fixed factors. In conjunction with the initial factor endowment these amounts transferred determine the final array of fixed factors. One may then define:

\( z_r^* = \) the final amount (after switching has been completed) of factor at \( u_r \), \( \forall r = 1, \ldots, n \).
\( I_r(\cdot) = \) the final amount of factor at \( u_r^* \), \( \forall r = 1, \ldots, n \).

Essentially, the final level of fixed factor at any position is the initial amount present at that position, minus the amount switched away to other positions, plus the amount of that factor which has been switched to that position from others.^2

Since the switching firm must be paid for its services, the costs incurred by switching are assumed to be dependent on the output of that firm, i.e., on the output of the switching process. Thus:
\[
K = K(g(\cdot))
\]

where:
\( K(\cdot) = \) the amount of switching costs
\( g(\cdot) = \) vector of the outputs of all switching processes.

The behavioral objective of the adaptive multiproduct firm is to minimize total costs, defined as the sum of variable, fixed and switching costs constrained by minimum

\( \text{More formally:} \)

\( z_r(\cdot) = \sum_{u_r} \Delta(u_r - u_r^*) \)

\( I_r(\cdot) = \sum_{u_r} g(\Delta(u_r - u_r^*)) \)

To simplify the notation the following definitions are introduced:
\( L_r(\cdot)^2 = \) the set of all possible destinations reachable from \( u_r \), \( \forall r = 1, \ldots, n \).
\( S_r(\cdot)^2 = \) the set of all possible sources from which \( u_r \) may be reached.

Here, by assumption the set of all possible destinations and the set of all possible sources for any point are every other factor in the same factor group.
output constraints, nonnegativity constraints on the levels of fixed factors at every position, and nonnegativity constraints on all independent variables. Formally, the solution of the program may be written:

$$\begin{align*}
C^* &= \min_{\mathbf{x}} C \\
&= \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} x_{ij} + K(\mathbf{g}(\mathbf{x})) + F
\end{align*}$$

(4)

such that

1. $$x_{ij} - f_{ij}(\mathbf{z}, \mathbf{g}(\mathbf{x})) \leq 0$$,
2. $$\mathbf{g}(\mathbf{x}) \geq 0$$,
3. $$\mathbf{h}(\mathbf{x}) \geq 0$$,
4. $$\mathbf{x} \geq 0$$,
5. $$\mathbf{z} \geq 0$$

where $$\mathbf{z}$$ is a vector of the inputs into the switching process.

Comparative Efficiency

In comparing the efficiency of the adaptive multiproduct firm with that of the simple multiproduct firm, and hence, with a group of n single product firms, the bundle of outputs, the production technology, the endowments of fixed factors, and the forms of the fixed, variable and switching cost functions are assumed to be the same. From previous discussions one may note that the initial factor endowment and the amount of switching determine the final array of fixed factors (i.e., $$\mathbf{z} = \mathbf{z}(\mathbf{*, D})$$ and $$\mathbf{g} = \mathbf{g}(\mathbf{*, D})$$). Then, if a firm does not switch, the initial and final array of fixed factors are the same (i.e., $$\mathbf{z} = \mathbf{z}$$ and $$\mathbf{g} = \mathbf{g}$$). Also note that in this case the firm incurs no switching costs ($$K(\mathbf{g}(\mathbf{0})) = K(\mathbf{0}) = 0$$).

One may now rewrite program (4) in order to compare relative efficiency more clearly. Consider the set of $$\mathbf{\Delta}$$'s which satisfy constraints 2, 3, 5 and for which the set $$\{y_{ij} \geq 0$$ and $$x_{ij} - f_{ij}(\mathbf{z}, \mathbf{g}(\mathbf{x})) \leq 0, v_j \}$$ is not empty. This constitutes the feasible set of switchers. That is, if $$\mathbf{\Delta} = 0$$, then there exists a $$\{x_{ij}, z\}$$ which satisfies all constraints.

Suppose the value of $$\mathbf{\Delta}$$ were fixed, then the array of fixed factors, and switching costs are determined. To minimize its costs the firm need only adjust its variable factors. That is, by choosing $$\mathbf{\gamma}$$ the firm could find $$C^*_\mathbf{\gamma}$$ which represents the minimum cost of production given $$\mathbf{\Delta}$$. Thus, $$C^*_\mathbf{\gamma}$$ is a function of the level at which $$\mathbf{\Delta}$$ is fixed, so that an alternative way of representing the solution to program (4) would be:

$$C^* = \min_{\mathbf{\gamma}, \mathbf{\Delta}} C^*_\mathbf{\gamma} (\mathbf{\Delta})$$

(5)

Now, consider the value of $$C^*_\mathbf{\gamma}$$ if $$\mathbf{\Delta} = 0$$. Since the initial and final array of fixed factors are the same, and since no switching costs are incurred, the program (4) reduces to the program (1). Thus, their solutions are the same, i.e.,

$$C^*_0 = C^*_\mathbf{\gamma} (\mathbf{\Delta})$$

(6)

From this it follows that:

$$C^* = \min_{\mathbf{\gamma}} C^*_\mathbf{\gamma} (\mathbf{\Delta})$$

(7)

Hence, the adaptive multiproduct firm is at least as efficient as a simple multiproduct firm and, by extension, a group of single product firms. Moreover, only if the adaptive multiproduct firm does not switch (i.e., can find no channel which lowers costs) will the group of single product firms be as efficient, so that in general they will be less efficient.7

A somewhat broader result may be derived (c.f. Haber [4], pp. 150-3). This states that the larger the set of feasible switches the lower the cost of production of a set of outputs. In particular, if new channels of switching are opened to the firm, then the firm is no worse off than it was previously, and it can find use for those channels, it is more efficient. This derivation hinges upon the fact that if the new channels of switching are not utilized, switching costs and total costs remain unaffected.

This efficiency results from the relaxation of the constraint which fixes the amount of capital in each use. Thus, it may be viewed as an application of Le Chatelier's principle. Now, instead of facing a completely molded, rigid capital stock, the adaptive firm, while in some sense it is presented with an array of fixed factors, has the freedom to transfer those factors among various alternative uses. That is, the firm may rational fixed factors rather than take the array as given. Essentially then, the efficiency results from a better utilization of a given capital stock. An example in which this freedom proved significant came at the time of the first Arab oil embargo, during which some of the automakers were able to resell (switch) their large car assembly lines so that they could be used to produce compact cars. This ability allowed the firms to adapt more easily to changing output-demand conditions.

Economic models depicting the benefits of production of more than one product have done so in terms of interrelated demand for the firm's outputs or in terms of the joint production of these outputs (see for example, Masur and Naylor [9], Panzaar and Willig [10], and Baumol [2]). This study suggests another efficiency of a quite different nature. The ability to transfer factors might have value as long as the source and the destination factors have different marginal revenue products, however they might be defined (c.f. Haber [6], and Masur and Naylor [9]). Further, this efficiency occurs without reference to the form of the firm's production function. Here, the production processes were assumed independent to show that factor transferability provides added efficiency even where there are no efficiencies derived from joint production.

While the efficiencies provided by factor transferability have been couched here in a short-run model, the importance of this sort of adaptability is not limited to the short-run. Factor transfer allows the firm to obtain input at a given destination from internal sources. That is, it creates internal markets for the factors of production in the absence of external markets (i.e. in the short-run). Then, even when external markets do exist but are imperfect (e.g., there exist search, transaction or other adjustment costs), these internal markets might still play a large role in the efficient allocation of resources. For example, suppose a firm had to purchase today a set of capital goods (i.e., a technology) which it would use to produce output in the future. That is, the investment decision is made ex ante to any production. Assume that future output prices are uncertain, but that the relevant, possible outcomes, are known and their probabilities estimated. In this risky situation it would not be uncommon for the firm to buy a higher priced technology which is adaptable to many uses in favor of a less adaptable cheaper technology. The increased adaptability provides a hedge against the price uncertainty.

References

The Classical Economics of the Pre-Classical Economists

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The argument of this paper is that much of classical economics was anticipated — so much and for so long that it ought to be considered an extension of the ideas that preceded it and not a radical departure from them or a distinctively new doctrine.

There are two ideas at its center. One describes what the market is and how it operates. It is an impersonal mechanism that sends people about their business in an efficient way. The other is a statement of what the market is for — the purposes it should serve.

The first is positive. It consists of statements that can be put to the test of fact or logic. The other is normative and consists of statements about values.

This paper is about the antecedents of both as disclosed in the writings done before the eighteenth century. Some goes back to antiquity. Most was done between 1500 and 1700, the period of mercantilism. I shall not say much about the anticipation of classical economists in the century when it began. Most of what should be said has been: that Smith was anticipated by the physiocrats and that they were influenced by Cantillon. Something ought to be added (and is here) about the critical place of utilitarianism in classical economics. The period before 1700 has received much less attention. Schumpeter described the ideas that contributed to the progress of analytical economics, whether or not they have a bearing on classicism, and (in his History) he had little interest in the development of normative economics. The present writer has described the classical elements in mercantilism and some of their origins in Stoicism. This paper draws on those studies and others made since they were published. The findings are summarized and related entirely to the classical conception of the market and its purpose.

1. The Market and its Operation

The classical conception of the market includes a theory of how relative prices are determined, how they cause resources to be allocated, and how they determine the distribution of the product. That is, classical economics explains that the market determines relative prices and they determine what is to be produced, how it is to be produced, and how it is to be distributed.

This is an important thing to have done. In every economic doctrine, whatever its normative elements are, there is the idea of an authority or power that coordinates the activities of individuals. The classicists contributed

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1Joseph A. Schumpeter, History of Economic Analysis (New York, 1954), Part II, esp. ch. 2.