

A Defense of Marshall on Substitutes and Complements in Consumption

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It is clear that in the first edition of his *Principles of Economics* Marshall implicitly assumed that a consumer's income-utility function is the sum of independent commodity-utility functions (Stigler 1950a, p. 326; p. 326, n. 103; p. 327, n. 104), a condition that will be described as the assumption of additive utilities. In the second edition, his assumption of additive utilities became explicit in reference to "the every-day facts of economic life" and to the demand for commodities in exchange situations (Marshall 1891, p. 756 [845]).¹

I. Substitutes

Additive utilities and substitutes

In addition to assuming an additive utility function, Marshall demonstrated repeatedly his cognizance of the existence of substitute consumption commodities. He mentioned them as early as 1879 in *The Pure Theory of Domestic Values* (Marshall 1879, p. 15), and repeatedly in the first edition of the *Principles* (Marshall 1890, p. 160 [100], p. 168 n. [105 n.], p. 740 [841-42]). In subsequent editions he added more remarks about substitutes. Stigler has maintained that by recognizing substitute commodities Marshall was incon-

sistent, for the reason that a definition of substitutes in terms of interrelated utilities² is inconsistent with an additive utility function (Stigler 1950a, p. 327; Stigler 1950b, p. 389).³

This paper will point out that Marshall's recognition of the existence of substitutes is not inconsistent with the assumption of additive utilities. It will be shown first that the weight of textual evidence indicates that Marshall believed that additive utilities and substitutes are consistent. Then the sort of economic behavior that Marshall probably had in mind will be discussed. Finally, the formal proof that definitively settles the matter will be cited. It should be emphasized that in presenting these considerations—even in suggesting why Marshall might have made his assumptions—this study is not directly

²Alford described the utilities of substitutes as interrelated "if, for two commodities x and y , a rise in qx [the quantity of x] lowers the uy [utility of y] function and a fall in qy raises the ux function" (Alford 1956, pp. 29, 31). Therefore, when the utilities of x and y are interdependent and the commodities are substitutes, a change in the price of y , for example, will change the consumption of y and shift the utility curve for x . That will shift the marginal utility curve for x and therefore the demand curve for x .

³Stigler's article on utility theory is a fundamental contribution to the history of economic thought. It deals with many matters other than Marshall's work on substitutes and complements, and the present study is in no sense an adverse commentary on Stigler's powerful and brilliant treatment of a vast range of topics and writers. Indeed, the present study clearly reveals that its subjects cannot be discussed without reference to Stigler's article, which has been the only place in which a systematic examination and evaluation of Marshall's work on substitutes and complements appears.

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¹The editions cited in this paper are the first (1890), second (1891), third (1895), fourth (1898), and eighth (1920). When editions other than the eighth are cited and the passage also appears in the eighth edition, page references in the eighth edition are shown in square brackets.

concerned with defending or criticizing the assumption of additive utilities.

In the light of his many references to substitutes and his treatment of utility, it seems evident that Marshall must have thought about the implications of substitutability in relation to preference functions, and concluded that the utilities of substitutes are independent and that utilities are additive. If he had believed that the utilities of substitutes are significantly interdependent, the inconsistency of such a supposition with his additive utility function could not have escaped him. In fact, he addressed the issue of the desirability of assuming such a function in 1891, stating explicitly that in most cases the utilities of commodities are independent. He implied that interdependence of utilities either does not exist or is so slight that it can be disregarded, and rejected the utility function that Edgeworth suggested to recognize the interdependence of utilities. "Edgeworth's plan of representing U and V as general functions of x and y has great attractions to the mathematician," he wrote, "but it seems less adapted to express the everyday facts of economic life than that of regarding, as Jevons did, the marginal utilities of apples as functions of x simply" (Marshall 1891, p. 756 [845]; and see Edgeworth 1881, p. 20). Marshall then proceeded to write the utility function as the sum of the total utilities of the two commodities in his example.

Many of the everyday facts of economic life are consistent with the view that the utilities of substitutes are independent. If we exclude, as we should, the dinner-table type of experiment in which a person sits down and judges the utility of different quantities of edibles within the space of fifteen minutes, then we must concede that there is no necessary relation between many experiences with commodities that are substitutes for each other. In some cases only one of a group of substitutes is purchased, such as one brand of cigarettes, and it may happen that a fall in the price of a

substitute leads to its consumption instead, but that would not affect the utility curve for the commodity initially in use. For example, it would not affect the utility curve for the brand of cigarettes smoked initially nor the curve for any other brand. The utility derived from drinking a cup of coffee is not related to the utility that is derived from drinking a cup of tea, unless they are consumed in a dinner-table experiment, or perhaps in the very general and vague sense that every experience of a person's life is related, a sense that is not germane to the present issue.

Similarly, the utility of a cup of Tide detergent does not differ depending upon whether it is one of two cups used when a cup of Cheer is used per month, or a single cup used when two of Cheer are used during a month. A rise in the price of Tide would result in a substitution effect in favor of Cheer, but that would not make Tide appear more or less useful and attractive than before. If the price of a commodity rises, its consumption decreases because it becomes relatively more expensive, and the consumption of its substitutes increases because they become relatively less expensive means of satisfying the same general want, but the utility curve for each such commodity is often unaffected, or affected only slightly. These considerations indicate that Marshall could have found some basis in economic behavior for the assumption that substitutes have independent utilities, even though that assumption is not of general validity.⁴

In any event, the logical consistency of Marshall's assumptions of additive utilities and of the existence of substitutes has been established by a formal proof, and Marshall's

⁴It is now generally recognized that the assumption that substitutes have independent utilities leads to some unacceptable conclusions about consumer behavior. Similarly, the assumption that the utilities of all substitutes are interrelated is untenable because it fails to recognize that the utilities of many commodities are unaffected by changes in the consumption of their substitutes.

remarkable theoretical intuition has been substantiated. It is well known that additive utilities and diminishing marginal utilities for all commodities are sufficient conditions to ensure that all demand curves are negatively sloped, and that the quantity demanded of each commodity increases with income (Liebhafsky 1969; and see Marshall 1890, p. 159–60 [99, 101 n.]). As an extension of that analysis, Eugene Silberberg (1972) has proved that, given the two foregoing conditions, all goods must be net substitutes, that is, there is a positive pure substitution effect of a change in the price of the j th commodity upon the quantity demanded of the i th commodity ($i, j = 1, \dots, n$). This result agrees with the intuitive notion that "all things are more or less imperfect substitutes for each other" (Chamberlin 1950, p. 65).

Adherence to the assumption of additive utilities

Stigler also argued that Marshall fell into inconsistency because he was, in Stigler's opinion, persuaded at one juncture in his career that the utilities of commodities may be interdependent, and consequently modified some of his conclusions in ways which depend on that notion, but failed to abandon his additive utility function (Stigler 1950a, p. 327). In support of his opinion, Stigler brought forward three considerations. First, he argued (1950a, p. 327) that Marshall appeared to have been influenced by Irving Fisher to attach more importance to the interrelationships of utilities in the third edition of the *Principles*, as evidenced by the introduction of a laudatory reference to Fisher's *Mathematical Investigations in the Theory of Value and Prices* (Marshall 1895, p. 460 n. [390–91 n.]). "Once persuaded," Stigler wrote, "Marshall modified his theory" (Stigler 1950a, p. 327). There is, however, no evidence that Fisher's work led Marshall to recognize the existence of interrelationships between the utilities of commodities that are

substitutes in consumption. In the relevant section of Fisher's book, the results of the interdependence of utilities are revealed by mechanical analogy through a change in the shapes of the cisterns in his pedagogical contraption. "However," Fisher argued,

in general the interdependence in the shapes of the cisterns is very slight. That is, the utility of a commodity usually varies so much more under a variation in the quantity of that commodity than under variations of other commodities that the relations discussed in Part I [where the commodities are treated as having independent utilities] may be regarded as good first approximations (Fisher 1892, p. 67).

It was only in the special cases of perfect substitutes and perfect complements that Fisher believed that the interrelationships are "really important," and, he pointed out, when related goods are treated as a single commodity, the problem of interrelated utilities disappears (ibid., pp. 66–67). To buttress his argument about the unimportance of interdependence of utilities, Fisher quoted Marshall's comment on Edgeworth's utility function (ibid., p. 67 n.). If Marshall was indeed influenced by Fisher, or vice versa, it would appear more reasonable to suppose that it was in the direction of believing that the interrelationships of utilities between both substitutes and complements can be disregarded.

Second, Stigler (1950a, p. 327) stated that Marshall gave "more weight to interrelationships of utilities in the third edition of the *Principles* (1895). Once persuaded, Marshall modified his theory on two points. The first was that he slightly modified his assertion of the universality of negatively sloping demand curves and, in fact, introduced the Giffen paradox as an exception."

Stigler's belief that Giffen's paradox is inconsistent with additive utilities is not accurate. Eugene Silberberg has shown (1972) that if one commodity exhibits increasing marginal utility, then additive utilities, income-inferior commodities, and substitu-

tion effects for those commodities are all consistent phenomena.⁵ It follows that if the price of one of the income-inferior commodities rises, the quantity demanded of that commodity tends to fall because of the negative substitution effect, but tends to rise because of the positive income effect. The income effect may outweigh the substitution effect, in which case the quantity demanded rises, and the commodity therefore displays Giffen's paradox.⁶

In any event, there is no evidence for Stigler's contention that Marshall knowingly abandoned his assumption of additive utilities when he dealt with Giffen's paradox. Marshall did not introduce the paradox in recognition of the interdependence of the utilities of commodities, but in recognition of the possibility that the marginal utility of income may vary. This is not an interpretation but a simple restatement of what Marshall wrote. He introduced the paradox, and then argued in relation to it that "it is seldom necessary to take account of changes in the purchaser's command of money" (Marshall 1895, 208, [132]), thus indicating the issue with which

⁵Marshall discussed income-inferior commodities with downward-sloping demand curves in the following passage: "It is even conceivable, though not probable, that a simultaneous and proportionate fall in the price of all teas may diminish the demand for some particular kind of it; if it happens that those whom the increased cheapness of tea leads to substitute a superior kind for it are more numerous than those who are led to take it in the place of an inferior kind" (Marshall 1895, p. 176 n. [100 n.]). Despite the reference to substituting one kind of tea for another, this passage does not refer to ordinary substitution effects, since the relative prices of the substitutes are constant. The fall in their absolute price levels gives rise to income effects. Some individuals regard a particular kind of tea as an income-inferior good, and their demand curves for it shift to the left; others regard it as an income-normal good, and their demand curves for it shift to the right. It should be noted that if utilities are additive, the behavior described in this passage, like Giffen's paradox, is inconsistent with the assumption that all marginal utilities are decreasing.

⁶I am indebted to Professor Eugene Silberberg for furnishing me in personal correspondence with mathematical proofs of the properties of the Giffen case in relation to additive utilities.

he was concerned. He went on to write:

There are however some exceptions. For instance, as Mr Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families and raises so much the marginal utility of money to them, that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and, bread being still the cheapest food which they can get and will take, they consume more, and not less of it (ibid., p. 208 [132], and see Marshall 1903, p. 382).

Clearly, Marshall was concerned about an exception to the constancy of the marginal utility of income, not an exception to the consequences of an additive utility function. The diminution in the consumption of relatively expensive foods occurs, Marshall believed, because the consumer has less income. Marshall did not change the prices of substitutes for bread and examine the effects upon the utility or demand curve for bread, nor did he assert that the change in the price of bread shifts or affects the utility or demand functions for its substitutes. He changed the price of bread, examined the impact on the quantity of bread consumed, and asserted that Giffen's paradox affects the form of the demand curve for bread because of the increased marginal utility of income.

Third, as evidence for his contention Stigler offered (1950a, p. 327) the sentence in which Marshall wrote that when varying the demand price of a particular commodity it is assumed that other things are constant, and "when the total utilities of two commodities which contribute to the same purpose are calculated on this plan, we cannot say that the total utility of the two together is equal to the sum of the total utilities of each separately" (Marshall 1895, p. 207 [131]).⁷ In an explan-

⁷D. M. Mayston (1976, pp. 498-99), like Stigler, uses this passage as a basis for his contention that Marshall treated utilities as interdependent. Mayston also believes, without furnishing any real evidence or controverting the evidence to the contrary, that Marshall's utility functions were ordinal.

atory footnote, written at the same time as the foregoing quotation and appended to it, Marshall added: "Some ambiguous phrases in earlier editions appear to have suggested to some readers the opposite opinion" (Marshall 1895, p. 207 n. [131 n.]). This was described by Stigler as "less than candid" (Stigler 1950a, p. 327 n.), evidently in the belief that the comment reinforces his view that Marshall had inconsistently recognized interdependent utilities. Stigler went on to observe that "these alterations were only patchwork repairs; Marshall did not rework his theory of utility. He retained to the last a theory constructed on the assumption of an additive utility function" (ibid., p. 327). The last sentence of that statement is true, but it is argued here, in contrast to the first sentence, that the changes Marshall made in the third edition were not alterations of his theory. Specifically, Marshall's comment about "ambiguous phrases" is not an admission that utilities are interdependent and that he had erred by writing phrases which mistakenly encouraged the interpretation that they are independent. Marshall's text and his footnote refer to a totally different issue, namely, to the conditions of summation of independent utilities.

The footnote is attached to a statement about the summation of utilities, and makes clear, by the sentences that immediately follow the comment, that Marshall's concern was that some ambiguous phrases in earlier editions might have led some readers to suppose that summing up all utilities is possible:

But the task of adding together the total utilities of all commodities, so as to obtain the aggregate of the total utility of all wealth, is beyond the range of any but the most elaborate mathematical formulae. An attempt to treat it by them some years ago convinced the present writer that even if the task be theoretically feasible, the result would be encumbered by so many hypotheses as to be practically useless (Marshall 1895, p. 207 n. [131 n.]).

The problems that make the summation difficult are not those introduced by interdependent utilities but those that arise in connection with the double-counting of utility (ibid., pp. 208 n. [132 n.], 795 [842]; Marshall 1898, p. 206 n. [132 n.]), and the related difficulty that will now be examined.

In the footnote just quoted Marshall went on to explain in greater detail the meaning of the statement in the text regarding "the total utility of the two together": "It is obvious that, if tea were inaccessible, people would increase their consumption of coffee, and vice versa. The loss that people would suffer from being deprived both of tea and coffee would be greater than the sum of their losses from being deprived of either alone" (ibid., p. 207 n. [131 n.]). In this passage, which is also quoted as evidence in support of his interpretation by Stigler (1950b, p. 385 n.), it is certainly true that Marshall was referring to substitute commodities, such as tea and coffee, which he described as contributing to the same purpose. Nevertheless, he did not imply that their utilities are interdependent, and, indeed, an examination of the passage leads to the opposite conclusion.

The correct interpretation of Marshall's remarks can be explained by examining them in application to an individual's utility functions for tea and coffee. Initially, as shown in Figure 1, the person is consuming C_0 of coffee and T_0 of tea, with the indicated respective amounts of utility derived from them. Thus "the total utility of the two together" is equal to $U(C_0) + U(T_0)$. The only possible meaning of "the sum of the total utilities of each separately" is that the total utility of one is calculated in the absence of the other and vice versa, and that the utilities calculated in this way are then added together. In fact, Marshall explained that the total utility of each commodity separately would be determined by the amount of the commodity consumed if the other were inaccessible. He also indicated that if one commodity were

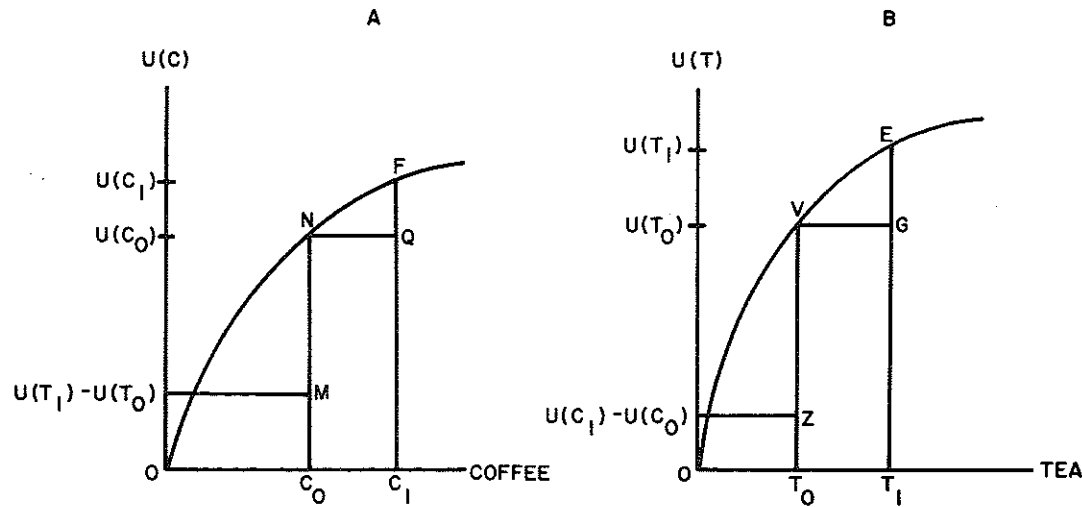


Figure 1

inaccessible a person would increase his or her consumption of the substitute. Accordingly, let it be assumed that the utility functions for tea and coffee are independent and additive, and that the individual would consume C_1 units of coffee if no tea is available. He would obtain $U(C_1)$ utils and would therefore gain $U(C_1) - U(C_0) = QF$ from coffee, as compared with the initial situation, but would lose $U(T_0)$ because tea becomes inaccessible. Marshall wanted to measure the loss of utility that occurs when the consumer is deprived of tea and consumes more coffee, and that is $U(T_0)$ minus the additional utility QF from the increased consumption of coffee. The net loss is therefore equal to ZV in Fig. 1B.

Similarly, if the individual has no coffee and consumes only tea, he or she would lose $U(C_0)$ in Fig. 1A, and would consume more tea, gaining $U(T_1) - U(T_0) = GE$ in Fig. 1B. Subtracting the latter amount from the loss $U(C_0)$ in Fig. 1A, the net loss to the individual if he is not able to drink coffee is seen to be MN . The loss that a person would suffer from being deprived of both tea and coffee, namely $U(C_0) + U(T_0)$, would be greater, Marshall wrote, than the sum of their losses

from being deprived of either alone. That is quite true, because $U(C_0) + U(T_0)$, must be greater than the sum of MN and ZV . Thus it can be seen that Marshall's recognition of substitute commodities in his consumer demand theory did not involve him in an inconsistency. Whether the utilities of substitutes are actually interdependent or not, his statements reveal that he treated their utilities as independent and additive like the utilities of the other commodities in his income-utility function.

Changes in the prices of substitutes

Marshall's treatment of changes in the prices of substitutes will now be examined. Stigler wrote that the purpose of Marshall's proviso that the prices of rival goods be held constant "is obvious . . . ; when p_i rises, consumers will shift to close rivals, and their prices will tend to rise even if the price level is stable, so the effect of changes only in p_i on purchases of X_i will be obscured" (Stigler 1950b, p. 389). Stigler observed that Marshall's insistence on this proviso "is troublesome to reconcile with his utility theory . . . because rival goods are defined in terms of

utility and cannot exist with an additive utility function" (Stigler 1950b, p. 389), a criticism that has already been rejected. Stigler went on to write, however, that "we can of course eliminate this difficulty by generalizing the utility function or shifting to a definition of rival products in terms of demand cross-elasticities" (ibid.). It has been seen that Marshall rejected the idea of generalizing the utility function, but, as will now be shown, he did in actuality follow the alternative procedure suggested by Stigler. Although Marshall indicated that a change in the price of a commodity may shift the demand curve for a substitute, he did not do so in the belief that the shift occurs because of a change in the position of the utility curve for the substitute, and therefore in effect he treated rival products in terms of demand cross-elasticities.

It is not at all obvious that when Marshall introduced his proviso the objective uppermost in his mind was to rule out the endogenous process described by Stigler whereby a rise in p_i induces a rise in p_j , and hence leads to a mitigation of the adverse substitution effect against X_i . The primary reason for Marshall's proviso, judging by the fact that it was the only reason that he gave explicitly, was that the quantity demanded of X_i at any given price may be altered by an exogenous change in the price of a substitute X_j —not by a change that results from variations in the price of X_i . Thus he argued that the prices of substitutes have to be held parametrically constant along the demand curve for X_i in order to trace out a given locus, and that changes of such prices will shift the demand curve. These observations first appeared in *The Pure Theory of Domestic Values*, where it is explained that the market demand curve will shift as a result of "the invention or the great cheapening of any other ware which comes to be used largely as a substitute for it" (Marshall 1879, p. 15). The introduction of a new commodity alters demand functions, but

the period of flux is followed by a new stable situation, in which a change in the price of a substitute must be great to have an impact. Another sort of change raises the price of a substitute, and shifts the demand curve for the given commodity, namely, "the deficiency of the supply of any ware for which the ware in question may be used as a substitute." The deficiency that gives rise to the shift of demand, Marshall explained, is of the magnitude induced by a major alteration of the conditions of supply of the substitute, such as would be "occasioned by bad harvests, by war, or by the imposition of customs or excise taxes" (ibid.). Then, in the first edition of the *Principles*, Marshall wrote that if parametric demand conditions "vary in any respect the figures of the schedule will probably require to be changed. One condition which it is especially important to watch is the price of rival commodities, that is, of commodities which can be used as substitutes for it" (Marshall 1890, p. 160). For instance, Marshall continued, the demand schedule for tea is drawn out on the assumption that the price of coffee is known, but, he noted, once again referring to an exogenously-induced and major change in the price of a substitute, a failure of the coffee harvest would raise the prices throughout the demand schedule for tea (Marshall 1890, p. 160 [100]).⁸ In yet another discussion of other things that must be constant along the demand curve (ibid., p. 170 [109]), Marshall stated again that "al-

⁸To this passage Marshall added the following sentence in the third edition: "The demand for gas is liable to be reduced by an improvement in electric lighting; and in the same way a fall in the price of a particular kind of tea may cause it to be substituted for an inferior but cheaper variety" (Marshall 1895, p. 176 [100]). By an improvement in electric lighting Marshall was referring to an innovation, not to a change in the price of electricity; and innovations can lead to shifts of the utility curves for a number of commodities. The second part of the statement, although attached to a paragraph that deals with the parameters of the market demand curve, nevertheless does not deal with a shift of demand but with the elasticity of a given stable demand curve.

lowance must be made . . . for the . . . cheapening of other things that can be applied to the same uses with it" (ibid., p. 171 [110]).

It is evident from the foregoing material that Marshall did not discuss the issue of the effects of a change in the price of a substitute from the standpoint of the possible interrelatedness of utilities. All his discussion of the matter in the passages just examined is couched in terms of shifts of the demand for the i th commodity as a consequence of exogenously induced changes in the price of the j th commodity, and none of it is couched in terms of shifts of the utility function of a commodity occurring as a consequence of changes in the amounts of its substitutes that are consumed as a result of changes in p_i , p_j , or any other variable.

An exception to Marshall's thesis

It would be hard to believe that none of Marshall's remarks can be interpreted as an exception to his central thesis. In fact, Marshall inserted such a remark into the second edition, stating that the desire for a commodity may be altered by the cheapening of the supply of a substitute (Marshall 1891, p. 157 [100]). This might be interpreted as recognition that a change in p_j , by changing X_j , shifts the marginal utility curve for X_i , which shifts the demand curve for X_i , a sequence that results from the interdependence of utilities. Marshall did not, however, present this chain of reasoning. He made no elaboration of the statement, no illustration of it, no reference to its implications; and I have been unable to find any repetition of the concept in his work. It is concluded, therefore, that the statement is an isolated exception, not a part of his systematic theory, and that it does not qualify Marshall's thesis that the interdependence of the utilities of substitutes is nonexistent or so slight that it can generally be disregarded.

II. Complements

Complements not part of Marshall's theory

Stigler observed that the utility theory of complementarity had to wait until a generalized utility function was used, because the usual relationships among the utilities of commodities cannot be defined if the utility function is additive (Stigler 1950b, p. 384). That is not strictly true, because complementarity can exist with an additive utility function (Silberberg 1972). The issue with which this paper is concerned, however, is that Stigler also maintained that Marshall was inconsistent because he recognized the existence of complementary commodities in the theory of consumer demand, despite his use of an additive utility function (Stigler 1950b, p. 385). In contrast, it will be demonstrated that complementary consumption commodities are not part of Marshall's demand theory.

None of the material cited by Stigler to show that Marshall recognized complementarity makes any mention of that relationship. Marshall's examples quoted by Stigler deal only with substitutes. The section of his article entitled "Complementarity" introduced his criticism of Marshall, but then referred immediately to Marshall's views on substitutes:

Marshall displayed greater inconsistencies than Pareto, for he implicitly followed the Auspitz-Lieben definition even though he employed an additive utility function which did not permit of complementarity. Thus he speaks of "rival commodities, that is, of commodities which can be used as substitutes for it" (ibid.).

Clearly, Marshall's comments on substitute commodities do not support Stigler's judgment about his treatment of complements. Stigler appended a footnote to the foregoing quote in which he cited the first edition of the *Principles* again: "See also pp. 438 and 178 n., with its accompanying Mathematical Note VI referring to 'several commodities which

will satisfy the same imperative want. . . ." (ibid., p. 385 n.). Page 438 of the first edition, however, has nothing to do with complements in the theory of consumer demand or even with complements in the theory of production. It deals exclusively with joint products and composite supply in the theory of production, substitute finished goods and factors of production, and increasing and diminishing returns. Similarly, page 178 n. of the first edition refers not to complements but to substitutes: "The desire for anything is much dependent on the difficulty of getting substitutes for it." Finally, Marshall made no mention of complements in Mathematical Note VI, and the phrase that Stigler quoted from it in his footnote obviously deals with substitutes. To make this perfectly clear, after the reference to "imperative wants" Marshall went on to write, "as e.g. water and milk, either of which will quench thirst" (Marshall 1890, p. 740 [842]).

Stigler also wrote that Marshall held the price of complementary commodities constant along the consumer's demand curve, adopting that condition in the second edition (Stigler 1950b, p. 389 n.). In fact, Stigler does not provide any evidence to support his contention, and I have been unable to find any. He gave page 158 of the second edition and page 100 n. of the eighth edition (ibid.) as references, all of which material appears on page 100 of the eighth edition, in the text and in a footnote. The text refers exclusively to substitutes, with tea and coffee, and gas and electricity, given as specific examples. The footnote contains two paragraphs, the first of which deals exclusively with substitutes, using once more the example of tea and coffee. The second paragraph in the footnote, which is presumably the material to which Stigler had reference, dates from the second edition, where it appeared in the text on page 158. Although inserted in a chapter on consumer demand, the paragraph deals exclusively with

the theory of production:

Again, a commodity may be simultaneously demanded for several uses (for instance there may be a "composite demand" for leather for making shoes and portmanteaus); the demand for a thing may be conditional on there being a supply of some other thing without which it would not be of much service (thus there may be a "joint demand" for raw cotton and cotton-spinners' labour).

The commodities to which Marshall referred as jointly demanded are complements in the process of production, not consumption. The final part of the footnote goes on to mention the demand for a commodity by professional dealers and has nothing to do with complements or substitutes. In short, there is nothing in the material cited by Stigler which states or implies that the prices of complements should be held constant along the demand curve for a consumption good, or that mentions the prices of complements, or consumption complements themselves, in any connection. The same is true of the passages in which Marshall took up the issue of holding the prices of other commodities constant as the price of the particular commodity is varied (Marshall 1920, pp. 100, 131). He repeatedly mentioned the prices of substitutes, but I have not been able to find in the *Principles* or elsewhere in Marshall's published work any mention of constancy of the prices of complements along the demand curve for a consumption commodity.

A dubious exception

There is, so far as I can determine, only one passage in the *Principles* in which the meaning of complementarity is used in a possible connection with the theory of consumer demand, but the passage is more easily explained as a statement about the theory of production which appears in proximity to a discussion of consumer demand. The place in question is Mathematical Note VII, which was introduced in the first edition. As has

been observed above, the note deals with a measure of the utility of income. The measure is explained as the sum of the definite integrals of the demand curves for the interval between the amount of each commodity that is necessary for existence and the amount consumed. In the third edition Marshall added the statement that the sum of those integrals could be used only "if we could find a plan for grouping together in one common demand curve all those things which satisfy the same wants, and are rivals; and also for every group of things of which the services are complementary (*see Book V, ch. VI*)" (Marshall 1895, p. 795 [842]).⁹ Why did Marshall mention things which satisfy wants in connection with rival commodities, but then refer to the *services* of complements? Why did he refer the reader to book V, chapter VI for further elaboration of his position?

The answer to both questions is that the reference to complementarity had no close connection in Marshall's mind with the subject of consumer demand. This is evident from the fact that the chapter to which he directed the reader is concerned almost exclusively with the theory of production, and

⁹Marshall went on to observe that related commodities cannot be grouped under a single demand curve (Marshall 1895, p. 795 [842]). He made this statement in reference to the problem of achieving a measure of the total utility of income, but for other purposes he repeatedly indicated elsewhere in the *Principles* that substitutes can be grouped together, although he did not again mention consumption complements. "It may be best," he wrote, "to group together commodities as distinct as beef and mutton, or even as tea and coffee, and to have a single schedule to represent the demand for the two combined" (Marshall 1890, p. 160 n. [100 n.]). He also remarked that if the causes which determine the production of rival finished commodities "are nearly the same, they may for many purposes be treated as one commodity. For instance, beef and mutton may be treated as varieties of one commodity for many purposes" (*ibid.*, p. 438 [391]). Again, as has been seen, Marshall observed that different kinds of fresh meat can be grouped under one demand curve (*ibid.*, p., 168 n. [105 n.]). Indeed, "for some purposes," he insisted, "such things as tea and coffee *must* be grouped together as one commodity" (Marshall 1895, p. 207 n. [131 n., italics added]).

discusses the services of complements only in the context of production. The chapter deals, among other things, with the joint demand for factors of production that are used together to make an output (Marshall 1890, p. 430 [381–82]). In some examples of joint demand Marshall assumed that variation of the proportions of the inputs is possible, but he also presented the demand by producers for knife handles and blades as a case of perfect complementarity. His extremely brief remarks in book V, chapter VI about related consumption commodities, of which there are only three, deal not with complements but with substitutes—beef and mutton, different kinds of sewing machines, and different kinds of electric lights (*ibid.*, pp. 438, 439 [391, 391–92 n.]). In the fourth edition Marshall introduced the word "complementary" into the chapter, observing that

hops and malt are complementary. . . . Thus the demand for each of several complementary things is derived from the services which they *jointly* render in the production of some ultimate product. . . . In other words there is a *joint demand* for the services which any of these things render in helping to produce a thing which satisfies wants directly and for which there is therefore a direct demand: the direct demand for the finished product is in effect split up into many derived demands for the things used in producing it (Marshall 1898, p. 453 [381]).

This is a statement about complementary inputs *in production*. A second comment in the same chapter reads: "There is . . . a composite demand on the part of upholsterer and shoemaker for leather; and also for cloth when the upper part of a shoe is made of cloth: the shoe offers a joint demand for cloth and leather, they offering complementary supplies" (Marshall 1898, p. 465 n. [393 n.]). Once again, the remarks regarding cloth and leather refer to complementarity of factors of production.

It has been observed earlier that Stigler believed "Fisher's discussion of competing

and completing goods seems to have been the stimulus to Marshall to give more weight to interrelationships of utilities," on the ground that he made reference "to Fisher's 'brilliant' book, precisely on this point" (Stigler 1950, p. 327 n.). In actuality, Marshall did not mention complements in his comment on Fisher. It appears in book V, chapter VI, which, as has been observed, is dominated by a concern with production. In the section related to the comment, Marshall explained that "we may pass to the problem of COMPOSITE SUPPLY which is analogous to that of composite demand. It is closely connected with the Law of Substitution which has been noticed already. We may consider that two things are *rivals* or competing commodities when they are capable of satisfying the same demand" (Marshall 1890, p. 438). In the third edition (1895, p. 460 n.) Marshall attached his remark about Fisher as a footnote to the phrase ending with the words "competing commodities," and credited Fisher with that term, thus indicating that Fisher's book had impressed him in regard to its treatment of substitutes. In the fourth edition the passage in the text was altered to read:

A demand can often be satisfied by any one of several routes, according to the principle of substitution. These various routes are rivals or competitors with one another; and the corresponding supplies of commodities are *rival*, or *competitive* supplies relatively to one another. But in relation to the demand they co-operate with one another; being "compounded" into the total supply that meets the demand (Marshall 1898, p. 462 [390]).

The number of the footnote referring to Fisher was moved to the last word of the foregoing quotation, and thus, once again, the footnote was clearly attached to a discussion of substitutes. The footnote itself has nothing to do with complements. On the contrary, it also deals with substitutes: "The latter phrase 'competing commodities' is used by Prof.

Fisher in his brilliant *Mathematical Investigations in the theory of value and prices*, which throw much light on the subjects discussed in the present chapter" (Marshall 1898, p. 462 n. [390–91 n.]). There is, therefore, no support in the passages that have just been examined for the belief that Marshall identified complementary consumer's commodities.

In summary, I have not been able to find in Marshall's works any discussion whatsoever of complementary consumers' goods, or any example of commodities that are complementary to a consumer, or any mention of data or behavior that would be explicable by reference to complements in the theory of consumer demand. Marshall's remark about grouping complementary commodities under a single demand curve is an isolated exception of debatable meaning and significance. It is not accompanied by an examination of complements; it is not derived from an underlying analysis; nor are any implications drawn from it. It is a fragmentary comment that is totally unintegrated into his theory of consumer demand, in which complementary commodities do not appear. Since Marshall was aware of the concept of complementarity in consumption—he presumably had read Fisher on the topic—it seems clear that complements did not find their way into his theory of consumer demand because, like Fisher and Samuelson (1947, pp. 183–87; Samuelson 1974, pp. 1261, 1266), he thought that they are not important.

III. Conclusion

It is clear that Marshall believed that the utilities of substitutes are ordinarily independent or so slightly interrelated that their interdependence can be neglected, and in examples and applications he treated them as independent. He should therefore be absolved of the charge of inconsistency in regard to

substitutes. Whatever the importance of complements may be, it should be recognized that they play no part in Marshall's theory of consumer demand. It also follows that there can be no grounds for alleging that his work contains an inconsistency in regard to them.

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