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A Note on Two-Stage Monetary Policy under Multiplier Uncertainty

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This note investigates the conditions under which the widely accepted two-stage procedure for monetary policy remains optimal if there is multiplier uncertainty. The answer depends on the value of a risk aversion parameter in the policy loss function.

The two-stage procedure involves, first, optimizing loss as a function of the variable of concern (referred to here as GNP) with respect to the money supply, thereby obtaining an optimal or "target" money supply (B. Friedman, 1975). Then the monetary base is chosen to minimize loss as a function of the money supply relative to its target value. This note compares the value chosen for the base with the truly optimal value of the base (the one which directly minimizes loss as a function of GNP).

Let the economy be described by

$$y = aM + u \qquad (1)$$

$$M = bH + e \qquad (2)$$

where y is the deviation of GNP from its lagged value, M is the deviation of the money supply from its lagged value, H is the deviation of the supply of high-powered money (the monetary base) from its lagged value, a and b are stochastic multipliers with positive means $\bar{a}$ and $\bar{b}$ and variances σ_a^2 and σ_b^2, and u and e are random intercept terms with means $\bar{u}$ and $\bar{e}$ and variances σ_u^2 and σ_e^2. Since lagged changes in the base may affect current changes in the money supply, these effects appear in the intercept term e so that, in general, $\bar{e}$ may be non-zero. By similar reasoning $\bar{u}$ may be non-zero because of current effects on y due to lagged changes in the money supply.[1]

For simplicity we consider the case in which a, b, u, and e are all uncorrelated. The key result below (that, in general, two-stage policy is optimal if and only if $\delta = 1$) remains true if we allow for covariance between the additive and multiplicative shocks.

To find the true optimal H we combine (1) and (2) to express y as a function of H:

$$y = (ab)H + (ae + u) \qquad (3)$$

The Fed is concerned with loss L as a function of y:

$$L = \delta_1(Ey - y^d)^2 + \sigma_y^2 \qquad (4)$$

where E is the expectation operator, σ^2 always refer to the variance of the subscripted variable, and y^d is the desired value of y. For simplicity we consider only a one-period loss horizon. $\delta_1 \geq 0$ is a policy-maker risk aversion parameter; a low δ_1 would imply a great concern with creating conditions of certainty. Equation (4) is a general loss function (Mitchell 1979) which has two widely used

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[1] Note that we have not ruled out the possibility of serially correlated multipliers a and b. While the analysis in the text involves only a single period, it is possible that the expected values of a and b for this period depend on the previous period's realizations of a and b.

special cases: $\delta_1 = 1$ (e.g., Brainard (1967)) and $\delta_1 = 0$ (e.g., M. Friedman (1953) and Fix and Sivesind (1978)).

The value of the directly controlled change in the base which minimizes loss is found by minimizing (4) subject to (3), to yield

$$H_1^* = \frac{(y^d - \bar{a}\bar{e} - \bar{u})\bar{a}\bar{b}\delta_1 - \text{cov}(ab,ae)}{\text{var}(ab) + \delta_1 \bar{a}^2\bar{b}^2} \qquad (5)$$

where the subscript "1" refers to the outcome of the one-stage optimization, and the asterisk indicates an optimal value. In (5) $\text{cov}(ab,ae) = \bar{b}\bar{e}\sigma_a^2$ and $\text{var}(ab) = \bar{a}^2\sigma_b^2 + \bar{b}^2\sigma_a^2 + \sigma_a^2\sigma_b^2$ (see Goodman (1960)).

In contrast, the two-stage procedure involves first optimzing loss as a function of y with respect to M. The first stage loss function is

$$L_{2i} = \delta_{2i}(Ey - y^d)^2 + \sigma_y^2 \qquad (6)$$

where the subscript "$2i$" refers to stage "i" of the two-stage procedure. δ_{2i} may or may not be the same as δ_1 in (4).

Minimizing loss in (6) subject to (1), we find the target value M^* of the change in the money supply:

$$M^* = \frac{(y^d - \bar{u})\bar{a}\delta_{2i}}{\sigma_a^2 + \bar{a}^2\delta_{2i}} \qquad (7)$$

Stage "ii" involves optimizing the change in the money supply, by minimizing the loss L_{2ii} in (8) subject to (2):

$$L_{2ii} = \delta_{2ii}(EM - M^*)^2 + \sigma_M^2 \qquad (8)$$

This yields the change in the base arrived at by the two-stage procedure:

$$H_2^* = \frac{(M^* - \bar{e})\bar{b}\delta_{2ii}}{\sigma_b^2 + \bar{b}^2\delta_{2ii}} \qquad (9)$$

where M^* in (9) is given by (7).[2]

By comparing (9) with (5) the conditions under which the two-stage result in (9) is equal to the true optimum in (5) can be identified. Substituting (7) into (9), tentatively equating (9) with (5), and cross-multiplying, we find that (9) and (5) are equivalent only if $\delta_1 = \delta_{2i} = \delta_{2ii} \equiv \delta$ and if

$$(1 - \delta)[(y^d - \bar{u} - \bar{e}\bar{a})\bar{a}\bar{b}\delta - \bar{e}\bar{b}\sigma_a^2] = 0 \qquad (10)$$

(10) states that the two-stage procedure is optimal only if the bracketed expression is zero (so the one-stage procedure would result in zero use of the base), or if the loss function parameter δ equals one.

Since $\delta = 1$ is a frequently used parameter value in papers on policy formulation, we have the comforting result that for this case the widely accepted two-stage policy procedure is not invalidated by the existence of multiplier uncertainty. On the other hand, if the policy maker has either less risk aversion ($\delta > 1$) or more risk aversion ($\delta < 1$) then the two-stage procedure yields a sub-optimal outcome if the true optimum involves a nonzero change in the base. It can be shown that if $\delta > 1$ the two-stage procedure results in over-use of policy ($|H_2^*| > |H_1^*|$), while if $\delta < 1$ that procedure results in under-use of policy ($|H_2^*| < |H_1^*|$). This can be illustrated by the other frequently used special case, in which $\delta = 0$ (so the policy maker is solely concerned with making GNP predictable). In this case, the two-stage procedure yields $H_2^* = 0$. In contrast the true optimum is $H_1^* = -\text{cov}(ab,ae)/\text{var}(ab) \neq 0$. The reason the two-stage procedure does not yield the optimal result here is that it fails to "exploit the covariance" between the base-to-GNP multiplier (ab) and the additive shock to GNP ($ae + u$) (see equation (3)).

[2]Several degenerate possibilities, involving no optimum because policy does not affect loss, should be noted. If $\delta_{2ii} = 0 = \sigma_b$ the second part of two-stage procedure is degenerate. If $\delta_{2i} = 0 = \sigma_a$ the first part of the two-stage procedure is degenerate, though if $\delta_{2ii} = 0$ and $\sigma_b \neq 0$ stage ii still yields the base value of zero. And the one-stage procedure is degenerate if $\delta_1 = \sigma_a = \sigma_b = 0$.

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