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The Combined Effect of Salary Restrictions and Revenue Sharing on Club Profits, Player Salaries, and Competitive Balance

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Abstract

This article provides a standard "Fort and Quirk"-style model of a professional team sports league and analyzes the combined effect of salary restrictions (caps and floors) and revenue-sharing arrangements. It shows that the invariance proposition does not hold even under Walrasian conjectures if revenue sharing is combined with either a salary cap or a salary floor. In leagues with a binding salary cap for large clubs but no binding salary floor for small clubs, revenue sharing will decrease the competitive balance and increase club profits. Moreover, a salary cap produces a more balanced league and decreases the cost per unit of talent. The effect of a more restrictive salary cap on the profits of the small clubs is positive, whereas the effects on the profits of the large clubs as well as on aggregate profits are ambiguous. In leagues with a binding salary floor for the small clubs but no binding salary cap for the large clubs, revenue sharing will increase the competitive balance. Moreover, revenue sharing will decrease (increase) the profits of large (small) clubs. Implementing a more restrictive salary floor produces a less balanced league and increases the cost per unit of talent. Furthermore, a salary floor will result in lower profits for all clubs.

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1 Introduction

Invariance principles are the golden eggs of economics. Franco Modigliani, Merton Miller, and Ronald Coase were awarded Nobel prizes for their formulations of important invariance principles. A predecessor of the famous Coase theorem is Rottenberg's invariance proposition. According to Rottenberg (1956), the distribution of playing talent between clubs in professional sports leagues does not depend on the allocation of property rights to players' services.El-Hodiri and Quirk (1971), Fort and Quirk (1995), and Vrooman (1995) extend this invariance proposition to gate revenue sharing. Based on their models, they claim that revenue sharing does not change the level of competitive balance within a league. This form of invariance proposition has become one of the most heavily disputed issues in sports economics because its centerpieces, revenue sharing and the uncertainty of outcome hypothesis, represent two of the most important idiosyncrasies in the professional team sports industry.

According to the uncertainty of outcome hypothesis, fans prefer to attend games with uncertain outcomes and enjoy close championship races. Unlike Toyota, which benefits from weak competitors in the automobile industry, Real Madrid and the New York Yankees need strong competitors to maximize their revenues. In sports, a weak team produces a negative externality on its stronger competitors. Revenue-sharing arrangements have been introduced as a measure to improve the competitive balance by (partially) internalizing this externality. If the invariance proposition held, revenue sharing would be worthless.

Current revenue-sharing schemes vary widely among professional sports leagues all over the world. The most prominent is possibly that operated by the National Football League (NFL), where the visiting club secures 40% of the locally earned television and gate receipt revenue. In 1876, Major League Baseball (MLB) introduced a 50-50 split of gate receipts that was reduced over time. Since 2003, all the clubs in the American League have put 34% of their locally generated revenue (gate, concession, television, etc.) into a central pool, which is then divided equally among all the clubs. In the Australian Football League (AFL), gate receipts were at one time split evenly between the home and the visiting team. This 50-50 split was finally abolished in 2000.

Other measures to increase competitive balance are salary caps and floors. A salary cap (floor) puts an upper (lower) bound on a club's payroll. Since most leagues compute their salary caps and floors on the basis of the revenues of the preceding season, caps and floors can be treated as fixed limits.

The North-American National Basketball Association (NBA) was the first league to introduce a salary cap for the 1984-1985 season. For the 2008-2009 season the (soft) salary cap is fixed at US$ 58.7 million. Today, salary caps are in effect in professional team sports leagues all over the world. In the National Hockey League (NHL), for example, each team had to spend between US$ 34.3 million and 50.3 million on player salaries in the 2007-08 season. In the National Football League (NFL), the salary cap in 2008 was approximately US$116 million per team, whereas the salary floor was 85.2% of the salary cap, which is equivalent to US$ 98.8 million. The Australian Football League (AFL) also operates with a combined salary cap and floor: for 2009, the salary cap was fixed at AU$ 8.81 million, the floor at 7.93 million. Another Australian league, the National Rugby League, has implemented a salary cap and floor system which forced each team to spend between AU$ 3.69 million and 4.1 million in 2008. In Europe, salary caps are in effect in the Guiness Premiership in rugby union and the Super League in rugby league.

In any industry other than the team sport industry, payroll caps would be regarded as an exploitation of market power and would be prohibited by anti-trust authorities. In professional team sports, however, salary cap (and floor) arrangements are usually granted anti-trust exemption whenever they are the result of collective bargaining agreements between representatives of club owners and players.

In the sports economic literature, the invariance proposition with regard to revenue sharing has been derived under two major assumptions: First, club owners are modeled as profit maximizers (rather than win maximizers). Second, talent supply is regarded as fixed. There is wide agreement that the invariance proposition does not hold in leagues with either win-maximizing owners or flexible talent supply (Atkinson et al. 1988; Szymanski, 2003 Késenne 2000, 2005, 2007). There is disagreement, however, over whether the invariance proposition holds in a league with profit-maximizing owners and a fixed talent supply. The models of El-Hodiri and Quirk (1971), Fort and Quirk (1995), and Vrooman (1995) show that the invariance proposition does hold with respect to revenue sharing, whereas the model of Szymanski and Késenne (2004) concludes that gate revenue sharing results in a more uneven distribution of talent between large- and small-market clubs and therefore contradicts the invariance proposition. Since all of these models use the same assumptions, namely, a fixed supply of talent and profit-maximizing club owners, the contradiction results from methodological differences. El-Hodiri and Quirk, Fort and Quirk, and Vrooman use "Walrasian conjectures," whereas Szymanski and Késenne employ "Nash conjectures."

Our contribution to the literature is to show that the invariance proposition does not hold even in a standard "Fort and Quirk" style (FQ-style) model if one considers the combined effect of salary restrictions (salary cap and floor) and revenue-sharing agreements. In particular, we analyze the joint effect of salary restrictions and revenue sharing on club profits, player salaries, and competitive balance. We show that in leagues with a binding salary cap for large clubs but no binding salary floor for small clubs, revenue sharing will decrease competitive balance and increase the profits of the small clubs as well as aggregate profits. The effect on the profits of the large clubs is ambiguous. In this case, a salary cap also results in a more balanced league. The effect of a stricter salary cap on the profits of small clubs is positive, whereas the effects on the profits of the large clubs and on aggregate profits are ambiguous.

Moreover, in leagues with a binding salary floor for the small clubs but no binding salary cap for the large clubs, revenue sharing will increase competitive balance. Moreover, revenue sharing will decrease (increase) the profits of large (small) clubs. Implementation of a more restrictive salary floor will produce a less balanced league and will increase the cost per unit of talent. Furthermore, a salary floor will result in lower profits for all clubs. Finally, our analysis shows that revenue sharing decreases the cost per unit of talent in all regimes except when either the salary cap or the salary floor is binding for all clubs.

The remainder of the article is organized as follows. In the next section we present our model setup with the main assumptions. In Subsection 2.1, we consider Regime A which represents the benchmark case without a (binding) salary cap/salary floor. In Subsection 2.2, we consider Regime B where the salary cap is only binding for the large-market club and the salary floor is not binding for the small-market club. In Subsection 2.3, we analyze Regime C where the salary floor is only binding for the small-market club and the salary cap is not binding for the large-market club. Subsection 2.4, represents Regime D where either the salary cap or the salary floor is binding for both clubs. Finally, Section 3 concludes.

2 Model

We model the investment behavior of two profit-maximizing clubs in a standard FQ-style league. Each club $i = 1, 2$ invests independently in playing talent t_i in order to maximize its own profits. Our league features a pool revenue sharing arrangement, and salary payments (payroll) are restricted by both a salary cap (upper limit) and a salary floor (lower limit).

The revenue of club i (R_i) depends on its market size (m_i) as well as its own win percentage (w_i) and the win percentage (w_j) of the other club. We assume that the revenue function has the following properties: $\frac{\partial R_i}{\partial w_i} > 0$ and $\frac{\partial^2 R_i}{\partial w_i^2} < 0$ for all $w_i \in [0, 1]$ or $\exists!\ (w_i)^* \in (0, 1)$ such that $\frac{\partial R_i}{\partial w_i} > 0\ \forall w_i \in (0, (w_i)^*)$ and $\frac{\partial R_i}{\partial w_i} < 0\ \forall w_i \in ((w_i)^*, 1]$. The following specification of the revenue function for club i satisfies the required properties:[1]

$$R_i = m_i(w_i + w_i w_j) = m_i(2w_i - w_i^2). \tag{1}$$

We assume that club 1 is the large market club with a higher drawing potential than the small market club 2 such that $m_1 > m_2$. For notational simplicity and without loss of generality, we normalize m_2 to unity and write m instead of m_1 with $m > 1$.

The win percentage w_i of club i is characterized by the contest-success function (CSF), which maps the vector (t_1, t_2) of talent onto probabilities for each club. We apply the logit approach, which is the most widely used functional form of a CSF in sporting contests.[2] The win percentage of club $i = 1, 2$ is then given by

$$w_i(t_i, t_j) = \frac{t_i^\gamma}{t_i^\gamma + t_j^\gamma}, \tag{2}$$

with $i, j = 1, 2$, $i \neq j$. For the sake of tractability, we normalize the "power parameter" γ in the following to unity.[3] Given that the win percentages must sum up to unity, we obtain the adding-up constraint: $w_j = 1 - w_i$. Since we consider a standard FQ-style model, we assume a fixed supply of talent given by $s > 0$ and adopt the so-called "Walrasian conjecture" $\frac{dt_i}{dt_j} = -1$. We compute the derivative of (2) as[4]

$$\frac{\partial w_i}{\partial t_i} = \frac{t_i + t_j - t_i(1 + \frac{\partial t_j}{\partial t_i})}{(t_i + t_j)^2} = \frac{1}{t_i + t_j}.$$

We measure the competitive balance in the league by the product of win percentages $w_i w_j$.[5] Moreover, we introduce revenue sharing in our league and assume that club revenues are shared according to a pool-sharing agreement. In a simplified pool-sharing agreement, each club contributes a certain percentage $(1 - \alpha)$ of their individual revenues in a pool that is managed by the league and equally distributed among the clubs.[6] In its simplest version, the after-sharing

[1] This specification of the revenue function is widely used in the sports economic literature: see, e.g., Hoehn and Szymanski (1999), Szymanski (2003), Szymanski and Késenne (2004), Késenne (2006, 2007) and Vrooman (2007, 2008).

[2] The logit CSF was generally introduced by Tullock (1980) and subsequently axiomatized by Skaperdas (1996) and Clark and Riis (1998). An alternative functional form would be the probit CSF (e.g., Lazear and Rosen, 1981; Dixit, 1987) and the difference-form CSF (e.g., Hirshleifer, 1989).

[3] See Dietl et al. (2008) and Fort and Winfree (2009) for a more detailed analysis of the role of the power parameter.

[4] For a discussion of the "Walrasian conjecture" vs. the "Nash conjecture", see, e.g., Szymanski (2004) and Fort and Quirk (2007).

[5] For an analysis of competitive balance in the North American Major Leagues, see, e.g., Fort and Lee (2007).

[6] Note that the results are robust also for a gate revenue-sharing agreement where club i obtains share α of its own revenues R_i and from the away match share $(1 - \alpha)$ of club j's revenues R_j. In this case, the after-sharing revenues of club i are given by $\widehat{R}_i = \alpha R_i + (1 - \alpha) R_j$ (for an analysis, see, e.g., Dietl and Lang, 2008).

revenues of club i can be written as

$$\widehat{R}_i = \alpha R_i + \frac{(1-\alpha)}{2}(R_i + R_j),$$

with $\alpha \in (0,1]$ and $i, j = 1, 2$, $i \neq j$. The limiting case of $\alpha = 1$ describes a league without revenue sharing, whereas $\alpha = 0$ describes a league with full revenue sharing.

We derive the following lemma:[7]

Lemma 1

Aggregate club revenues $\widehat{R}_1 + \widehat{R}_2$ are maximized for $(w_1^*, w_2^*) = \left(\frac{m}{m+1}, \frac{1}{m+1}\right)$.

Proof. Straightforward and therefore omitted. ■

The win percentages that maximize aggregate after-sharing revenues are independent of the revenue-sharing parameter α. This is due to the fact that the after-sharing aggregate club revenues are given by the sum of the individual revenues of club 1 and 2, i.e. $\widehat{R}_1 + \widehat{R}_2 = R_1 + R_2$.

Moreover, as is standard in the literature, we assume constant marginal costs c of talent such that the salary payments (payroll) of club i, denoted by x_i, are given by $x_i = c \cdot t_i$.[8]

The profit function of club $i = 1, 2$ is then given by after-sharing revenues minus salary payments

$$\pi_i(t_i, t_j) = \widehat{R}_i(t_i, t_j) - c \cdot t_i,$$

with $i, j = 1, 2$, $i \neq j$.

As mentioned above, we introduce both an upper limit (salary cap) and a lower limit (salary floor) for each club's payroll. The sizes of the salary cap and salary floor, which are the same for each club, are based on the total league revenue in the previous season, divided by the number of clubs in the league. The salary cap and the salary floor are therefore exogenously given in the current season.[9]

Each club invests independently in playing talent such that its own profits are maximized subject to the salary cap and salary floor constraints. That is, salary payments $x_i = c \cdot t_i$ must be at least as high as $floor > 0$, given by the salary floor, but must not exceed $cap > 0$, given

[7] For a comparison of the noncooperative outcome and the socially optimal outcome, see, e.g., Cyrenne (2001), Whitney (2005) and Dietl, Lang and Werner (2009).

[8] For the sake of simplicity, we do not take into account non-labor costs and normalize the fixed capital cost to zero. See Vrooman (1995) for a more general cost function where clubs have different marginal costs or Késenne (2007) for a cost function with a fixed capital cost. Idson and Kahane (2000) analyze the effect of team attributes on player salaries.

[9] See, e.g., Késenne (2000*a*) and Dietl, Lang and Rathke (2009).

by the salary cap. The maximization problem for club $i = 1, 2$ is given by

$$\max_{t_i \geq 0} \left\{ \alpha R_i(t_i, t_j) + \frac{(1-\alpha)}{2}(R_i(t_i, t_j) + R_j(t_i, t_j)) - c \cdot t_i \right\}$$

$$\text{subject to } \mathit{floor} \leq c \cdot t_i \leq \mathit{cap}.$$

The corresponding first-order conditions are derived as[10]

$$\begin{aligned}
&\frac{\partial \widehat{R}_i}{\partial t_i} - c - \lambda_{i1}c + \lambda_{i2}c \leq 0, \quad \mathit{cap} - ct_i \geq 0, \quad ct_i - \mathit{floor} \geq 0, \\
t_i &\left(\frac{\partial \widehat{R}_i}{\partial t_i} - c - \lambda_{i1}c + \lambda_{i2}c \right) = 0, \quad \lambda_{i1}(\mathit{cap} - ct_i) = 0, \quad \lambda_{i2}(ct_i - \mathit{floor}) = 0,
\end{aligned} \tag{3}$$

where $\lambda_{ij} \geq 0$ are Lagrange multipliers. The equilibrium in talent (t_1^*, t_2^*) is characterized by (3) and the market-clearing condition $t_1^* + t_2^* = s$ due to the fixed supply of talent.

We must distinguish different regimes depending on whether the salary cap and/or salary floor is binding or not.

2.1 Regime A: neither salary cap nor salary floor is binding

In this section, we assume that the salary cap and salary floor are ineffective for both clubs; i.e., we consider the benchmark case that no (binding) salary cap/floor exists. In Regime A, the equilibrium demand for talent and the cost per unit of talent are computed from (3) as[11]

$$\begin{aligned}
\left(t_1^A, t_2^A\right) &= \left(\frac{m}{m+1}s, \frac{1}{m+1}s \right) = \left(w_1^A s, w_2^A s\right), \\
c^A &= \frac{2\alpha m}{s(m+1)}.
\end{aligned} \tag{4}$$

We derive that the large club demands more talent in equilibrium than does the small club, because the marginal revenue of talent is higher for the large club. Furthermore, note that the equilibrium win percentages in Regime A, given by $(w_1^A, w_2^A) = \left(\frac{m}{m+1}, \frac{1}{m+1}\right)$, coincide with the revenue-maximizing win percentages (w_1^*, w_2^*) from Lemma 1.[12] It follows that aggregate club revenues are maximized in Regime A.

[10] It can easily be verified that the second-order conditions for a maximum are satisfied.

[11] Note that the equilibrium demand for talent before calculating the equilibrium cost per talent is given by $\left(t_1^A(c), t_2^A(c)\right) = \left(\frac{m(1+\alpha)s - cs^2}{m(1+\alpha)+(1-\alpha)}, \frac{(1+\alpha)s - cs^2}{(1+\alpha)+m(1-\alpha)}\right)$.

[12] This is due to the well-known result that the first-order conditions in (3) collapse to $\frac{\partial R_i}{\partial w_i} = \frac{\partial R_j}{\partial w_j}$ which is independent of the revenue-sharing parameter α.

The equilibrium salary payments in Regime A, denoted $\left(x_1^A, x_2^A\right)$, are computed as

$$\left(x_1^A, x_2^A\right) = \left(\frac{2\alpha m^2}{(m+1)^2}, \frac{2\alpha m}{(m+1)^2}\right).$$

Thus, we are in Regime A if $floor < x_2^A$ and $cap > x_1^A$.

In the following proposition, we summarize the effect of changing the revenue-sharing parameter α in Regime A:

Proposition 1

(i) The invariance proposition holds in Regime A: revenue sharing has no effect on the distribution of talent.

(ii) Revenue sharing decreases the cost per unit of talent in Regime A.

(iii) Revenue sharing increases the profits of the small club and aggregate club profits. The profits of the large club only increase if the difference between both clubs in terms of market size is not too big, i.e., if $m < m' \approx 2.83$.

Proof. See Appendix. ■

In accordance with the literature, we derive that the well-known "invariance proposition" with respect to revenue sharing holds in our FQ-style model when neither the salary cap nor the salary floor is binding.[13] That is, revenue sharing has no effect on the win percentages and thus does not change the league's competitive balance in Regime A.

To illustrate this result, Figure 1 depicts the downward-sloping marginal after-sharing revenue curves as functions of the win percentages for the two clubs. The two topmost lines indicate the case of no revenue sharing, i.e., $\alpha = 1$. When revenue starts to be shared, the marginal revenue curves shift down for both clubs. Instead of receiving all the additional revenue from an extra unit of talent, the clubs receive only $(1 + \alpha)/2$ of the additional revenue. This results in a downward shift of both marginal revenue curves, where the shift is more pronounced for the large club.

Moreover, increasing the win percentage of club i is tantamount to reducing the win percentage of club j. As a result, club j's contribution to the shared pool is shrinking. It follows that club i loses $(1 - \alpha)/2$ of club j's reduced revenue when increasing its win percentage. Note that the contribution to the pool increases with the degree of revenue sharing. Since the large club's contribution to the pool is always greater than the small club's contribution, it follows that the small club loses more through a higher degree of revenue sharing. As a consequence, more revenue sharing implies that marginal revenue is decreasing faster for the small club, whereas the

[13] See, e.g. El-Hodiri and Quirk (1971), Fort and Quirk (1995), and Vrooman (1995).

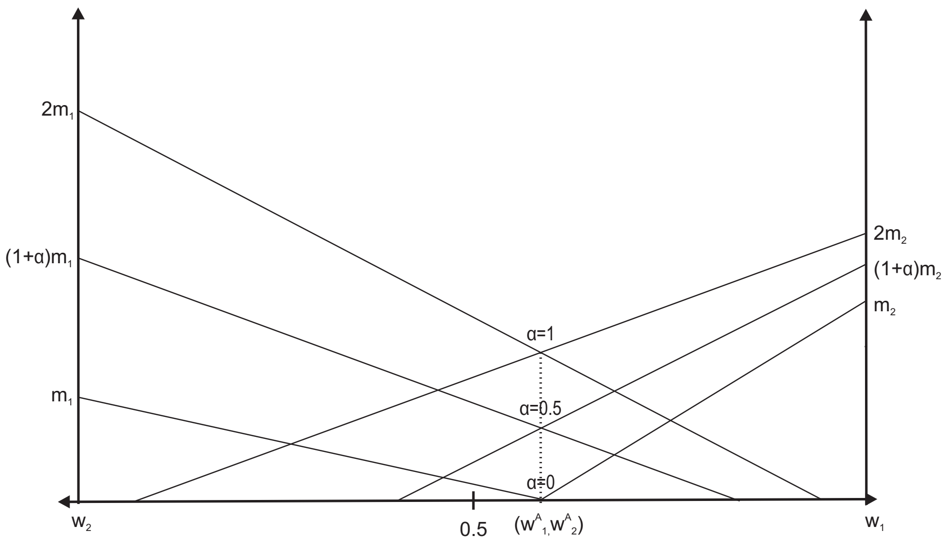

Figure 1: Effect of revenue sharing on marginal revenues

marginal revenue curve of the big club is getting flatter. Overall, even though the intercept of the big club shifts down more than the intercept of the small club, the two curves still intersect at the same pair of win percentages (w_1^A, w_2^A) for all values of α because the changing slopes offset the change of the intercepts.

Moreover, the proposition shows that a higher degree of revenue sharing, i.e., a lower value of α, lowers the equilibrium cost per unit of talent. As argued above, marginal revenue decreases for both clubs and with it talent demand $t_i^A(c)$ (depending on c). Hence, the market-clearing cost per unit of talent c^A set by the "Walrasian auctioneer" also has to be lower.

Even though revenue sharing leaves the distribution of talent unchanged and therefore also the revenues of both clubs, it has implications for club profits. A higher degree of revenue sharing will increase the profit of the small club in Regime A, because revenue sharing lowers the cost per unit of talent and redistributes some of the money to the small club. As a result, the small club's after-sharing revenues $\widehat{R}_2$ and profits increase through revenue sharing.

Despite the fact that salary payments x_i^A will decrease for both clubs, revenue sharing decreases the profit of the large club if the difference between both clubs in terms of market size is too big, i.e., $\frac{\partial \pi_1^A}{\partial \alpha} > 0 \Leftrightarrow m > m' \approx 2.83$. Note that the large club's after-sharing revenues $\widehat{R}_1$ decline as a result of the redistribution to the small club. If the market size is greater than m', the lower costs cannot compensate for the lower revenues.[14]

On aggregate, however, club profits increase because aggregate revenues $R_1^A + R_2^A$ are independent of α and thus remain constant but costs decline through revenue sharing. Due to the contest structure, the maximum level of aggregate club profit would be attained in a league

[14]Note that the large club's salary payments x_1^A are an increasing function in the market size m.

with full revenue sharing, i.e., for $\alpha = 0$, because in this case both clubs would fully internalize the externality they impose on the other club when hiring an additional unit of talent.[15]

2.2 Regime B: salary cap is binding for large club, but salary floor is not binding for small club

In this section, we assume that the salary cap is only binding for the large-market club and that the salary floor is not binding for the small-market club. In Regime B, the equilibrium demand for talent and the cost per unit of talent are computed as[16]

$$\begin{aligned} \left(t_1^B, t_2^B\right) &= \left(\frac{2cap}{(\alpha-1)m+\phi^B}s, \left(1-\frac{2cap}{(\alpha-1)m+\phi^B}\right)s\right) = \left(w_1^B s, w_2^B s\right), \\ c^B &= \frac{(\alpha-1)m+\phi^B}{2s}, \end{aligned} \tag{5}$$

with $\phi^B := \sqrt{(\alpha-1)^2m^2 + 4cap(1+\alpha+m(1-\alpha))}$.[17] The equilibrium salary payments in Regime B are computed as

$$\left(x_1^B, x_2^B\right) = \left(cap, \frac{1}{2}((\alpha-1)m+\phi^B-2cap)\right).$$

Thus, we are in Regime B if $cap \in \left(\underline{cap}, \overline{cap}\right) = \left(\frac{1+\alpha-m(1-\alpha)}{4}, x_1^A\right)$ with a sufficiently low salary floor.[18] The condition for cap guarantees that the salary payments of the small club are lower than cap. Otherwise the salary cap would be binding for both clubs and we would be in Regime D.[19] Moreover, the condition $cap \in \left(\underline{cap}, \overline{cap}\right)$ implicitly defines the interval of feasible revenue-sharing parameters α for Regime B with $\alpha \in (\underline{\alpha}^B, \overline{\alpha}^B) = \left(\frac{cap(1+m)^2}{2m^2}, \frac{4cap+m-1}{1+m}\right)$.[20]

2.2.1 The effect of a salary cap in Regime B

In this subsection, we analyze the effect of changing the salary cap parameter given that the league has set a certain degree α' of revenue sharing. We derive the following results:

[15] However, we assume that players have a certain reservation wage $c^w > 0$ such that $\alpha = 0$ is not a feasible solution.

[16] The equilibrium demand for talent before calculating the equilibrium cost per talent is given by $\left(t_1^B(c), t_2^B(c)\right) = \left(\frac{cap}{c}, \frac{(1+\alpha)s-cs^2}{(1+\alpha)+m(1-\alpha)}\right)$.

[17] $(\alpha-1)^2m^2 + 4cap(1+\alpha+m(1-\alpha)) > 0$ since $cap \in \left(\underline{cap}, \overline{cap}\right)$.

[18] Note that $\underline{cap}$ is less than zero if the difference between both clubs is too big, i.e. $\underline{cap} < 0 \Leftrightarrow m > \frac{1+\alpha}{1-\alpha}$.

[19] Formally, if $cap < \underline{cap}$, then $x_1^B < x_2^B$ and we are in Regime D. If $cap > \overline{cap}$, then $x_1^B > x_1^A$ ($x_2^B < x_2^A$) and we are in Regime A.

[20] Suppose that the league has set a certain $cap' \in \left(\underline{cap}, \overline{cap}\right)$. Decreasing (increasing) the revenue-sharing parameter α induces both $\underline{cap}$ and $\overline{cap}$ to decrease (increase). If α decreases below $\underline{\alpha}$, then $cap' > \overline{cap} = x_1^A$, and we would be in Regime A because the cap would not be binding anymore. If α increases above $\overline{\alpha}^B$, then $cap' < \underline{cap}$ and we would be in Regime D.

Proposition 2

The introduction of a binding salary cap increases competitive balance and decreases the cost per unit of talent in Regime B.

Proof. See Appendix. ■

The salary cap forces the large club to cut back on expenses, lowering the overall demand for talent, and thus the market-clearing cost per unit of talent c^B set by the Walrasian auctioneer is lower. As a consequence, the small club will hire a greater amount of talent.

Hence, a more restrictive salary cap (i.e., a lower value of *cap*) induces a reallocation of talent from the large to the small club. That is, the large club decreases its talent demand by the same amount by which the small club increases its talent demand, i.e., $0 < \frac{\partial t_1^B}{\partial cap} = -\frac{\partial t_2^B}{\partial cap} > 0$. As a consequence, a more restrictive salary cap increases the win percentage w_2^B of the small club and decreases the win percentage w_1^B of the large club in Regime B. Since the large club is the dominant team, competitive balance increases and thus a salary cap produces a more balanced league. It follows that the individual revenues R_1^B of the large club decrease and that the individual revenues R_2^B of the small club increase through a more restrictive salary cap. Aggregate club revenues $R_1^B + R_2^B$, however, will decline because the league departs from the revenue-maximizing win percentages (w_1^*, w_2^*). Thus, the after-sharing revenues $\widehat{R}_1$ of the large club decline, and the after-sharing revenues $\widehat{R}_2$ of the small club increase (see also Figure 1).

The second part of the proposition states that the cost per unit of talent will be lower in equilibrium through the introduction of a salary cap, i.e., $\frac{\partial c^B}{\partial cap} > 0$. It is therefore clear that a more restrictive salary cap helps the large club to control costs, because the large club decreases its salary payments, i.e., $\frac{\partial x_1^B}{\partial cap} > 0$. But will a salary cap also help the small club to lower costs? We derive that the effect of a more restrictive salary club on the small club's salary payments is ambiguous because

$$\frac{\partial x_2^B}{\partial cap} = \frac{1}{2}\left(\frac{2(1+\alpha+m(1-\alpha))}{\phi^B} - 2\right) \begin{cases} > 0 \text{ if } cap \in (\underline{cap}, \widetilde{cap}), \\ = 0 \text{ if } cap = \widetilde{cap}, \\ < 0 \text{ if } cap \in (\widetilde{cap}, \overline{cap}), \end{cases}$$

with $\widetilde{cap} = \frac{1+2m+a(2+a(1-2m))}{4(1+a+m(1-a))}$.[21] That is, if the salary cap is not too restrictive, i.e., $cap \in (\widetilde{cap}, \overline{cap})$, the increase in talent demand offsets for the decrease in the cost per unit of talent such that salary payments x_2^B of the small club increase. However, if the salary cap is relatively restrictive, i.e., $cap \in (\underline{cap}, \widetilde{cap})$, the decrease in the cost per unit of talent outweighs the

[21] Note that depending on the parameters (α, m), the threshold $\widetilde{cap}$ can be bigger than $\overline{cap}$. In this case, the salary payments of the small club always decrease through a tighter salary cap.

increase in talent demand, and salary payments x_2^B decrease. Moreover, we derive that a salary cap always decreases aggregate salary payments, i.e., $\frac{\partial(x_1^B+x_2^B)}{\partial cap} > 0$.[22] That is, the increase in the small club's salary payments never offsets the decrease in the large club's salary payments.

In the next proposition, we analyze how changes in the salary cap affect club profits:

Proposition 3

A more restrictive salary cap increases the profits of the large club and aggregate club profits until the maximum is reached for $cap = cap^*$ and $cap = cap^{**}$, respectively, whereas the profits of the small club will always increase through a more restrictive salary cap.

Proof. See Appendix. ■

Figure 2 illustrates the proposition's results. A more restrictive salary cap increases aggregate club profits π^B until the maximum is reached for $cap = cap^{**}$. Intuitively, a salary cap has two effects on club profits. On the one hand, a more restrictive salary cap lowers aggregate club revenues because the league departs from the revenue-maximizing win percentages from Regime A. On the other hand, it lowers the cost per unit of talent. Suppose that the league has set a relatively loose salary cap. By implementing a more restrictive salary cap, the marginal (positive) effect of lower aggregate club costs $x_1^B + x_2^B$ outweighs the marginal (negative) effect of lower aggregate club revenues $R_1^B + R_2^B$ such that aggregate club profits increase. Both effects balance each other out for $cap = cap^{**}$. By implementing a more restrictive salary cap than cap^{**}, the lower club costs cannot compensate for the lower aggregate club revenues, and therefore aggregate club profits will decrease.[23]

For a relatively loose salary cap, the profits of both clubs will increase through the introduction of a salary cap. The small club, however, will always benefit, independent of the size of the salary cap, whereas the large club has an interest in the salary cap not being too restrictive. Formally, a more restrictive salary cap increases the profit of the large club π_1^B until the maximum is reached for $cap = cap^*$. The intuition is as follows. Remember that a more restrictive salary cap will increase (decrease) the small (large) club's after-sharing revenues. For the small club, even in the case that a more restrictive salary cap increases the club's costs (i.e., for $cap \in (\widetilde{cap}, \overline{cap})$), the higher revenues offset for the higher costs and the profits of the small club will increase. For the large club the reasoning is similar that for aggregate profits above. The lower costs can only outweigh the lower club revenues if the salary cap is not set to be too restrictive, i.e., if $cap > cap^*$. Otherwise, the profits of the large club will decrease through a

[22]To see this note that $x_1^B + x_2^B = c^B(t_1^B + t_2^B) = c^B s$ and $\frac{\partial c^B}{\partial cap} > 0$.

[23]Note the equilibrium cost per talent $c^B(cap)$ is a convex function in cap, i.e., $\frac{\partial^2 c^B(cap)}{\partial cap^2} > 0$. Thus, tightening the salary cap for high values of cap decreases the aggregate salary payments more than for low values of cap.

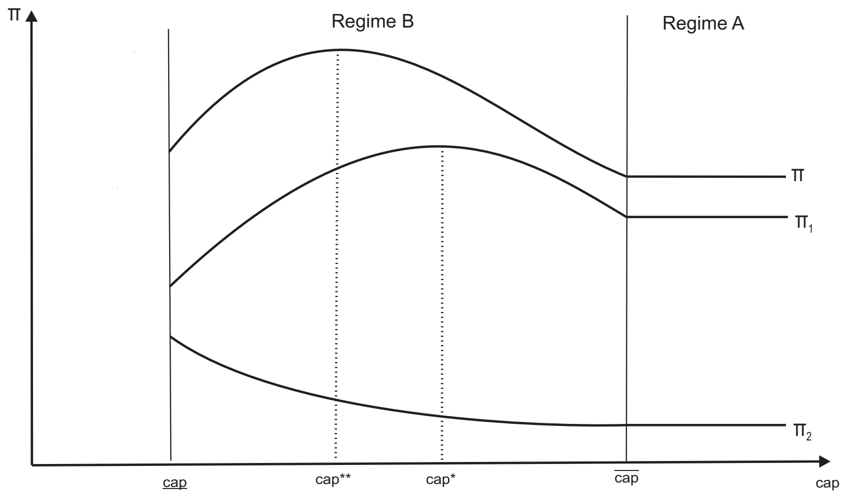

Figure 2: Effect of a salary cap on club profits

more restrictive salary cap and can even be lower than in Regime A.

Moreover, note that the salary cap that maximizes the profits of the large club is less restrictive than the salary cap that maximizes aggregate club profits, i.e., $cap^{*} > cap^{**}$. If $cap < cap^{*}$, the profits of the large club already start to decrease, but the additional profits of the small club exceed the losses of the large club, and aggregate profits thus still increase until $cap = cap^{**}$.

2.2.2 The effect of revenue sharing in Regime B

In this subsection, we analyze the effect of changing the revenue-sharing parameter α in Regime B, given that the league has set a certain $cap' \in \left(\underline{cap}, \overline{cap}\right)$.

The effect of revenue sharing on talent demand and the cost per unit of talent is derived in the following proposition:

Proposition 4

(i) The invariance proposition does not hold in Regime B: revenue sharing decreases competitive balance.

(ii) Revenue sharing decreases the cost per unit of talent in Regime B.

Proof. See Appendix. ■

The proposition shows that the invariance proposition with respect to revenue sharing does not hold when a revenue-sharing arrangement is combined with a (binding) salary cap. A higher degree of revenue sharing (i.e., a lower value of α) induces a reallocation of talent from the small to the large club. That is, the large club increases its talent demand by the same amount by which the small club decreases its talent demand, i.e., $0 > \frac{\partial t_1^B}{\partial \alpha} = -\frac{\partial t_2^B}{\partial \alpha} < 0$.

As a consequence, revenue sharing increases the win percentage w_1^B of the large club and decreases the win percentage w_2^B of the small club, producing a more unbalanced league. It follows that the individual revenues R_1^B (R_2^B) of the large (small) club increase (decrease) through a higher degree of revenue sharing. In the aggregate, club revenues $R_1^B + R_2^B$ in Regime B will increase through more revenue sharing because the league approaches the revenue-maximizing win percentages (w_1^*, w_2^*). Thus, revenue sharing counteracts the salary cap's positive effect on competitive balance in the league.[24]

As discussed in Section 2.1, more revenue sharing inevitably decreases marginal revenue and thus the talent demand for both clubs. As the talent demand of the small club decreases, the cost per unit of talent also has to decrease in order to clear the labor market. This in turn reduces aggregate salary payments. Note that the salary payments of the small club also decrease, i.e., $\frac{\partial x_2^B}{\partial \alpha} > 0$, whereas the salary payments of the large club are fixed to the salary cap.

The effect of revenue sharing on club profits is analyzed in the following proposition:

Proposition 5

Revenue sharing increases the profits of both clubs and thus also aggregate club profits.

Proof. Straightforward and therefore omitted. ■

The proposition shows that both the small and the large club benefit from revenue sharing. On the one hand, the introduction of a revenue-sharing arrangement increases aggregate club revenues $R_1^B + R_2^B$ in the league. Note that the large club's individual revenues R_1^B and thus its after-sharing revenues $\widehat{R}_1^B$ also increase. Even though the individual revenues of the small club decrease R_2^B through revenue sharing, this club's after-sharing revenues $\widehat{R}_2^B$ increase due to the higher aggregate club revenues. On the other hand, revenue sharing decreases the costs of the small club due its lower salary payments and does not change the cost side of the large club. As a consequence, revenue sharing increases the profits of both clubs.

What would happen if in addition to a binding salary cap (for the large club), a binding salary floor (for the small club) was also introduced? The salary floor would have an effect opposite to that of the salary cap. The salary floor would artificially boost the demand of the small club. This would increase the cost per unit of talent and reallocate talent from the large to the small club. Aggregate revenues would deteriorate as the distribution of win percentages would move further away from the optimal allocation. As a consequence, profits of the large

[24] See also Vrooman (2007, 2008).

club would shrink as revenue decreases and costs rise. For the small club, the effect is ambiguous and would depend on whether the additional revenue exceeds the increased costs.

2.3 Regime C: salary cap is not binding for large club, but salary floor is binding for small club

In this section, we assume that the salary floor is only binding for the small-market club and the salary cap is not binding for the large-market club. In Regime C, the equilibrium demand for talent and the cost per unit of talent are computed as[25]

$$\begin{aligned}(t_1^C, t_2^C) &= \left(\left(1 - \frac{2floor}{(\alpha-1)+\phi^C}\right)s, \frac{2floor}{(\alpha-1)+\phi^C}s\right) = (w_1^C s, w_2^C s),\\ c^C &= \frac{(\alpha-1)+\phi^C}{2s},\end{aligned} \tag{6}$$

with $\phi^C := \sqrt{(\alpha-1)^2 + 4floor(1-\alpha+m(1+\alpha))}$.[26] The equilibrium salary payments are computed as

$$(x_1^C, x_2^C) = \left(\frac{1}{2}((\alpha-1)+\phi^C - 2floor), floor\right)$$

Thus, we are in Regime C if $floor \in \left(\underline{floor}, \overline{floor}\right) = \left(x_2^A, \frac{\alpha-1+m(1+\alpha)}{4}\right)$ with a sufficiently loose salary cap. The condition for $floor$ guarantees that the salary payments of the large club are higher than $floor$. Otherwise, the salary floor would be binding for both clubs and we would be in Regime D.[27] As with Regime B, the condition $floor \in \left(\underline{floor}, \overline{floor}\right)$ implicitly defines the interval of feasible revenue-sharing parameters α for Regime C with $\alpha \in (\underline{\alpha}^C, \overline{\alpha}^C) = \left(\frac{1+4floor-m}{1+m}, \frac{floor(1+m)^2}{2m}\right)$.

2.3.1 The effect of a salary floor in Regime C

In this subsection, we analyze the effect of changing the salary floor parameter given that the league has set a certain degree α'' of revenue sharing. We derive the following results:

Proposition 6

The introduction of a binding salary floor increases both competitive balance and the cost per unit of talent in Regime C.

Proof. See Appendix. ■

[25]Note that the equilibrium demand for talent before calculating the equilibrium cost per talent is given by $\left(t_1^C(c), t_2^C(c)\right) = \left(\frac{m(1+\alpha)s-cs^2}{m(1+\alpha)+(1-\alpha)}, \frac{floor}{c}\right)$.

[26]Note that $(\alpha-1)^2 + 4floor(1-\alpha+m(1+\alpha)) > 0$ since $floor \in \left(\underline{floor}, \overline{floor}\right)$.

[27]Formally, if $floor < \underline{floor}$, then $x_1^C > x_1^A$ ($x_2^C < x_2^A$) and we are in Regime A. If $floor > \overline{floor}$, then $x_1^C < x_2^C$ and we are in Regime E.

The reasoning for this result is similar to that for Regime B. The salary floor forces the small club to enhance expenses thereby raising the overall demand for talent and thus the market clearing cost per unit of talent. Despite this, the small club hires a larger amount of talent.

Hence, implementing a more restrictive salary floor induces a reallocation of talent from the large club to the small club, i.e., $0 < -\frac{\partial t_1^C}{\partial floor} = \frac{\partial t_2^C}{\partial floor} > 0$.[28] A higher value of $floor$ decreases the win percentage w_1^C of the large club and increases the win percentage w_2^C of the small club. As a result, competitive balance increases in Regime C. Moreover, the large club's individual revenue R_1^C will decrease, and the small club's individual revenue R_2^C will increase. Aggregate club revenues $R_1^C + R_2^C$, however, will decrease because the league departs from the revenue-maximizing win percentages from Regime A.

Moreover, a more restrictive salary floor will increase the salary payments for both clubs in equilibrium, i.e., $\frac{\partial x_i^C}{\partial floor} > 0$, $i = 1, 2$. This is obvious for the small club, as price and quantity of talent increase. For the large club, the decrease in talent demand cannot compensate for the increase in cost per unit of talent. As a result, salary payments will also increase for the large club.

The effect of a salary floor on club profits is stated in the following proposition:

Proposition 7

A more restrictive salary floor decreases the profits of both clubs and thus also aggregate club profits.

Proof. See Appendix. ■

It is clear that the profits of the large club will decrease because this club's revenues decrease and its costs increase. However, the effect of a more restrictive salary floor on the profits of the small club is also negative. Note that in Regime A, the condition that marginal revenue equals marginal cost holds for the small club. Moreover, a more restrictive salary floor yields a higher win percentage for the small club and thus induces a decrease in the marginal revenue of the small club. Additionally, cost per unit of talent increases. All together this implies that additional revenues cannot compensate for the higher costs.

2.3.2 The effect of revenue sharing in Regime C

In this subsection, we analyze the effect of changing the revenue-sharing parameter α in Regime C given that the league has fixed a certain $floor' \in \left(\underline{floor}, \overline{floor}\right)$.

We analyze the effect of revenue sharing on talent demand and the cost per unit of talent in the following proposition:

[28]Note that a more restrictive salary floor is characterized by a higher level of $floor$.

Proposition 8

(i) The invariance proposition does not hold in Regime C: revenue sharing increases competitive balance.

(ii) Revenue sharing decreases the cost per unit of talent in Regime C.

Proof. See Appendix. ■

As noted above, revenue sharing always decreases marginal revenue and thus the talent demand of the large club. This implies that the market-clearing cost per unit of talent decreases and that the large club hires less talent. The lower cost per unit of talent reduces aggregate salary payments.[29]

As in Regime B, the invariance proposition does not hold when revenue sharing is combined with a (binding) salary floor. In contrast to Regime B, a higher degree of revenue sharing produces a more balanced league in Regime C because the large club decreases its talent demand by the same amount by which the small club increases its talent demand, i.e., $0 > -\frac{\partial t_1^C}{\partial \alpha} = \frac{\partial t_2^C}{\partial \alpha} < 0$. As a result, more revenue sharing increases (decreases) the win percentage w_2^C (w_1^C) of the small (large) club and therefore increases (decreases) the individual revenues R_2^C (R_1^C) of the small (large) club. Moreover, we depart from the revenue-maximizing win percentages from Regime A, and aggregate club revenues $R_1^C + R_2^C$ will thus decrease through revenue sharing.

Both mechanisms - a salary floor and a revenue-sharing arrangement - contribute to producing more balanced competition. However, the revenue-sharing arrangement achieves this goal with lower costs (salary payments), because it lowers the costs of the large club, whereas a salary floor increases the costs of both clubs.

The effect of revenue sharing on clubs' profit is analyzed in the following proposition:

Proposition 9

Revenue sharing increases the profits of the small club and decreases the profits of the large club.

Proof. See Appendix. ■

The proposition shows that only the small club benefits from a revenue-sharing arrangement in Regime C. The positive effect of revenue sharing through lower costs and higher individual revenue R_2^C for the small club compensates for the lower aggregate revenues $R_1^C + R_2^C$. For the large club, the effect is different, because the lower costs cannot compensate for lower (individual and aggregate) revenues, and thus profits decrease.

[29]Note that the salary payments of the small club are unaffected by revenue sharing.

2.4 Regime D: either salary cap or salary floor is binding for both clubs

In this section, we assume that either the salary cap or the salary floor is binding for both clubs. For notation's sake, we write $\Phi \in \{floor, cap\}$.[30] In Regime D, the equilibrium demand for talent and the cost per unit of talent are computed as

$$\begin{aligned} \left(t_1^D, t_2^D\right) &= \left(\frac{s}{2}, \frac{s}{2}\right) = (w_1^D s, w_2^D s), \\ c^D &= \frac{2 \cdot \Phi}{s}. \end{aligned} \tag{7}$$

The equilibrium salary payments are then given by $\left(x_1^D, x_2^D\right) = (\Phi, \Phi)$ with $\Phi \in \{floor, cap\}$, depending on whether we consider a binding salary floor or salary cap for both clubs. Thus, we are in Regime D if either $floor > \overline{floor}$ or $cap < \underline{cap}$. In the first case, the salary floor is binding for both clubs, and in the second case, the salary cap is binding for both clubs.[31]

From (7), we derive that a change in the salary cap (salary floor) does not change the distribution of talent in Regime D. However, by implementing a more restrictive salary cap, the cost per unit of talent c^D decreases, whereas c^D increases through a more restrictive salary floor.

A salary cap is therefore beneficial for club profits because it lowers the costs of both clubs and club revenues remain unchanged. The opposite is true for a more restrictive salary floor, because it raises clubs' costs and leaves clubs' revenues unchanged.

Moreover, we see that talent demand and the cost per unit of talent are independent of the revenue-sharing parameter α if the salary floor (cap) is binding for both clubs, i.e., for $\Phi \in \{floor, cap\}$. Thus, the invariance principle holds in Regime D because revenue sharing has no effect on the distribution of talent and thus does not affect individual club revenues. Moreover, the cost per unit of talent c^D is alsoare unaffected by revenue sharing.

As in Regime A, revenue sharing redistributes revenues from the large to the small club. As a consequence, the profits of the large club decrease and the profits of the small club increase through a higher degree of revenue sharing. Aggregate club profits, however, are not affected by revenue sharing in Regime D.

3 Conclusion

In this article, we have analyzed the combined effect of salary restrictions (salary cap and floor) and revenue-sharing agreements on club profits, player salaries, and competitive balance. For

[30] Note that we consider both cases at the same time because the analyses are very similar.

[31] Note that the salary cap has to be sufficiently large (first case) and the salary floor has to be sufficiently small (second case).

our analysis, we used a standard FQ-style model with Walrasian conjectures.

We show that in the well-known case of a league without a binding salary cap or floor, the famous invariance proposition holds. Although revenue sharing has no effect on the distribution of talent it does have implications for the distribution of benefits between clubs and players. Revenue sharing inevitably lowers the market-clearing cost per unit of talent and increases the profits of the small clubs and aggregate club profits. The effect on the profits of the large club is ambiguous and depends on the difference between the clubs in terms of market size. This means that revenue sharing can be used to redistribute rents from clubs to players and vice versa.

However, the invariance proposition does not hold even under Walrasian conjectures if revenue sharing is combined with either a salary cap or a salary floor. Introducing a salary cap has the intended effect of increasing competitive balance and increasing the profits of the small club. A salary cap therefore effectively supports the small clubs. However, the increased competitive balance is detrimental to aggregate league revenues, because talent is removed from its most productive use. In this situation, adding a revenue-sharing arrangement helps to reallocate talent back to its most productive use. Additionally, increased revenue sharing lowers costs and increases profits. Therefore, far from being invariant, revenue sharing is a very effective tool for cross-subsidization.

Introducing a salary floor is beneficial to players but achieves this by departing from the productive allocation of talent and lowering the profits of the clubs. In this case, revenue sharing will worsen the misallocation. We conclude that the mixture of revenue sharing and salary caps is preferable.

The analysis has shown that both a salary cap and a salary floor contribute to improving competitive balance in the league. From the perspective of a league planner, however, a fully balanced league is not desired, i.e., a certain degree of imbalance is favorable. In our model, the league optimum (w_1^*, w_2^*), defined as the allocation of talent that maximizes aggregate league revenues, is characterized by an allocation of talent where the large club is the dominant team that has a higher win percentage than the small club. According to our analysis, this league optimal degree of imbalance, which increases in the difference between clubs, is already achieved in a league with revenue sharing that has implemented neither a salary cap nor a salary floor.[32] Every intervention to improve competitive balance like salary caps and salary floors combined with revenue sharing arrangements, is counter-productive, because it will result in an unproductive allocation of talent.

[32]Remember that due to the invariance principle, revenue sharing has no effect on the league optimal allocation.

Appendix

A Proofs

A.1 Proof of Proposition 1

The proof of Part (i) and (ii) is straightforward by inspection of(4) which represents the demand for talent and the cost per unit of talent in equilibrium.

In order to prove Part (iii), we compute the equilibrium after-sharing revenues of club $i = 1, 2$ in Regime A as follows:

$$\begin{aligned} \widehat{R}_1^A &= \frac{1 + m + m(1+\alpha)(1+m)^2 - \alpha(3m+1)}{2(1+m)^2} \\ \widehat{R}_2^A &= \frac{(1+m)(1+m+m^2) - \alpha(m-1)(1+m(3+m))}{2(1+m)^2} \end{aligned}$$

We derive the derivatives with respect to α as: $\frac{\partial \widehat{R}_1^A}{\partial \alpha} = \frac{m(1+m)^2-(3m+1)}{2(1+m)^2} > 0$ and $\frac{\partial \widehat{R}_2^A}{\partial \alpha} = -\frac{(m-1)(1+m(3+m)}{2(1+m)^2} < 0 \ \forall \alpha \in (1, 0)$. Thus after-sharing revenues of the large (small) club decrease (increase) through a higher degree of revenue sharing, i.e. a lower value of the parameter α.

The equilibrium profits of club $i = 1, 2$ in Regime A are then given by $\pi_1^A = \widehat{R}_1^A - x_1^A$ and $\pi_2^A = \widehat{R}_2^A - x_2^A$ with the corresponding derivatives

$$\frac{\partial \pi_1^A}{\partial \alpha} = \frac{m(-2 + m(m-2)) - 1}{2(1+m)^2} \text{ and } \frac{\partial \pi_2^A}{\partial \alpha} = \frac{-m(1+m)^2 + (1-m)}{2(1+m)^2}.$$

It follows that $\frac{\partial \pi_1^A}{\partial \alpha} > 0 \Leftrightarrow m^3 - 2m(1+m) - 1 > 0$. Thus, $\frac{\partial \pi_1^A}{\partial \alpha} > 0 \Leftrightarrow m > m' \approx 2.83$. Moreover, $\frac{\partial \pi_2^A}{\partial \alpha} < 0 \ \forall \alpha \in (1, 0)$ and $m > 1$. Thus revenue sharing always increases the profits of the small club π_2^A whereas the profits of the large club π_1^A only increase if the difference between both clubs in terms of market size is not too big, i.e. if $m < m'$. It is obvious that aggregate club profits increase through revenue sharing because aggregate revenues are independent of α whereas the clubs' costs (aggregate salary payments) decrease. This proves the proposition.

A.2 Proof of Proposition 2

First of all, remember that we are in Regime B, i.e. $cap \in (\underline{cap}, \overline{cap}) = \left(\frac{1+\alpha-m(1-\alpha)}{4}, x_1^A\right)$. In order to prove that a more restrictive salary cap produces a more balanced league by increasing the win percentages of the small club and decreasing the win percentage of the large club, we derive the equilibrium win percentages in Regime B as

$$w_1^B = \frac{t_1^B}{t_1^B + t_2^B} = \frac{2cap}{m(\alpha-1)+\phi^B} \text{ and } w_2^B = 1 - w_1^B \tag{8}$$

with $\phi^B = \sqrt{(\alpha-1)^2 m^2 + 4cap(1+\alpha+m(1-\alpha))}$. The corresponding derivatives are given by $\frac{\partial w_1^B}{\partial cap} = \frac{1}{\phi^B} > 0$ and $\frac{\partial w_2^B}{\partial cap} = -\frac{1}{\phi^B} < 0$. It follows that a more restrictive salary cap, i.e. a lower value of *cap*, produces a more balanced league by increasing competitive balance.[33] This proves the first part of the proposition.

The derivative of the equilibrium cost per unit of talent $c^B = \frac{(\alpha-1)m+\phi^B}{2s}$ in Regime B with respect to *cap* is given by $\frac{\partial c^B}{\partial cap} = \frac{1+\alpha+m(1-\alpha)}{\phi^B s} > 0$. This proves the second part of the proposition.

A.3 Proof of Proposition 3

In order to prove the claim, without loss of generality, we normalize the supply of talent to unity, i.e. $s = 1$. Moreover, we consider a league without revenue sharing, i.e. $\alpha = 1$.[34] In this case, the maximum of aggregate club profits π^B and the profits of the large club π_1^B are given by

$$\begin{aligned}
\max_{cap>0} \pi^B \quad &: \quad \frac{\partial \pi^B}{\partial cap} = \frac{\sqrt{2}(ms-1) - \sqrt{cap}(1+m)s}{2\sqrt{caps}} = 0 \Leftrightarrow cap^{**} = \frac{2(ms-1)^2}{s^2(m+1)^2} \\
\max_{cap>0} \pi_1^B \quad &: \quad \frac{\partial \pi_1^B}{\partial cap} = -1 + m\left(\frac{1}{\sqrt{2cap}} - \frac{1}{2}\right) = 0 \Leftrightarrow cap^* = \frac{2m^2}{(2+m)^2}
\end{aligned}$$

We derive $cap^* > cap^{**}$. The derivative of the small club's profits π_2^B with respect to *cap* is computed as

$$\frac{\partial \pi_2^B}{\partial cap} = \frac{1}{2} - \frac{1}{\sqrt{2cap}} < 0 \; \forall cap \in \left(\underline{cap}, \overline{cap}\right) = \left(\frac{1+\alpha-m(1-\alpha)}{4}, x_1^A\right)$$

This proves the claim.

A.4 Proof of Proposition 4

Ad (i) In order to prove that the invariance proposition does not hold in Regime B, we compute the derivative of the clubs' talent demand (t_1^B, t_2^B) with respect to the revenue sharing parameter as follows

$$\frac{\partial t_1^B}{\partial \alpha} = -\frac{2cap\left(m + \frac{-2cap(m-1)+m^2(\alpha-1)}{\phi^B}\right)}{((\alpha-1)m+\phi^B)^2} = -\frac{\partial t_2^B}{\partial \alpha},$$

[33] Remember that club 1 is the dominant team which has a higher win percentage than club 2.
[34] It can be shown that the result holds true for all $\alpha \in [0, 1]$.

with $\phi^B = \sqrt{(\alpha-1)^2 m^2 + 4cap(1+\alpha+m(1-\alpha))}$. We deduce that $\frac{\partial t_1^B}{\partial \alpha} < 0$ and $\frac{\partial t_1^B}{\partial \alpha} > 0$, because $\alpha \in (\underline{\alpha}^B, \overline{\alpha}^B) = \left(\frac{cap(1+m)^2}{2m^2}, \frac{4cap+m-1}{1+m}\right)$.[35] Thus, revenue sharing changes the allocation of talent in Regime B because it induces the large club to increase its demand for talent and the small club to decrease its demand for talent. As a consequence the large (small) club's win percentage w_1^B (w_2^B) increases (decreases). Since the large club is the dominant team, competitive balance decreases as a result of revenue sharing.

Ad (ii) In order to prove that revenue sharing decreases the cost per unit of talent c^B in Regime B, we derive the derivative of c^B with respect to α as

$$\frac{\partial c^B}{\partial \alpha} = \frac{1}{2s}\left(m + \frac{-2cap(m-1)+(\alpha-1)m^2}{\phi^B}\right).$$

We deduce that $\frac{\partial c^B}{\partial \alpha} > 0$, because $\alpha \in (\underline{\alpha}^B, \overline{\alpha}^B)$. Thus, more revenue sharing (i.e. a lower value of α) decreases c^B which proves the claim.

A.5 Proof of Proposition 6

First of all, remember that we are in Regime C, i.e. $floor \in \left(\underline{floor}, \overline{floor}\right) = \left(x_2^A, \frac{\alpha-1+m(1+\alpha)}{4}\right)$. In order to prove that a more restrictive salary floor produces a more balanced league by increasing the win percentages of the small club and decreasing the win percentage of the large club, we derive the equilibrium win percentages in Regime C as

$$w_1^C = \frac{t_1^C}{t_1^C + t_2^C} = \frac{(\alpha-1) - 2floor + \phi^C}{(\alpha-1)+\phi^C} \text{ and } w_2^C = 1 - w_1^C, \tag{9}$$

with $\phi^C := \sqrt{(\alpha-1)^2 + 4floor(1-\alpha+m(1+\alpha))}$. The corresponding derivatives are given by $\frac{\partial w_1^C}{\partial floor} = -\frac{1}{\phi^C} < 0$ and $\frac{\partial w_2^C}{\partial floor} = \frac{1}{\phi^C} > 0$. It follows that a more restrictive salary floor produces a more balanced league by increasing competitive balance.[36] This proves the first part of the proposition.

The derivative of the equilibrium cost per unit of talent $c^C = \frac{(\alpha-1)m+\phi^B}{2s}$ in Regime C with respect to $floor$ is given by $\frac{\partial c^C}{\partial floor} = \frac{1+\alpha(m-1)+m}{\phi^C s} > 0$. This proves the second part of the proposition.

A.6 Proof of Proposition 7

It is straightforward to prove that the profits of the large club π_1^B decrease through a more restrictive salary floor: On the one hand, revenues (individual and aggregate revenues) decrease

[35]Remember that we are in Regime B where $cap \in \left(\underline{cap}, \overline{cap}\right)$ which determines implicitely the corresponding interval of feasible α.

[36]Remember that club 1 is the dominant team which has a higher win percentage than club 2.

and on the other hand costs (salary payments) increase for the large club. As a consequence, profits decrease. A similar argument holds true to show that aggregate club profits π^B decrease.

In order to prove that also profits of the small club decrease we derive the derivative of π_2^B with respect to $floor$ as

$$\frac{\partial \pi_2^B}{\partial floor} = \frac{a^2(m-1)-(1+m)(3\phi^C-1)+a(\phi^C-m(\phi^C-6))}{2\phi^C(1+\alpha(m-1)+m)}$$

with $\phi^C := \sqrt{(\alpha-1)^2+4floor(1-\alpha+m(1+\alpha))}$. We derive that $\frac{\partial \pi_2^B}{\partial floor} < 0$ for all $floor \in \left(\underline{floor}, \overline{floor}\right)$. This proves the claim.

A.7 Proof of Proposition 8

Ad (i) In order to prove that the invariance proposition does not hold in Regime C, we compute the derivative of the clubs' talent demand (t_1^C, t_2^C) with respect to the revenue sharing parameter α as follows

$$\frac{\partial t_1^C}{\partial \alpha} = -\frac{2floor\left(\alpha-1+2floor(m-1)+\phi^C\right)}{\phi^C(\alpha-1+\phi^C)^2} = -\frac{\partial t_2^C}{\partial \alpha},$$

with $\phi^C := \sqrt{(\alpha-1)^2+4floor(1-\alpha+m(1+\alpha))}$. We deduce that $\frac{\partial t_1^C}{\partial \alpha} > 0$ and $\frac{\partial t_1^C}{\partial \alpha} < 0$, because $\alpha \in (\underline{\alpha}^C, \overline{\alpha}^C) = \left(\frac{1+4floor-m}{1+m}, \frac{floor(1+m)^2}{2m}\right)$. Thus, revenue sharing changes the allocation of talent in Regime C, because it induces the large (small) club to decrease (increase) its demand for talent. As a consequence the large (small) club's win percentage w_1^C (w_2^C) decreases (increases). Since the large club is the dominant team, competitive balance increases as a result of revenue sharing.

Ad (ii) In order to prove that revenue sharing decreases the cost per unit of talent c^C in Regime C, we derive the derivative of c^C with respect to α as

$$\frac{\partial c^C}{\partial \alpha} = \frac{1}{2s}\left(1+\frac{\alpha-1+2floor(m-1)}{\phi^C}\right)$$

We deduce that $\frac{\partial c^C}{\partial \alpha} > 0$, because $\alpha \in (\underline{\alpha}^C, \overline{\alpha}^C)$. Thus, more revenue sharing (lower value of α) decreases c^C which proves the claim.

A.8 Proof of Proposition 9

First, we claim that the after-sharing revenues $\widehat{R}_2^C$ of the small club increase through a higher degree of revenue sharing. To prove this claim, we derive that $\widehat{R}_2^C$ is maximized for $\widehat{w}_2 = \frac{1+\alpha}{1+\alpha+m(1-\alpha)}$. The equilibrium win percentages $\left(w_1^C, w_2^C\right)$ in Regime C are given by (6). We deduce that $\widehat{w}_2 > w_2^C \Leftrightarrow floor < floor' := \frac{4\alpha(1+\alpha)m}{(1+\alpha+m(1-\alpha))^2}$. Since $floor' > \underline{floor} = \frac{2\alpha m}{(m+1)^2}$ and

$\frac{\partial w_2^C}{\partial \alpha} < 0$, it follows that after-sharing revenues $\widehat{R}_2^C$ increase through a higher degree of revenue sharing (lower α). Remember that the small clubs' salary payments in Regime C are given by $x_2^C = floor$ and are unaffected by revenue sharing. As a consequence, revenue sharing increases the profits of the small club.

Second, we prove the claim for the large club: On the one hand, more revenue sharing induces the after-sharing revenues $\widehat{R}_1^C$ of the large club to decrease because both individual revenues R_1^C and aggregate revenues $R_1^C + R_2^C$ decrease. On the other hand, the large club's costs also decrease due to lower salary payments. However, the decrease in costs cannot compensate for the decrease in after-sharing revenues and it follows that profits of the large club decrease.

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