A Problem in the Measurement of Capital Embodied Productivity Change

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Most published estimates of capital productivity and total factor productivity rely on perpetual inventory methods for their capital stock estimates (Denison [1974], [1979], Jorgenson [1980a], Gollop and Jorgenson [1980], Kendrick [1973], Kendrick and Grossman [1980], American Productivity Center [1980]). In this paper, it will be shown that perpetual inventory capital stock estimates contain a bias which will typically reverse the sign of the capital productivity. For capital embodied productivity change, an acceleration in the rate of productivity growth will actually result in a decline in measured capital productivity.

This is a much stronger and more important point than merely showing that there are large errors in the perpetual inventory capital stock estimates. The latter is well known to the specialists in the area. What is not realized is that the errors in the standard methods are correlated with changes in the variable being measured in such a way that any true change in best practice technology will typically cause an error in the sign of the reported short run change in capital input. Thus, the current methodology is worse than useless for measuring short run changes in productivity. The perpetual inventory method is simple. Instead of actually measuring the stock of capital, one estimates it by taking the initial year's inventory (which may be zero) and adds to it additions to the stock and subtracts out retirements and the amount of depreciation (if any) considered appropriate (Miller [1983a]). In current applications there are no direct measures of retirements or depreciation. Instead, depreciation and retirements are estimated as a function of age. These functions are based on historical data on length of life (usually developed some years ago for tax purposes).

For any year, the retirements and depreciation estimates are a function of the capital stock at the beginning of the year and the historically-based retirement and depreciation assumptions. No use is made of the current year's data. Thus, for any particular year, the capital stock estimate is the initial stock (a fixed number) minus retirements and depreciation (also a fixed number) plus investment. To simplify, the capital stock estimate is a constant (depending on historical data) plus investment.

Thus, the key question is: How is investment affected by changes in capital productivity? If investment increases as capital productivity goes up, estimated capital productivity will decline, causing the estimates to have a sign opposite to the actual change in productivity.

Let us start by analyzing a long-established enterprise with machines of all ages. As machines age, their variable inputs increase due to wearing out and the need for repairs. A
machine can be thought of as having an annual rent which is equal to the value of its output minus its variable costs. Rent will be assumed to start at value R and to decline by d (for depreciation) per year. Thus, the rent in any year will be R-dt. After R/d years, this value has declined to zero, and the machine is replaced by purchasing of a new machine and scrapping of the old.

Let us assume that technology has been stable and that the enterprise has settled into a steady state where each year it purchases a machine and scrap a machine. At any time the enterprise will have a total of R machines (where R/d is the life of the machine) with the rentals earned distributed uniformly between R and zero. At any time the enterprise has some equipment that is almost new and some equipment that is almost ready for the scrap heap.

Suppose a new model machine appears with a higher capital productivity in the form of a lower capital input per unit of output while the other inputs are left unchanged. The reason would be essentially unchanged if the output increased while holding capital input constant.

A lower capital input per unit of output implies that the rent of new capital has increased to R. This implies that all existing capital with rents lower than R should be replaced through acquiring the new model capital and retiring the old.

Thus, improved capital productivity results in both increased investment and increased retirements of old capital. The perpetual inventory method incorporates the increased investment but fails to incorporate the increased retirements. Thus, improved capital productivity embodied in new capital leads to an increase in estimated capital stock and a corresponding decline in measured capital productivity. Actual and measured capital productivity move in opposite directions.

Of course, a very similar effect occurs when improved labor productivity is embodied in new capital. If the new models of capital equipment require less labor, energy, or materials per unit of output, the rent of the new models per unit of output will be above that of much of the old equipment. Old capital will be made obsolete, and both investment and retirements will increase. Again, the historically based perpetual inventory method incor- porates the increased investment but fails to discover the increased retirements. The apparent capital stock increases, and measured capital productivity decreases. Unexpected changes in capital embodied labor, energy, or material productivity lead to errors in the opposite direction in capital productivity.

Very frequently, new models of capital incorporate reductions in both capital and labor inputs per unit of output. The reason is that technological progress often takes the form of increasing the output from a production unit (such as a piece of equipment) by making it larger or making it run faster. The innovation is both capital and labor saving. Examples would be making ships larger, planes faster, or rolling mills wider. In such cases old capital is made obsolete, resulting in a decrease in measured capital productivity at a time when capital productivity is actually increasing.

The historical experience used for perpetual life studies includes previous experience with productivity increases (both capital and labor) embodied in capital. There is an implicit assumption that capital embodied productivity increases will occur at rates consistent with the retirement and depreciation assumptions. There is an obvious logical inconsistency in trying to measure changes in capital productivity by assuming that its rate of growth will remain unchanged. There are of course other problems with the perpetual inventory capital stock method which the author discusses elsewhere (1936).

If capital embodied productivity growth is greater than the historical rate implicit in the length of life assumptions, the capital stock will be overstated, and the growth in capital productivity will be underestimated. If the rate of productivity growth embodied in capital (causal impact) is less than the historical rate, the capital stock will be underestimated, and capital productivity will be overstated. Since the rate of productivity growth in the last decade has been below historical levels, it is likely that less capital has been retired than was assumed in perpetual inventory capital stock studies. As a result, the capital stock has probably been underestimated, and capital productivity overstated. Total factor productivity is probably somewhat worse than current estimates indicate.

Possible Solutions

The above argument has shown that measuring productivity using a perpetual inventory capital stock estimate depends on assumptions about the relative productivity of the new stock relative to the old, which is what one is trying to measure. Having identified the problem, the second question is what can be done about it. There are no perfect answers, but there are a few possibilities.

The obvious solution is somehow to obtain information on retirements and incorporate it into the estimation process. At this point one makes the usual call for better data. However, obtaining better data will not be easy. Measurements of capital investment are comparatively simple because data on sales of investment goods are available from the producers. In contrast, retirements are made by a large number of firms and there is no standardization as to accounting treatment. If the item is fully depreciated (as most retired equipment is) the accounting records may not pick up its retirement. If it is partially depreciated, its book value will bear no consistent relationship to its original cost (and even less to its original costs in constant dollars).

While there are no good data on retirements, McGraw-Hill [1982] has included in its survey of business investment plans questions about planned "replacement" investment, and at times questions about how much such investment was actually made. Feldstein and Foote [1971] used this data to construct a perpetual inventory capital stock estimate by adding gross investment to last year's capital stock and subtracting replacement investment (a formula Jorgenson [1980b] applied with replacements estimated as proportional to the capital stock). For their purpose (a study of investment), this was a reasonable procedure.

Each year a certain proportion of the capital stock becomes worn out or obsolete and hence a candidate for replacement. One 'replaces' a machine providing a certain output with a new machine serving the same function and able to replace the output of the machine retired. The capital stock is serving as a surrogate for capacity, and a replacement investment can be interpreted as one that is accompanied by retirement of an equivalent capacity of old capital. Thus, if one conceptual framework is that investment demand is determined by the difference between desired capacity and current capacity, such a capital stock is reasonable.

However, for productivity studies, the capital stock is being used as a surrogate not for capacity but for the ability to provide capital services (rent). Even when the replacement investment replaces a machine of equivalent capacity (i.e., output), the quantity of capital investment is comparatively simple because data on sales of investment goods are available from the producers. In contrast, retirements are made by a large number of firms and there is no standardization as to accounting treatment. If the item is fully depreciated (as most retired equipment is) the accounting records may not pick up its retirement. If it is partially depreciated, its book value will bear no consistent relationship to its original cost (and even less to its original costs in constant dollars).
terial. However, in either case, statistical series of replacement investment contain information which could be used to improve series on retirements or loss of capital services.

Another possibility is to use the book value data from corporate annual reports and various government publications based on tax returns or answers to questions on the Annual Survey of Business. For discussion and application of such data base (for 450 four digit manufacturing firms derived from the Annual Survey of Manufacturers) see Miller [1981].

The advantage of data from standard accounting methods is that equipment and building are included on the balance sheet only if they still exist and have not been sold to another firm or industry. Equipment that has not reached the end of its normal life, but which was made obsolete by recent technical progress and retired will not be included as it would be with a perpetual inventory approach. The disadvantage until recently has been that all assets were carried at their original cost (adjusted by accounting depreciation), and no correction for price changes was made. However, replacement accounting has recently been adopted for large firms and should soon make better data available.

Making Retirements a Function of Investment

Since total investment is the sum of investment for expansion and for replacement, the gross investment series should contain information on the level of replacement investment which could be used to improve the estimates of retirements as discussed above. Feldstein and Foot (1971) have shown that there is a positive correlation between expansion and replacement investment because both respond to common factors (cash flow and capacity utilization). A partial offset comes from varying replacement investment to level gross investment.

Currently not enough is known about the replacement decision to know exactly how to relate retirements to new investment, although theory clearly indicates that technological progress should increase both. The current procedure, by assuming that retirements depend only on the total capital stock, implicitly assumes that retirement is independent of gross investment (even though half of gross investment is replacement investment). An alternative procedure might be to assume that some specified fraction (perhaps one half) of investment beyond that required to hold the capital output ratio constant was undertaken as replacement investment, indicating a corresponding offset in the form of retirements of capital.

Making Retirements a Function of Cost Reductions

Another alternative is to make retirements a function of other relevant variables within the model. The improvement in productivity possible from the introduction of new capital is a key determinant of the replacement rate and hence the retirements of old capital.

Standard replacement theory shows that an item should be replaced when the new machine has total costs (fixed plus variable) that are less than the variable costs alone of the old machine. In theory if there are data enough to calculate the change in productivity of the new capital, it should be possible to estimate the change in variable costs per unit of output. All machines whose rental was less than the decrease in variable costs would then be retired. If the distribution of machines by variable costs is known, it should be possible to calculate the amount of capacity ready for retirement. The simplest way to estimate the decline in rental for capital is to assume that it declines uniformly between the time of purchase and retirement. For instance, a type of capital that is normally presumed to last twenty years would be assumed to lose 5 percent of its rental each year. The length of life is used in the current methodology for calculating retirements if no new data are needed to approximate the distribution of rentals. For instance, if new capital appeared to have variable costs that were lower by a dollar amount equal to one year's decrease in rental, the retirements of one year's equipment would be presumed.

Knowing the rate of increase in productivity of the variable factors (available from current productivity statistics) and knowing (or assuming) the fraction of the productivity increase due to new capital, the variable cost reduction is determinable in theory. This makes possible estimation of which vintages of capital are obsolete and elimination of them from the capital stock.

The reaction of the reader to the above suggestion is probably that its implementation would involve too many assumptions to be very accurate. Yet by including the theoretically relevant variables, the above procedure should be more accurate than merely assuming that retirements induced by obsolescence occur at a uniform rate, ignoring the obvious fact that the rate of obsolescence depends on the rate of the technological advance. This serves to show how serious the problems are in the current procedure.

Information Content of Stock Prices

In theory, changes in the relationship between a perpetual inventory capital stock and stock market prices can provide information about the obsolescence of existing capital. Theoretically, the value of stock in a company is the present value of future income streams. Assuming that all income streams are rents from capital (obviously some are rents for monopoly power or goodwill), that rental income streams for all future years move in parallel with those of the current year, and that discount rates do not change, it follows that changes in stock prices are proportional to changes in gross rents from capital. If each year the depreciation charges reduced the book value of capital by the same proportion as its rent was expected to decline, book value would remain equal to market value. If technical progress then caused the rents to decline more rapidly than expected, market value would fall below book value. A decline in technical progress below that expected would cause market values to rise above book values.

If expected rates of capital embodied technological progress are based on historical rates, an acceleration in capital embodied technical progress would cause market values to fall below book values. Thus, changes in the ratio of market value to book value could indicate whether the adjustment for obsolescence in capital stock estimates is too large or too small.

As with most such theoretical schemes, many of the necessary assumptions are unrealistic. The decline in capital value (depreciation) will be equal to the decline in capital services for only one pattern of decline: a steady exponential decline that is expected to continue at the same rate [Jorgenson 1960b]. Since this implies a steady (and anticipated) rate of obsolescence, it appears logically inconsistent with the rate of capital embodied productivity advance changing so unpredictably that it is necessary to measure it.

Security markets capitalize income streams other than rents of physical capital. Some examples are monopoly income, goodwill and natural resources. These fluctuate independently of technological progress. Likewise, changes in the discount rate can cause changes in asset values that are unrelated to changes in present or future rents from capital. Capital value changes may be caused by differing expectations of future incomes or even speculative manias which need not be related to productivity changes.

Finally, obsolescence or declines in capital values due to changing conditions may be caused by events other than capital embodied productivity increases. This is well shown by
the work of Martin Bailey [1981]. He develops a model using "putty-putty" technology in which Tobin's q or the ratio of market value to book value is a measure of the rate of obsolescence of the existing capital stock. He then notes that the market to book ratio has declined and interprets this decline as evidence that the rate of obsolescence has increased. This would seem to be the exact opposite of the argument of this paper that the slowdown in the rate of productivity logically implies that the rate of obsolescence (and retirements induced by obsolescence) is less than assumed in the perpetual inventory capital stock analysis.

However, the apparently opposite conclusions can be reconciled. The obsolescence Martin Bailey is discussing is not due to technical progress but due to changing conditions which make existing capital unsuitable for the current situation. This may be due to changing public demands for the products produced by the capital or to changing factor prices. In particular, Bailey believes that the rapid increase in energy prices during the seventies made much of the existing capital stock obsolete, causing a sharp decline in the ratio of capital services to the capital stock. While this appears implausible to this author, acceptance of Bailey's argument is not logically consistent with acceptance of the argument of this paper.

Use of Security Price Data as Capital Stock Data

Although security price data used in conjunction with a traditional capital stock measure may be able to provide information on the obsolescence rate, security values alone cannot replace a traditional measure of the capital stock. This may at first appear paradoxical since capital rents are frequently considered a measure of the services of a piece of capital.

However, even if the value of securities is taken to be a good measure of the rent of capital, there is a problem. For simplicity, let's consider an industry where the only variable cost item is labor (the same argument applies where there are other variable cost items).

Let $I$, the quantity of labor, $W$, the wage rate, $P$, an index of the price level, $Q$, quantity of output, $K$, quantity of capital, $p$, productivity, $R$, rent.

Now $R = PQ - WL$ But $Q = pf(K, L)$.

Thus: $R = Pf(K, L) - WL$.

Inspection shows that knowing $W, L, P$, and $R$ is not sufficient to determine $p$. There are too many unknowns. To determine productivity, one still must know the quantity of capital.

Looking at the problem another way, fluctuations in total rent can be caused by changes in either productivity or the quantity of capital. A firm's profits (rent) will go up by the same amount if it purchases with its own funds a new machine that reduces labor input or if it succeeds in getting the same output from the old machine through higher productivity. One cannot separate changes in productivity from changes in the quantity of capital.

Hoping That Errors Offset Each Other

In some circumstances, inability to measure retirements may not be critical. Different vintages of capital should be weighted on the basis of marginal product, but if the statistics were uncorrected for quality, offsetting errors may occur. With no quality correction (the usual situation with current statistics), later vintage machines will represent more capital, but this will not be picked up in the capital stock estimates. However, if new machines have higher marginal productivity, they will also cause this return to new investment. This has obvious empirical problems, not only in adjusting for risk and in knowing how much of the observed return on debt represents an inflation adjustment which should be subtracted in determining the real return. However, this approach has an advantage in that the necessary assumptions are apparent and can be debated, while the assumptions needed for perpetual inventory "residual" estimates are concealed in the derivation.

Conclusion

The perpetual inventory method for measuring capital gives the correct sign for capital productivity changes only if unexpected capital saving innovations result in less investment. However, investment is the mechanism by which innovations come into use and technical progress results in increased investment even if the innovation is capital saving. Even though gross investment is the sum of replacement investment and expansion investment, current perpetual inventory methods treat replacement investment (and the associated retirements) as independent of both the determinants of gross investment and gross investment itself. Several possible alternative procedures exist, but they present their own problems.

References

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A Remark on the Graphical Exposition of Neo-Classical Two-Sector Growth Models

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I. Introduction

Graphical techniques for neo-classical two-sector growth models, as in Uzawa (1961, 1963), have been widely used by several authors in the literature, including Professor Allen in his macro-text (1970), and Professor Johnson (1973), and his joint work with Professor Krauss (1975). In particular, the two techniques used in Professor Johnson's various works are known as the Lerner-Pearce diagram and the Edgeworth-Bowley box diagram. Although these methods have proved very useful in illustrating many important aspects of the two-sector economy in a static framework, they do not appear as helpful for a comparative-static or a dynamic analysis of economic growth. The simpler treatment for the latter subjects would seem to involve the presentation of the cross-section production functions relating per-capita output to capital-labor ratio. This alternative was in fact used in a well-known survey article on the theory of economic growth by Professors Hahn and Matthews (1964). Unfortunately, their graphical technique was not developed to its full capacity and therefore, was unable to give a complete geometrical version even for the simplest case of the classical saving assumption where workers consume all wages and capitalists save all profits (or using notation below, $s_1 = 0$, and $s_2 = 1$). Using the Hahn and Matthews diagram, Professor Allen remarked in his text (1970, p. 278) that, in the case where $x_1 = 0$ and $0 < s_2 < 1$, the allocation of capital and labor between the two sectors, and the determination of the over-all capital-labor ratio as well as per-capita output, for a given level of wage-rental ratio, cannot be graphically represented in any simple way.

The major purpose of this note is to demonstrate that such a diagrammatic presentation can be made in a simple fashion for the case of $x_1 = 0$ and $s_2 = 1$, and that of $x_2 = 0$ and $0 < s_2 < 1$.

Moreover, in the course of discussion, we shall indicate how some of the well-known propositions on existence, stability, and uniqueness of equilibrium can be graphically shown. Thus, the present method should also serve as a useful alternative to the Johnson and Krauss technique mentioned above.

II. The Model: The Determination of Short-run Equilibrium

The production functions of consumption-goods sector (C), and capital-goods sector (I), which are assumed to be well-behaved and exhibit constant returns to scale, can be shown...