Keynes on the Supply of Gold: A Statistical Test

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In 1936 John Maynard Keynes hypothesized that the South African supply curve of gold is backward sloping in the short-run and normally sloped in the long-run. This paper, using data from 1974 to 1981, reports on a statistical test of these hypotheses. It is seen that they can both be accepted at a high level of confidence.

John Maynard Keynes, best known as the architect of general macroeconomic theories, was not without insight into more locally enlightening micro-matters. A brilliant case in point is his 1936 essay concerning the supply curve of gold (Keynes, 1936). In this essay he carefully outlined his view that the short-run supply curve, at least from South African sources, is backward bending, which reflects the negative response of gold output to the early depression devaluations. Specifically, Keynes stated:

"...the effect of devaluation in raising the price of gold, so far from increasing the output of the South African Mines, has actually diminished it.... This is due to technical reasons. The higher price makes it profitable to mill lower grade at once, whereas a corresponding increase in the milling plant takes time. Nevertheless an increase in the output is only a question of time. Moreover, the high price of gold stimulates prospecting all over the world, and the supply from miscellaneous sources is steadily increasing." (Bib., p.414).

Keynes' statement actually contains four hypotheses: (1) the short-run supply elasticity from any given set of gold mines is negative; (2) the long-run supply elasticity from that same set is positive; (3) the long-run supply elasticity from all primary sources is positive and greater than that of the fixed set of mines; (4) the elasticity from all primary and secondary (boarding) sources is positive and larger still.

Two years later, Paish presented the same hypotheses in somewhat more elaborate detail (Paish, 1938). The fixed input argument was expanded to include heavy mining equipment, deep shafts and other assets as well as the capacity of the milling plant. He went on to argue the uniqueness of the South African industry, pointing out that the vast deposits make the short-run a much lengthier proposition than that of producers elsewhere in the world.

Both Keynes and Paish present strong evidence of a decline in South Africa's output during their period. It is striking that the same trend is present during the rapid price increases of the Seventies. In a panel of thirty mines, ore output expanded slightly from 1972 to 1980, while gold output dropped precipitously from 816 thousand kg in 1971, to about 600 thousand in 1975 and 547 thousand in 1980 (Chamber, 1972-80). These data neither refute nor confirm any of the hypotheses; nor did the data assembled by Keynes and Paish. In comparing changes over time, too many of those troublesome "other
things" are not held constant; hence, changes in the price of gold cannot have been the only causative factor.

Their hypotheses thus remain untested. The present paper is an attempt to fill this gap. Its scope is limited to the first two of Keynes' hypotheses for reasons of simplicity and data availability. In section II, the technical features of the mining industry which are relevant to an understanding of the presentation are discussed on the assumption that not all readers are thoroughly familiar with this industry. Section II also discusses a set of feasible assumptions about managerial behavior which are necessary for the validity of the model. Section III which is the theoretical heart of the paper rigorously explains the model and its coefficients. Care has been taken to show the relationship between the assumptions and the coefficients. Section IV contains a formal statement of the two hypotheses to be tested and a description of the data series which makes it possible. In Section V and Table I, the statistical results are presented and explained. It is seen that the two hypotheses are strongly supported. Some subsidiary estimates suggested by the data are also discussed. The concluding section returns to the question of significance and validity of the assumptions and hypotheses.

II

It is well known that South African gold mines maintain inventories of blocked-out, ready ore reserves which are held at levels averaging about two to five times yearly ore output. These reserves are held underground; i.e., they are sections of the reefs which have been made completely accessible to the miners and have been sufficiently sampled to yield the necessary information as to grade, thickness, etc. They are produced by tunneling and sampling. In such productive operations, the miners become aware of their total holdings, whether payable or substandard. Although published data present figures only for payable reserves and grades, internal records reveal the extent of subpayable reserves as well. These reserves become increasingly valuable as the price of gold rises. In fact, in a period in which continued price increases are expected with sufficient certainty, it is rational to maintain inventories of both payable and subpayable reserves at levels determined by expectations. Hence, if the anticipated gold price occurs, some of the subpayable reserves could be quickly shifted to the payable column. This implies the existence of an "underground" supply curve of reserve ore and of its gold content, which should bear some resemblance with the supply curves about which Keynes and Paish had written.

The latter statement involves implicit assumptions which should be made explicit. First, we assume that ore output is constant in the short-run no matter what happens to the price of gold. Second, we assume that the average grade of ore output is proportional to the average grade of reserves. Third, we assume that the desired ratio of payable ore reserves to ore output is constant within each firm. This is equivalent to assuming that each mine manager has a distinct inventory policy, which, in the form of the constant ratio, may be rigid but still a fair first approximation of reality. The final assumption is that mine managers consciously strive to maintain the inventory of subpayable as well as the inventory of payable reserves at some desired level. The desired inventory of subpayable reserves is expected to be zero if no gold price increase is anticipated in the foreseeable future. Otherwise it is an increasing function of the manager's price expectations.

The meaning of these assumptions is not difficult to understand. If an expected gold price increase is realized, part of the desired subpayable reserves becomes payable. The new total payable reserves represent what the manager desires, under the given change in conditions. Hence, the percentage change in payable reserves, as calculated from the data, is equal to the desired percentage change. Actual ore output will, of course, lag behind by several years due to the above-mentioned constraints. But, because of the assumed constant ratio between desired reserves and output, the percentage increase in reserves should mirror the desired percentage increase in ore output. The conclusion that the long-run supply elasticity of gold from the given set of mines is equal to the "underground" supply elasticity of reserves is derived from these probable assumptions.

II

The elasticities can now be more rigorously defined. By definition, the quantity of gold produced, G (in kg) is equal to the product of the quantity of ore, O (in metric tons) and the grade of ore, G (in $ per metric ton), viz.,

\[ G = O \times G \]  

(1)

Similarly, the quantity of gold in reserves, G, is equal to the product of the ore reserves, R, and the grade of those reserves, G:

\[ G = R \times G \]  

(2)

If the two equations are expressed in log-linear form, and derivatives are taken with respect to the logarithm of the price of gold, each term becomes an elasticity:

\[ \frac{\Delta G}{\Delta G} = \frac{\Delta O}{\Delta G} + \frac{\Delta G}{\Delta G} \]  

(3)

\[ \frac{\Delta G}{\Delta G} = \frac{\Delta R}{\Delta G} + \frac{\Delta G}{\Delta G} \]  

(4)

The above argument leads to the conclusion that these two equations are identical in the long-run. In particular, taking the terms a pair at a time,

the long-run elasticity of \( \frac{\Delta G}{\Delta G} = \frac{\Delta O}{\Delta G} + \frac{\Delta R}{\Delta G} \) is supply of ore output and of ore reserves.

This equality depends upon the assumption that sub-payable reserve accumulation is intentional rather than accidental, and that the R/O ratio is constant.

Next,

the long-run elasticity of \( \frac{\Delta G}{\Delta O} = \frac{\Delta G}{\Delta O} \) supply of gold metal both as output and in reserves.

Finally,

\[ \frac{\Delta G}{\Delta O} = \frac{\Delta R}{\Delta O} \]  

(5)

Because \( \frac{\Delta G}{\Delta O} \) is measurable, this extraordinarily simple equation allows us to estimate the short-run elasticity of gold supply, via proxy. As argued above, \( \frac{\Delta G}{\Delta O} \) < 0, hence Keynes' first hypothesis, that \( \frac{\Delta G}{\Delta O} \) < 0, appears to be correct.

IV

The two hypotheses can now be stated more formally. They are as follows:

1. \( H_1 \frac{\Delta G}{\Delta G} = 0 \)  
2. \( H_2 \frac{\Delta G}{\Delta O} = 0 \)  
3. \( H_3 \frac{\Delta G}{\Delta O} = 0 \)  
4. \( H_4 \frac{\Delta G}{\Delta O} > 0 \)

The test statistic will be the t ratio computed from data described subsequently.

The Chamber of Mines of South Africa regularly publishes data on reserve tonnages and grades of the principal mines (Chamber, 1974-81, p. 6). Since 1974 these tables have also included the gold price at which the reserves were evaluated. These published prices are not ordinary market prices, year
end or period average. Instead, they are point estimates of the average price that might prevail in the coming year. Hence, they are hypothetical. The importance of these hypothetical prices is that the published reserve tonnage and grade figures are based directly upon them. We always know the exact price that is behind the reserve and grade quantities. Furthermore, fifteen of the mines began publishing grade and reserve tonnage figures at two or three hypothetical gold prices or at two hypothetical prices on each of their reefs.

These price-quantity observations, which are valid within a particular time frame, constitute points on the underground supply curve. The next step is to take data points, an adjacent pair at a time, and substitute their values into an arc elasticity formula (Lewith, p. 43). The relevant formulae are as follows. For the short-run supply elasticity, we have

$$ e_{\text{arc}} = \frac{V_{P_2} - V_{P_1}}{V_{P_2}} = \frac{P_{E_2} - P_{E_1}}{P_{E_2}} $$

where $P_{E_1}, P_{E_2}$ = hypothetical gold prices and $V_{P_1}, V_{P_2} = $ the corresponding gold grades as defined above.

For the long-run supply elasticity, we have

$$ e_{\text{arc}} = \frac{R_{E_2} - R_{E_1}}{R_{E_2}} = \frac{P_{E_2} - P_{E_1}}{P_{E_2}} $$

where $R_{E_2}, R_{E_1}$ are the reserve tonnages at each price.

These elasticity formulae measure the elasticity at a point halfway between two reported data points. Hence, for mines reporting reserves and grade at three data points per reef, we can obtain two elasticity estimates per reef. In total, fifteen mines yielded twenty-nine estimates for each of the years 1975–1979, thirteen mines yielded twenty-four estimates in 1980, and four mines yielded eight estimates in 1981. Because $V_{P_2} - V_{P_1} < 0$ and $R_{E_2} - R_{E_1} > 0$ in all cases, the short-run elasticity of formula (6) and the long-run elasticity of (7) are always, respectively, negative and positive. This fits Keynes’ hypotheses.

The statistical objective, however, is to show that $e_{\text{arc}}$ and $e_{\text{arc}}$ are significantly different from zero. This problem is discussed in the next section.

Table 1 contains the relevant statistical summaries. With the exception of 1980 and 81, the coefficients are remarkable for their consistency. The short-run elasticities range from $0.422$ to $0.522$ and are all significantly less than zero at $\alpha = 0.01$. Similarly, the long-run elasticities range from $0.078$ to $0.149$ and are significantly above zero, also at $\alpha = 0.01$. The figures for 1980 and 1981 differ because in 1980 the two mines which consistently were associated with the largest (in absolute value) elasticities in 1975–79 did not report the required data, and because most of the mines ceased reporting the data in 1981. Although both the long and short-run elasticities remain significantly above and below zero (at $\alpha = 0.01$), their absolute values are clearly below the ranges for 1975–79 (except the 1981 short-run elasticity of 0.460).

The results of two subsidiary tests are also reported in Table 1. First, the hypothesis that the short-run elasticity is equal to $-0.500$ was tested. In all years except 1980, this hypothesis is accepted. In 1980 it is strongly rejected ($-0.01$). This, again, is more attributable to the missing data for the Grootvlei and Marievale mines than any other cause. The second hypothesis tested is that the long-run elasticity is equal to $-1.000$. This, too, is acceptable in all years but 1980 and 1981.

In the interest of consistency, two other subsidiary tests were performed which are not reported in Table 1. First, Grootvlei’s and Marievale’s elasticities were deleted from all the yearly subsamples. The resulting estimates and t-ratios for each year consistently resemble the pattern exhibited for 1980. Grootvlei’s and Marievale’s elasticities of 1979 were then added to the data for 1980. These results in estimates and t-ratios for 1980 which closely resemble the patterns of other years. From this it may be concluded that the overpricing reason for differences in the 1980 results is the missing data. Because so many data points were missing in 1981, a similar consistency test would have no meaning.

VI

What we have learned from this exercise is that two of Keynes’ hypotheses about the supply curve of gold should be accepted. In the short-run its elasticity is surely negative; in the long-run it is positive. We have also learned something about the probable magnitude of these elasticity coefficients. The short-run elasticity is approximately equal to $-0.500$, while the long-run coefficient is near $-1.000$.

In spite of the positive tone of these conclusions some cautions are in order. First, the results are certainly valid for the fifteen mines which reported sufficient data, but are valid for the fifteen that did not? We have seen the effect that the deletion of just two mines can have on the results. The addition of the other fifteen, although they average considerably less than half the output of the included fifteen, would surely affect the results quite significantly. It is true that a few of the excluded mines sporadically reported enough data to calculate elasticities. These individual coefficients were invariably in the same area of magnitude as those of the included mines. Based on these occasional figures the guess is that the conclusions about the signs of the elasticities would not be altered, but that the estimates would both be lower in absolute value.

A more serious problem involves the validity of our assumptions. Two of them are particularly crucial: that ore output and the ratio of reserve grade to ore output remain constant in spite of gold price changes. This first assumption can be relaxed if the mine has brought new capacity on line during the period. Free State Goldfield, for example,
expanded fixed assets in 1977. Before expansion, quarterly ore output varied around 500 thousand kg, whereas after the mean was about 780 thousand. However, with each flush capacity, there was little variation of output. Others, such as Marievale and Western Deep Levels stayed within the same scale of plant. That is, Marievale’s average quarterly output went from 284 thousand kg in 1973 to 205 thousand in 1981, while Western Deep Levels’ went from 802 thousand to 789 thousand in the same period. Nine other mines showed a similar variation. The only real exception was Vaal Reefs whose output and scale of plant expanded aggressively throughout the period. However, a statistical analysis of the significance of the difference between the mean output in 1981 (fourth quarter) and that in 1973 (first quarter), corrected for plant scale changes, showed that within a given scale of plant output variations are negligible. In short, for the included mines, this assumption is valid.

Secondly, a simple test of the ratio of reserve to output grade also showed insignificant change between these two time periods.

Although both grades fell in response to the rising gold price, their ratios remained roughly constant. Hence, the assumptions are acceptably valid. This clears the way to a more confident acceptance of Keynes’ hypotheses.

Although small in the general scheme of Keynes’ economics, this inquiry illustrates the breadth and depth of his insights. In this most recent period of reappraisal, obscure as well as prominent hypotheses merit repeated testing. This is the stuff out of which progress in economic thought is made.

References

Clearly, if government and private groups are to implement policies to narrow the differences in racial incomes, they need to know how changes in institutional, demographic, and socioeconomic factors influence the relative distributions of nonwhite and white incomes. We believe the present study provides several important contributions to our understanding of these factors and of the changes that occurred in the inequality in racial incomes among states over the decade of the 1960’s. First, we report in this study that the inequality in racial incomes, as measured by the nonwhite/white median family income ratio (NWIR), actually grew worse in nearly one-third of our state observations over the 1960’s. Second, we develop a model to explain the variations in the percentage changes in both the NWIR and the percentage ratio measures of nonwhite to white family incomes that occurred at various points over the relative nonwhite/white income distribution during the 1960’s. Consequently, in addition to identifying the factors that contributed to changes in the central tendencies of the nonwhite income distribution relative to the distribution of white incomes, we further analyze how these factors influenced the relative position of families over different segments of the distributions and thus provide additional information on the changes in the relative shape of the nonwhite/white income distribution over the 1960’s.

An Interstate Analysis of Changes in Nonwhite and White Family Incomes 1960 to 1970

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