The Dubious Case for Decreasing Costs: A Correction

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In a previous issue of this journal, William Shropshire and Barbara Zoloth [4] noted that textbook authors frequently attribute long-run decreasing costs in a competitive industry to the purchase of inputs from monopolistic suppliers who experience internal economies and sell larger quantities at lower prices. Shropshire and Zoloth then argued that a general case for decreasing costs cannot be made on this circumstance because there is no output-supply curve for the competitive industry.

If this assertion were true it would call for a revision of nearly all intermediate economics texts (e.g., [1 pp. 87-88, 2 pp. 312-318, 5 p. 202]) and the rethinking of applied studies. In this note I show that the output supply curve for a competitive industry buying from a monopolist supplier with decreasing costs is defined under standard general assumptions.

Deriving the Long-Run Supply Curve

I largely follow the notation of Shropshire and Zoloth except for explicitly introducing the number of firms, \(N\). Firms are assumed to be identical, and the indivisibility of firms is ignored. The production function for each firm is \(x = h(a, b)\) where \(x\) is output sold in a competitive market and \(a\) and \(b\) are inputs. Shropshire and Zoloth took a wrong turn in introducing the demand function into the derivation of supply. Their equation (3) introduced the zero-profit condition for the point of competitive market equilibrium, equating total industry revenue and total industry costs. The demand function for the competitive output was a component of revenues and was involved in the subsequent analysis. In fact, the zero-profit condition is a result of entry (a supply-side phenomenon) and prevails everywhere on the long-run supply curve. That is, unless there is zero economic profit there will be a tendency for firms to enter, and firms including potential entrants will not be in equilibrium. Long-run competitive supply requires zero profit whether the industry shows decreasing costs or not. Market equilibrium and demand are unnecessary constructs in deriving the supply. Competitive entry forces all firms to produce at the minimum of long-run average costs and, ignoring the indivisibility of firms, replication of firms expands output to whatever level is required. Each firm maximizes profit, equating marginal cost to price at the minimum of average costs.

Through the process of entry and competitive pressure on each firm the industry is led to produce any output at minimum cost. Hence, the long-run (inverse) supply curve is defined by the price that equals average cost of production with the minimum cost number of firms and \((a, b)\) for each firm. For output \(X\)

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the total costs of production are \( N(p_a a + p_b b) \) where each firm takes \( p_a \) and \( p_b \) as exogenous. The long-run inverse supply function can be derived by minimizing total cost subject to an output constraint \( X = N(a, b) \).

The Lagrangian is

\[
L = N(p_a a + p_b b) - ((N(a, b) - X))
\]

with first-order conditions

\[
\begin{align*}
N_a a &- t N = 0 \\
N_b b &- t N = 0 \\
N_a a &- t N = 0 \\
N_b b &- t N = 0
\end{align*}
\]

The second-order condition is consistent with decreasing marginal costs \( \frac{\partial^2 C}{\partial A^2} < 0 \) when marginal revenue falls faster than marginal costs. Invoking the implicit function theorem, the second-order condition guarantees that the first-order condition will yield a price function \( p_a = f_a(p_a, X) \).

The optimal profit function can be substituted into \( a, b \) and \( N \) to give

\[
\begin{align*}
a &= \hat{a}(p_a, X) \\
b &= \hat{b}(p_a, X) \\
N &= \hat{N}(p_a, X)
\end{align*}
\]

and these can be used to derive average cost for the competitive industry,

\[
AC = \frac{(p_a + p_b)N}{X}
\]

Long-run competitive supply requires zero profit or that price equal average industry cost (with identical firms), so that

\[
p_a = \frac{(p_a + p_b)N}{X}
\]

The conditional demands of the competitive industry are

\[
A = \hat{N} a = A(p_a, p_b, X)
\]

and

\[
B = \hat{N} b = B(p_a, p_b, X)
\]

Now, let input \( a \) be supplied by a monopolist. The monopolist will maximize \( X = A(p_a, A) - C(A) \) where \( C(A) \) is the cost function for producing input \( a \). The first and second-order conditions are,

\[
\frac{d^2 X}{dA^2} = \frac{\partial^2 C}{\partial A^2} < 0
\]

\[
\frac{d^2 A}{dp_a^2} = \frac{\partial^2 C}{\partial A^2} \frac{\partial A}{\partial p_a} \\
- \frac{\partial^2 C}{\partial A^2} \frac{\partial A}{\partial p_a} < 0
\]

X that will be supplied . . . for any given price, as required, in the definition of supply" [4, p. 114]. Equation (10) can be equated to market demand, \( X = D(p_a) \), to derive the market equilibrium price and quantity and the correct counterpart in Shapiro and Zodrow's equation (3). The supply function differs from the usual case only in that the price \( p_a \) does not appear.

It remains to show that the model is consistent with a decreasing cost industry, i.e., \( \frac{dp_a}{dx} < 0 \). Differentiating (8) totally with respect to \( X \) gives

\[
\frac{dp_a}{dx} = \frac{\frac{\partial p_a}{\partial X} \frac{\partial X}{\partial p_a} + \frac{\partial p_a}{\partial A} \frac{\partial A}{\partial p_a} + \frac{\partial p_a}{\partial N} \frac{\partial N}{\partial p_a}}{\frac{\partial X}{\partial p_a}}
\]

where

\[
\frac{dA}{dx} = \frac{\frac{\partial A}{\partial p_a} \frac{\partial p_a}{\partial X} + \frac{\partial A}{\partial N} \frac{\partial N}{\partial p_a}}{\frac{\partial X}{\partial p_a}} + \frac{\partial A}{\partial p_a} \frac{\partial p_a}{\partial X} + \frac{\partial A}{\partial N} \frac{\partial N}{\partial p_a}
\]

\[
\frac{dB}{dx} = \frac{\frac{\partial B}{\partial p_a} \frac{\partial p_a}{\partial X} + \frac{\partial B}{\partial N} \frac{\partial N}{\partial p_a}}{\frac{\partial X}{\partial p_a}} + \frac{\partial B}{\partial p_a} \frac{\partial p_a}{\partial X} + \frac{\partial B}{\partial N} \frac{\partial N}{\partial p_a}
\]

The competitive industry expands to produce any output with fixed input prices by the replication of firms, each producing at the minimum of average costs, so that \( \frac{dA}{dx} = \frac{\partial A}{\partial p_a} \frac{\partial p_a}{\partial X} + \frac{\partial A}{\partial N} \frac{\partial N}{\partial p_a} \). Hence, (13) can be simplified, and substituting into (12),

\[
\frac{dp_a}{dx} = \frac{A + \frac{\partial A}{\partial p_a} \frac{\partial p_a}{\partial X} + \frac{\partial A}{\partial N} \frac{\partial N}{\partial p_a}}{X \frac{\partial X}{\partial p_a}}
\]

From the first-order condition of (5) above, \( A = p_a b / p_b < 0 \), and it can be shown in the usual fashion that \( A \) and \( B \) are homogeneous of degree zero in \( p_a \) and \( p_b \) and \( \frac{\partial A}{\partial p_b} < 0 \), \( \frac{\partial A}{\partial p_b} < 0 \), and \( \frac{\partial A}{\partial p_b} > 0 \). Therefore, the first factor in (14) is ambiguous in sign. From (14) it would appear that \( dp_a / dx \) < 0 can occur with an input price from the monopolist that increases or decreases with the competitive industry output, depending on the sign of the first factor.

It is possible, however, to rule out \( dp_a / dx < 0 \) when \( \frac{\partial p_a}{\partial X} \). Recall that in the competitive industry any output is produced by replication of firms with each firm producing a minimum average cost output. The competitive price is the minimum average cost for each firm. Let \( C(x, p) \) be the cost function for an individual firm and let \( x \) be the minimum average cost output when input prices are \( p = (p_a, p_b) \). Let \( p_x > p_a \) be associated with a higher industry output in accordance with \( \frac{\partial p_a}{\partial X} > 0 \), and let \( x \) be the minimum average cost output for prices \( p = (p_a, p_b) \). The minimum average cost for the price vectors \( C(x, p) / x \) and \( C(x, p) / p \) is the minimum of average costs for \( p \) and the property that the cost function is nondecreasing in input prices if it follows that

\[
\frac{C(x, p)}{x} > \frac{C(x, p)}{x}
\]

Hence,

\[
\frac{dp_a}{dx} > 0
\]

is inconsistent with decreasing costs in the competitive industry. For the source of decreasing costs in a competitive industry to be purchases of an input from a monopolist, the price of that input must fall as the output of the competitive industry expands. In that case, decreasing costs result because the industry decreases its expenditures per unit of \( X \) on the relatively more expensive other input, "b", more than it increases its expenditures on the relatively cheaper input, "a".
This establishes that if the source of decreasing costs in a competitive industry is purchases of an input from a monopolist it is necessary that the monopolist have decreasing marginal costs.

Although it is not central to the main argument, it is interesting to note that the comparative static responses of firm input demands and the number of firms to input prices are ambiguous. Differentiating the system of identities (2) totally with respect to \( p_u \) and using Cramer's rule it can be shown that the responses of an individual firm's input demands and the number of firms to an increase in the price of an input are of indeterminate sign. This ambiguity arises because a change in \( p_u \) may increase or decrease the minimum average cost output of the individual firm and the input quantities associated with it depending on the production technology. A change in an input price will change relative input prices and produce substitution among inputs and a change in the shape of the average cost curve as well as a vertical shift in the curve. Since the scale effect on each firm is indeterminate, the change in the numbers of firms also is ambiguous. A similar and more detailed discussion of the question of changes in scale can be found in Hughes [3].

**Conclusion**

The standard textbook rationalization of a competitive decreasing cost industry as a result of input supply from a decreasing cost monopolist has been shown to be internally consistent. The long-run supply function for the competitive industry is defined under standard assumptions that solutions to simultaneous equations and inverses exist. The supply function differs from the usual competitive supply function only in that the price of the monopolized input is not an argument. If purchases of an input from a monopolist are