WHY DID MARSHALL TRANSPOSE THE AXES?

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Introduction

The teacher of elementary economics puts a schedule of prices and quantities demanded (or supplied) on the blackboard on which price is represented to be the independent variable and quantity the dependent one. He may follow this by writing the notation $Q = f(p)$. But then he draws a diagram in which quantity is put on the abscissa and price on the ordinate! Chances are that some student will challenge this, saying that the axes are wrong-way-round since mathematicians always put the independent variable on the abscissa. The lecturer might reply that, if the student will be patient, it will eventually be made plain that price and quantity are mutually dependent variables, just like the positions of three balls in a bowl, so it doesn't matter how we initially label the axes; or he might say that this will be fully explained in a more advanced class of microeconomics where the dynamics of market adjustment processes is examined; or he might simply say that it is just a matter of convention: mathematicians have their conventions and we economists have ours.

This graphic convention was established by Alfred Marshall, who used it in his lectures from which half the academic economists of England had learned their craft by the late 1800's (Foxwell, 1887, 92), and whose Principles of Economics taught even a larger number, and over a wider area, from its publication in 1890 down to the second world war.¹

Prior to Marshall’s Principles, Cartesian coordinate diagrams were not much used in economic analysis as such even as there was typically put price on the abscissa: Cournot (1838), Dupuit (1844), Lardner (1850), Jenkin (1870), Walras (1874-77).² F. Y. Edgeworth, not one of Marshall’s students, in his Presidential Address to Section F of the British Association “On the Application of Mathematics to Political Economy” in 1889, presented a diagram of “Simple Exchange” in which the abscissa was price and the ordinate quantity, noting that “this is Cournot’s construction. The converse construction in which the abscissa stands for quantity of commodity, the ordinate for price,” Edgeworth added, “is employed by Mr. Wicksteed in his excellent Alphabet of Economic

¹The present writer was introduced to Marshall’s Principles in his first course in economics, at Dalhousie University in 1941. This experience, while not repeated by many, was not unique: a student who took elementary economics at the University of Toronto in the same year recalls that Marshall’s Principles was the text, though students having difficulty with it were advised to read F. W. Taussig’s Principles instead.
²Wicksteed consistently put quantity on the ordinate even when, in Lesson 8 of the Elements, he discussed the relationship between the quantity of a commodity and the utility derived from the consumption of it. In some diagrams the abscissa was labelled “worth” which distinguished it from what we now call marginal utility (see Walras, 1944, Joffe’s note [9], p. 308), with only a cryptic parenthetical remark calling the reader’s attention to the fact that the ordinate is the independent variable in this case (p. 119).
 Marlborough's Practice

Due to Whittaker's excellent work on the
Marshall papers we now have access to
Marshall's unpublished writings on econom-
ics dating back almost to his earliest interest
in the subject (EEMAM). The materials,
which include such things as Marshall's anno-
notations of Cournot's Recherches and a note-
book of some early exercises in mathematical
economics, enable one to study the origins
and development of Marshall's economics much
more fully than hitherto. These papers throw
considerable light on the early development
of Marshall's graphic method, his relation to
predecessors, and the kinds of problems
Marshall focused upon in using, and experi-
menting with, geometrical tools of analysis.

One of Marshall's first attempts to use
graphic methods (if not the first) was a
diagrammatic formulation of Ricardo rent
theory which succeeded so well that Marshall
noted that it "decided me to adopt curves as
an engine" (EEMAM, p. 40-41). In view of
the similarity between the theory of rent and
the theory of consumer and producer
surpluses (which Marshall called "rents"
in the early editions of the Principles) this
graphic exercise has some relevance for the
explanation I will offer of Marshall's decision
to use the price-function form in graphing
demand and supply curves.

So far as the materials in Whittaker's
volumes enable one to determine, it appears
that Marshall used the price-function form
from the very beginning of his work on
demand and supply. Especially notable are his
notebook exercises on excise taxes and bridge
tolls, written probably in the late 1860s or
very early 70s, in which Marshall draws
demand-supply diagrams in the price-func-
tion form, employs the same form in the
algebraic notation and, significantly, uses the
analysis to measure the effects of taxes and
tolls on consumer surplus (ibid. 2, 297-
83).

At the time he made these notes, Marshall
could not have been aware of Walras's work
(1874-77) and their composition probably
antedates Fleming Jenkin's diagrams of
supply and demand (1870). He could have
known of Dupuit (1844) and Lardner (1850)
but apparently did not.

One might be tempted to argue that Marshall
continued to use the price-function form because
he happened to begin with it. But he was too good a
mathematician to make such a decision without careful
consideration and his development of the theory of
costumer surplus in these notes indicates that his main
reason for using the price-function form in the Principles
was proven in his earlier analytical work.

Highton (1985) objects to assigning the idea
of consumer surplus to so strongly with Marshall, pointing out
that Ausgut and Lief have published a well-worked out
type of theory in it in 1899. But, by this date, Marshall's own
analysis was twenty years old and he had lectured on it,
and taught students in it, during that period. Price
labourers, if it anyone other than Marshall, in Dupuit,
from whom Ausgut and Lief derived the foundations
for their own analysis.

Marshall's early paper on bridge tolls (EEMAM, 2,
281-283) is so similar to Dupuit's (1844) that internal
evidence alone would suggest that Marshall knew
Dupuit's work at this time. All the other evidence, how-
ever, indicates that this is highly unlikely (ibid., 312-31)
-and in the first edition the Principles Marshall included a
note to authors who had used mathematical methods
in economics: Jevons, Cournot, Gossen, Menger, Riemen-
bur, Walras, and Lardner are mentioned but not Dupuit or
Lardner (EEMAM, 2, 247-248). Marshall's only
referecnces to Dupuit were made in the second edition of
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edged in the preface to the first edition of the *Principles* (PE, 1, ix–x).

In his algebra, Cournot expressed quantity as a function of price, except in his treatment of deeply where he noted that "in this case it will be convenient to adopt the inverse notation \( p = f(D) \)" (1960, 80). In his diagrams, Cournot used the quantity-function form only.

Marshall’s notes on Cournot (E6WAM, 2, 240–248) are revealing. In giving graphic renderings of Cournot’s arguments Marshall uses both the quantity-function form (as Cournot had) and the price-function form, drawing three diagrams in the former mode and one in the latter. The main point of these notes is the geometric measurement and analysis of consumer and producer surplus, which Marshall carries out in both diagrammatic forms. These notes show that the geometric analysis of consumer and producer surplus, and the use of it to assess welfare effects, was a main focal point of Marshall’s analytic interest at least as early as his reading of Cournot; and that at the time he read, and made his notes on, Cournot’s *Recherches*, Marshall worked comfortably in either function form, both algebraically and geometrically. Shortly thereafter apparently, he decided to use the price-function form exclusively in his own geometric work. Clearly, it would be confusing for an author’s readers, if not for himself, if he drew supply-demand diagrams both ways, instead of choosing one mode and sticking with it. But the price-function form is the more useful of the two for some problems while the quantity-function form is more useful for others. Marshall decided, quite early apparently, to adopt the price-function form as his uniform geometrical and algebraic mode, but to vary the form employed in verbal discussion as necessary for the purpose of the problem at hand.13

The adoption of such a practice requires that one inform the reader that a geometric demand or supply curve may be read both ways. So, for example, in the "primer book" that Marshall and his wife wrote for junior students one finds the initial statement of the "Law of Demand" worded as follows: "It is a matter of common experience that the larger the stock which sellers determine to sell, the lower will be the price at which it can be got rid of. Vice versa, the lower the price at which anything is offered for sale, the greater the amount of it which can be sold off" (LE, 69). Marshall undoubtedly learned from tutoring and examining students that this point required some emphasis. In the *Principles* there are many passages in which both function forms are stated alternately. In the initial discussions of demand, for example (PE, 1, 96–100), he gives numerical schedules of prices and quantities, draws a demand curve, and in his verbal discussion shifts back and forth between the two modes several times in the space of four pages. In view of the care with which Marshall composed the *Principles* and the extensive revisions it was subjected to through eight editions, the only explanation which seems plausible is that Marshall regarded it necessary that the student of economics accustom himself to reading demand and supply curves both ways since they would have to be used in different forms for different purposes. There is one passage in the *Principles* in which he says that businessmen customarily think of supply in terms of the price at which a quantity can be sold (the price-function form) and the demand in terms of sales which will be forthcoming at different prices (the quantity-function form) and notes that "economists commonly follow this practice" (PE, 457b), but obviously he was not recommending this in retaining the use of both forms of verbal analysis. On the contrary, he repeatedly emphasized the necessity of treating demand and supply in consistent and commensurable terms.

Why did Marshall transpose Cournot’s axes? The answer, in my view, lies in Marshall’s early, and continuing, interest in consumer and producer surplus,11 that is, in what we today call "normative welfare economics," as well as one of the main objects of economic analysis. There is no substantive difference, as such, in labelling coordinate axes one way or the other, but Marshall, in my opinion, put quantity on the abscissa and price on the ordinate because in doing the analysis of "maximum satisfaction" this would be in accord with the mathematical convention of using the abscissa for the independent, and the ordinate for the dependent, variables.

The "Doctrine of Maximum Satisfaction" Whicker’s edition of Marshall’s unpublished writings (E6WAM) shows that from the very beginning of his theoretical work on economics, Marshall interpreted the demand curve as showing the prices consumers are willing to pay, and the supply curve as showing the prices producers are willing to accept, for various quantities of a commodity. To map such curves in rectangular coordinates

1Note that I avoid the "apophasis problem" by these expressions. Marshall always used an apophasis with an "x" but sometimes put the apophasis before and sometimes after the "x." It would be nice to record that his purpose depended upon the aggregation implied in the analysis but, in fact, there was no consistency in his position on this point. Sumner is definitely incorrect in asserting that "with Marshall the apophases in commensurate surplus is always after the x" (1955, 206).

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1For a discussion of Cournot on this point, see Nichol (1934).
not the only reason why Marshall chose to use the price-function form and, in my opinion, not his main reason. This other reason has to do with the doctrine of "maximum satisfaction" which was the central pillar of the analytical edifice he commenced constructing in the late 1860s. The previous section emphasized that Marshall's early analytical efforts were directed at measuring consumer and producer surpluses, starting with a geometrical representation of the Ricardian theory of rent. One of Marshall's notable innovations in economic theory was to use the concept of "surplus" in a very different way from his predecessors. The Physiocrats identified it ("produit net") with agriculture, and attributed it to the existence of free factors of production (sunlight, rain, etc.); the classical economists regarded surplus as the production above "subsistence," which was available for capital investment; Marx used the concept to define "exploitation" as labor's receiving less value than it produces. Marshall's surplus is a measure of the difference between the benefits accruing from productive effort and the costs incurred by it. Assuming a positive monotonic relationship between quantity of productive effort and quantity of output, costs and benefits can be described as functions of output; so, in accordance with mathematical convention, output quantities should be represented on the abscissa and some index of benefits and costs on the ordinate. In Marshall's paradigmatic case of a boy picking blackberries (PE, I, 331, 844; EE, I, 197) this is unproblematic since, for a single individual, benefits and costs can be regarded as measureable in commensurable terms—utilities and disutilities—and the boy's surplus is maximized when the marginal utility of blackberries (i.e., picking) equals the marginal disutility of picking (i.e., blackberries).

A great deal of Marshall's work was directed at extending the blackberries case to a world of division of labor and market exchange. Like Walras (1954, 256–257) Marshall was totally dissatisfied with those who followed Bastiat in simply accepting intuitively the "famous doctrine that free competition tends to make the aggregate satisfaction a maximum" (PE, I, 470, II, 409). One of the main tasks of economic theory, in his view, was to formulate the theory of "maximum satisfaction" for an exchange economy in precise analytical terms so that its conditions could be clearly seen and its exceptions investigated. The final descendent of this line of Marshall's thought were the studies of exceptions by the next generation of Cambridge economists: Pigou on welfare economics, Robinson on imperfect competition, and Keynes on unemployment. (12)

"This may have been the main for Marshall's strange neglect of Henry Sidgwick's remarkable discussion of market failure (1883, Book III). He cannot have disagreed much with Sidgwick's propositions, but they were erected upon wild analytical foundations. Marshall did not object to intuitive ethics, but Sidgwick demanded the use of barrier material. In his obituary of Sidgwick (1912), however, Marshall returned to this point of Sidgwick's Principle as "the best thing of its kind in any language" (EWWAM).

At his comprehensive survey of utility theory from Adam Smith to B. E. Denning, George Stigler (1967) omit the proposition of utility theory as welfare economics, and places on the ground that "most economists of the period at least can understand and discuss economic behavior (particularly demand behavior) and only secondarily (when it all) to explain or justify economic policy."

He notes Marshall, Pigou, and as, others, as exceptions to (96).

"It is worth noting, without overrating the significance of the point, that the Marshallian treatment of the imperfect competition theory and employment theory followed the Marshallian function form tradition in expressing value variables as functions of quantity variables. In John Robinson's theory of imperfect competition (1932) the marginal revenue curve is derived directly from the demand curve in the price-function form and is itself formulated in analytical terms, with market as the independent variable. Keynes' formulation of the aggregate demand function in the General Theory are in the Marshallian form, expressing supply price and demand price (Shawe 1942 and his Employment (1942, 1973, 25). The only diagram in the General Theory, however, is the output investment and the rate of interest in the quantity-function form. Keynes notes that this diagram was suggested by Harrod (1937, 180).

In examining this aspect of Marshall's work, it is important to recognize that he was not a Benthamite utilitarian, in either his ethics or his psychology. He did not believe that "pleasure" is the highest good, that people's wants should be regarded as data, that people are motivated only by self-interest, or that the efficiency of an economy can be determined by reference to the extent to which it satisfies currently perceived wants. On the contrary, the "Victorian moralizing" which sprinkles the pages of his analytical economics and punctuates his other writings, is distinctly non-utilitarian, contending that wants are products of cultural conditioning, that many of them are tawdry and ignoble, and that one of the main purposes of intellectual work (including theoretical economics) is to contribute to improvement in human character and the construction of a world suitable for, and inhabited by, better people. In this respect Marshall was more in agreement with the nineteenth-century romantics than the utilitarians and his frequent references to the "nobler aspirations" of socialist thinkers reflected strong convictions which he held unchanged from early youth into old age.

How did such views harness with his scientistic work and, specifically, with his doctrine of maximum satisfaction, which clearly seems to be based on a simple utilitarian calculus of benefits and costs? The answer is that, unlike the romantics and the socialists, Marshall believed that science, as hard as one could make it, is the essential foundation for social improvement. It was not the end, but it was the necessary beginning: the "mecca of economists," "economic biology," was a long way down a road which must be built patiently and carefully, one bit at a time. The first task was to construct a theory of surplus which would permit one to pass from the simple case of picking blackberries to the complex market economy where people are engaged in selling and buying instead of picking and eating. The analytical passage from the simple case to the complex one was made possible, in Marshall's view, by the role of money in an exchange economy, since the economist could employ money values, under certain conditions, as measurements of benefits and costs.

Money Measurement

In 1896 Marshall addressed the first meeting of the newly-formed Cambridge Economic Club on the topic: "The Old Generation of Economists and the New." One of his main themes was that the older generation had made great strides in qualitative analysis, but quantitative analysis, the sine qua non of applied science, had not yet made much headway.

More qualitative analysis . . . may show gain here and loss there; but it will not show whether the gain is sufficient to overbalance the loss; whether the gain should be pursued in spite of the loss. . . . It is useless to say that various gains and losses are inconceivable, and cannot be weighed against one another. For they must be, and in fact are, weighed against one another before any deliberative decision is or can be reached on any issue (MAH, 301–302).

Fortunately, in Marshall's view, it was not only useless but unnecessary to regard benefits and costs as incommeasurable. Twenty-years earlier he had noted that:

The pure science of Ethics lacks for a

(12) This section deals only with Marshall's use of money as a measuring unit for real benefits and costs, but it is also worth noting that Marshall's general monetary theory was based upon a demand curve for money as an asset (the cah-balance approach) which was formulated in the price-function form, i.e., the demand curve was the price (the amount of other things per unit) that people are willing to pay for various quantities of money. Marshall used this approach to monetary theory in his earliest work on the subject (see EWWAM, 1, 164–177).
system of measurement of efforts, sacrifices, desires, etc., fit for his wide purposes. But the pure science of Political Economy has found a system that will subservie her narrower aims (Ibid., 156).

That system is provided by money:

[The true philosophic raison d'être of [economic] theory is that it supplies a machinery to aid us in reasoning about these motives of human action which are measurable. In the world in which we live, money, as representing general purchasing power, is so much the best measure of motives that no one can compete with it (Ibid., 158).]

In the opening chapters of the Principles, Marshall discussed the characteristics of the modern economy and the springs of human action which lie behind market behavior, noting the play of religion, custom, collective sentiments and altruistic motives, but leading up through these considerations to a clear statement of the meaning of "Economic Generalization or Law":

[A law of social science, or a Social Law, is a statement of social tendency; that is, a statement that a certain course of action may be expected under certain conditions from the members of a social group. Economic Laws, or statements of economic tendencies, are those social laws which relate to branches of conduct in which the strength of the motives chiefly concerned can be measured by a money price (PE, I, 35).]

Mary Marshall, recalling Alfred’s early classes, noted that he regarded Bentham as having had "more influence on Economics than any other non-economist" due to his emphasis on measurement (EEWM, I, 11). Concerning Adam Smith, Marshall wrote:

His highest claim to have made an epoch in thought is that he was the first to make a careful and scientific inquiry into the manner in which value measures human motive, on the one side measuring the desire of purchasers to obtain wealth, and on the other the efforts and sacrifices (or "Real Cost of Production") undergone by the producers (PE, I, 758-759; see also AAM, 157).

Marshall's application of his views on money as a measure of motives is quite clear. In a demand-supply diagram, the ordinate is not a scale of motives in themselves, or of (subjective) costs and benefits in themselves, but a money measurement of these. The justification for this construction is that if, in a free market, consumers are willing to pay as much as Y for a commodity, then they reveal by their actions that they value its benefits at equal to Y units of money; the demand curve is a money measurement of the marginal utility of the commodity at different rates of consumption. Similarly, for the supply curve, the ordinate is a money measurement of marginal costs at different rates of production. Benefits and costs now being measured in commensurable units, the difference between the two functions can be integrated; the area enclosed between the demand curve and the supply curve is a money measurement of the aggregate surplus of total benefits over total costs. In relation to Marshall's adoption of the price-function form, the significance of this view on the role of money measurement in economics is that price not only plays the part of a causal variable but serves as a common objective measuring unit of heterogeneous subjective entities. With respect to the latter role it belongs on the ordinate in a conventionally constructed diagram.

Any system of measurement must meet one fundamental requirement: the unit of reference must be invariant with respect to the quality being measured. A yardstick would not serve as a device for measuring (comparative) lengths if it were inconsistent in its own length. With respect to Marshall's unit of measure in his analysis of surplus, two problems are involved: the potential inconsistency of the marginal utility of money at different points on the supply or demand curves, and the potential differences in the marginal utility of money for different participants in market transactions, whether as consumers or producers. Marshall was acutely conscious of these two problems, and there are innumerable references in the Principles, and in his other writings, to the necessity of taking them into account. In addition, if surplus is to be aggregated over commodities in order to make the "doctrine of maximum satisfaction" applicable to market processes in general, then the interdependence of commodities in consumer utility functions must be taken into account. Staiger notes that Marshall increasingly recognized the significance of commodity interdependence in successive editions of the Principles but that he only made "patch-work repairs" in his analysis and "retained to the last a theory constructed on the assumption of an additive utility function" (1965, 107). Economists have become equally dissatisfied with Marshall's handling of the constancy of the marginal utility of money.

Since the concern of this paper is to explain Marshall's diagrammatic practice, there is no need to evaluate his treatment of surplus or his use of money prices for measurement purposes. We should carry the study a step further, however, and look more closely at Marshall's demand and supply curves in relation to the surplus analysis.

The Demand Curve and Consumer Surplus

The literature on this issue is so vast, and complex, that any brief comment is bound to be unsatisfactory. This section is restricted to providing some brief amplification of Marshall's concept of money measurement that is germane to its understanding of the role of the price-function form of the demand curve.

In Milton Friedman's celebrated paper on the Marshallian demand curve (1953), he argues that Marshall should be interpreted as drawing the demand curve so that positive compensation takes place which has the opposite effect on consumer real income. Two such compensating changes are considered by Friedman: a fall (rise) in money income as the price of X falls (rises); and a rise (fall) in other prices as the price of X falls (rises). Friedman interprets Marshall as adopting the latter compensating device, arguing that this harmonizes with one of Marshall's "basic organizing principles" in theory construction: the separation of the theory of relative prices from the theory of the
general price level (1953, 65-68). But, else-
where, Friedman acknowledges that Marshall
seemed to be assuming compensating money
income changes rather than compensating price
changes (1953, 82, 92-94).

So far as Marshall's consumer surplus
argument is concerned, the burden of evidence
seems to be that Marshall assumed compensating
money income changes in drawing up his demand curve. His favorite
expository device was to picture a regime of
perfect price discrimination, what Pigou later
classified as discrimination of the "first
degree" (1959, 267). In such a model,
consumer real income is held constant in the
demand curve for X by eliminating the
income effect of any change in the price of X
on intra-unitary-elasticities. Consumer surplus is
then measured by the excess of the integral
of the prices that would be paid under price
discrimination over the integral of uniform
prices at the equilibrium point on a constant
real income demand curve. Once again, we
see Marshall's price-function form at work.

As Friedman notes (1953, 70), there is a
problem here. Perfect price discrimination
keeps real income constant, but it is not
constant at the level defined by the market
equilibrium; consumer surplus is consistently
underestimated, due to holding real income
constant at a level corresponding to a price of
X so high that no X would be purchased. This
is obviously unsatisfactory for goods of low
income-elasticity, and especially so if one
wishes to apply the consumer surplus analysis to
a wide category of commodities such as
"food."

Marshall seems to have been aware of this
difficulty, but he did not consistently deal
with it. He frequently referred to the
consumer surplus analysis as being confined
to individual commodities on which the
expenditure is a small part of a consumer's
budget (e.g., PE, I, 842). He noted, more
generally, that "the ordinary demand and
supply curves have no practical value except
in the immediate neighborhood of the point of
equilibrium" (ibid., I, 384n; see also 131n,
133n), but he clearly did not stick to this in
his applications of consumer surplus theory
(see, e.g., the tax-bounty analysis discussed
below) and, on occasion at least, he
defined consumer surplus as the loss that
would be sustained if the commodity in ques-
tion were not available at all (ibid., I, 830).

The Supply Curve and Producer Surplus

In the case of the boy picking blackberries, the
supply curve can be interpreted as a
mapping of disutility incurred against quan-
tities of blackberries produced. To translate
this into a model of a market equilibrium, the
cost of production must be appropriately
interpreted as a money measurement of dis-
utility. This is the part of Marshall's econom-
ic in which he clung most tenaciously to his
classical predecessors. He was reluctant to
adopt the concept of opportunity cost (which
was coming increasingly into vogue in the
later nineteenth century) in part because it
seemed to incorporate rent into cost of produc-
tion (PE, I, 436-437); II, 459-460), but
more generally, because it failed to focus
attention upon the disutility character of
"real" costs.

In the primer book Marshall wrote with his
wife and published eleven years before the
Principles, the discussion of the "Law of Supply
is introduced in opportunity cost terms but then a formal definition is given
and highlighted: "The Cost of Production of a thing... consists of the efforts and absten-
ience required for producing it" (EI, 73).

Marshall retained this conception of cost
throughout the remainder of his work,
together with his early definition of "Ex-
\penses of Production" as the money
measurement of those real costs in commensurable
terms (ibid., 97). In the Principles he defined the supply curve in terms of the money
compensation necessary to induce the quantities
of labor and waiting required for different
quantities of production, and repeatedly
referred to the supply curve as a money
measurement of such real costs (see, e.g., PE,
I, 339, 352-353, 366). The only significant
qualification Marshall made in this was that
social real costs might well be higher than the
"expenses of production" incurred by produc-
ers when young children are employed (PE, II,
412-413); IT, 183), though he also noted the
more general point that the marginal utility of money might not be the same for all
suppliers of factors (EI, 97).

On this basis Marshall was able to con-
struct a theory of producer surplus that
was homologous with his theory of consumer
surplus. The two surpluses taken together
were a money measurement of the "satisfac-
tion" which a well-functioning economy
should maximize.

While this interpretation of Marshall's
theory of producer surplus is amply supported
by a comprehensive reading of his work, there
are occasional remarks in his writings which
seem to run counter to it. One remark which
clearly seems to reject the treatment of the
supply curve as measuring disutility cost is
contained in a letter of 28 April 1892 to F. Y.
Edgeworth:

I think Jevons did great harm by talking of supply
price as measuring disutility curve [sic]. In picking
blackberries, the disutility curve of effort and the
supply curve are practically the same thing and they
are in partial material with the demand curve or
utility curve. But in the case of animal factomes,
etc., the economic supply curve has but the
slightest connection with the laws of disutility; for
the greater part they are not in part material at all
(PE, II, 811).

When interpreted in the light of Marshall's
conception of the long-run supply curve, and
his tax-bounty theorem, this will be seen to be
perfectly consistent with the interpretation
given above. Marshall used the animal bar-
ometers illustration in connection with
production characterized by increasing re-
turns due to economies of scale. In the same
letter, Marshall added:

I have always held and taught in lectures year after
year that Producer's rent cannot be represented in
the supply curve except in curves in which one can
ignore the economies of organization and produc-
tion on a large scale (ibid., 812).

Marshall's treatment of economies of scale
costs entirely in the framework of his
theory of long-run equilibrium so the
explanation of his criticism of Jevons is to be
found by noting the critical difference, in
Marshall's view, between long-run and short-
run supply curves. Before examining this
directly, however, additional light may be
supplied by noting Marshall's application of
his surplus analysis to taxes and bounties on
decreasing taxes and increasing returns
industries; that analysis is restricted to the
long-run since, in Marshall's treatment,
increasing returns is strictly a long-run
phenomenon.

The tax-bounty theorem has frequently
been cited as one of the most notable
contributions of Marshall's Principles (see, e.g.,
Schumpeter, 1941, 245; Whitaker, in Mar-
shall, EEWAM, 2, 334n). The essence of the
theorem is that an excise tax, if levied on an
increasing cost industry, will yield more rev-
ue than the reduction in consumer surplus
which results from its imposition, in which an
output subsidy, awarded to a decreasing cost
industry, will cost less than the increase in
EASTERN ECONOMIC JOURNAL

consumer surplus which ensues. Thus, the intersection of free market demand and supply cannot be regarded as invariably defining optimum output since a judiciously designed mixture of taxes and bootlegs can alter outputs in such a way as to increase the aggregate consumer surplus.

Marshall's use of the consumer surplus device for tax analysis in ways unforeseen by the tax-bounty theorem of the Principles dates from his earliest work in economic theory in the late 1860's and early 1870's (see REVIEW OF ECONOMIC AND POLITICAL STUDIES p. 79, 1879, and 289-302). The theorem was pretty fully worked out in his manuscript on "The Pure Theory of Domestic Values" composed in the mid-1870's and privately printed by Sidgwick in 1879 (Ibid., 223-236). The notes for Marshall's 1880 lectures at Bristol on "The Economic Influence of Government" made reference to similar propositions (Ibid., 2, 384-385). In the Principles the tax-bounty theorem is contained in a chapter entitled "Theory of Changes in Normal Demand and Supply in Relation to the Doctrine of Maximum Satisfaction" where Marshall notes the case of decreasing cost industries as one of the two important qualifications to the proposition that the equilibrium of demand and supply defines the welfare optimum, the other being that the marginal utility of money is not the same to rich and poor consumers (PE, 1, 63 V, Ch. XII).

The crucial point to note about Marshall's tax-bounty theorem is that the argument is carried out solely in terms of the effects on consumer surplus; producer surplus is not taken into account. Why not? Producer surplus is included in Marshall's general discussion of the doctrine of maximum satisfaction; why is it absent in this case? The answer lies in the fact that the tax-bounty theorem was restricted to the circumstances where some industries operate under increasing cost conditions and others under decreasing cost conditions, but the latter can only be true of long-run cost. It is obvious that in the case of decreasing cost, if the cost curve is treated as mapping the first derivative of a total cost function against output, the integration of the function shows what, in Marshallian terms, would be a deficit rather than a surplus. But this was not Marshall's difficulty. In his view, the concept of producer surplus is inappropriate to long-run cost, whether increasing or decreasing.

Beginning with the second edition of the Principles, Marshall adopted the practice of referring to the short-run supply curve as a "particular expenses" curve. In his letter to Edgeworth quoted above, Marshall notes that he adopted the term "particular expenses" specifically to apply to supply curves from which an estimate of producer surplus may be derived. He never used the term in connection with the long-run supply curve. The main reason seems quite plain, though Marshall never stated it as explicitly as one would like. The short-run supply curve depicts the locus of marginal cost with some input factors held constant, while the long-run supply curve depicts the locus of average cost with all factors variable. This simple point is not clearly expressed in the Principles because in Marshall's treatise in the second edition of the Principles, in discussing the conditions of a stationary state, Marshall described the supply price as "the marginal price, the expectation of which just induces persons to enter the trade..." but deleted the word "marginal" from this passage in the fourth edition (PE, 1, 507; II, 507). In another passage (PE, 2, 373) dating from the second edition, (PE, II, 389), however, he refers to costs of production as "marginal" in both short and long periods as long as the wage or rent of labor and land is held constant. Mishan (1968) has noted that Marshall's concept of marginal cost was inappropriate when applied to a long-run supply curve with all factors variable. He does not note that Marshall himself had similar reservations.

Marshall's analysis of the long-run and of increasing returns to scale was transitory. Marshall himself was not happy with it, recognizing it to involve, in an acute form, time: "the center of the chief difficulty of almost every economic problem" (PE, I, viii) and to require the extension of economic theory beyond statics to dynamics and, indeed, beyond dynamics to "biology" and the study of "economic evolution." In his discussion of long-run decreasing cost in Appendix H of the Principles, Marshall suggested the use of three-dimensional geometric models and time as one of the variables, but he was, in private, very critical of Cunynghame's efforts in this direction (PE, I, 809; II, 804-810, 412). In a letter to Cunynghame on April 7, 1904, he said that his "case...of increasing returns never seemed to me of much practical use," noting also that the circumstances which make the long-run supply curve undefined in real time also applied to the demand curve (MAM, 449).

Marshall's treatment of these matters was exceedingly unclear, but on the point at issue here he seems to be clearly enough: the short-run supply curve can be treated as a static mapping of "particular expenses" against output, and therefore, it is a differential equation which can be integrated to show producer surplus. The long-period supply curve is not so static; it does not unambiguously map "particular expenses," and its integral would be meaningless. These, it seems to me, were his grounds for rejecting Jevons' identification of the long-run supply curve with disability cost.

Conclusion

A demand (supply) curve can be read two ways: as a statement of the quantities consumers (producers) will buy (sell) at different prices, or as a statement of the prices consumers (producers) are willing to pay (accept) for different quantities. According to the first reading, mathematical convention requires that quantity be treated as the dependent variable and price as the independent variable, and diagrammed accordingly.

In this letter to Cunynghame, Marshall was specifically referring to his analysis of tariffs and bounties in international trade. The point concerning the non-essentiality of the demand curve is equally damaging to his tax-bounty theorem though Marshall did not give more than vague and passing acknowledgment of this (see PE, I, 807). Throughout his writings, Marshall displayed much greater doubt as to the wisdom of state intervention in international trade than in domestic markets. It is worth noting that Marshall's theory of economic development made heavy use of the concept of increasing returns in a domestic context (see, e.g., TETJ, 45, 106, 138; MAM, 266; also Stigler, 1941, 76).

42