The Effect of Luxury Taxes on Competitive Balance, Club Profits, and Social Welfare in Sports Leagues

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Abstract

This paper provides a game-theoretic model of a professional sports league and analyzes the effect of luxury taxes on competitive balance, club profits and social welfare. We show that a luxury tax increases aggregate salary payments in the league as well as produces a more balanced league. Moreover, a higher tax rate increases the profits of large-market clubs, whereas the profits of small-market clubs only increase if the tax rate is not set inadequately high. Finally, we show that social welfare increases with a luxury tax.

JEL Classification Codes: L11, L83

Keywords: Sports League, Luxury Tax, Social Welfare, Competitive Balance

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1 Introduction

A "luxury tax," or competitive balance tax, is a surcharge on the aggregate payroll of a sports team that exceeds a predetermined limit set by the corresponding sports league. The luxury tax was essentially designed to slow the growth of salaries and to prevent large-market teams from signing all of the top players within a league. The money derived from this tax is distributed among the financially weaker teams. The luxury tax thus aims to create a more balanced league, because redistribution among clubs counteracts financial imbalances.

In North America, the National Basketball Association (NBA) and Major League Baseball (MLB) operate with a luxury tax system. In 1984, the NBA became the first league to introduce salary cap provisions. The NBA’s salary cap is a so-called "soft cap", meaning that there are several exceptions that allow teams to exceed the salary cap in order to sign players. These exceptions are mainly designed to enable teams to retain popular players. In 1999, the NBA also introduced a luxury tax system for those teams with an average team payroll exceeding the salary cap by a predefined amount. These teams have to pay a 100% tax to the league for each dollar their payroll exceeds the tax level.

The first luxury tax in professional sports was introduced in 1996 by MLB as part of its Collective Bargaining Agreement (CBA). This agreement imposed a luxury tax of 35% for the first two years and 34% for the third year on the teams with the top five payrolls during the 1997, 1998 and 1999 seasons. Between 2000 and 2002, the luxury tax system was replaced by a revenue-sharing system. MLB reintroduced a luxury tax system in 2003 and set fixed limits on payrolls for every year. For instance, the limit was $137 million in 2006, $148 million in 2007 and $155 million in 2008. The excess payroll is taxed at 22.5% for first-time

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1A salary cap is a limit on the amount of money a club can spend on player salaries. The cap is usually defined as a percentage of average annual revenues and limits a club's investment in playing talent. For a more detailed analysis, see e.g., Fort and Quirk (1995), Késenne (2000a), Szymanski (2003) and Vrooman (1995, 2000).
offenders, 30% for the second offence and 40% for three or more offences.

The following table shows recent luxury tax payments in the NBA and MLB:

<table>
<thead>
<tr>
<th>League</th>
<th>Club</th>
<th>Season 2006-07 ($100'000)</th>
<th>Season 2007-08 ($100'000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NBA</td>
<td>New York Knicks</td>
<td>45'100'000</td>
<td>19'700'000</td>
</tr>
<tr>
<td></td>
<td>Dalas Mavericks</td>
<td>7'200'000</td>
<td>19'600'000</td>
</tr>
<tr>
<td></td>
<td>Cleveland Cavaliers</td>
<td>-</td>
<td>14'000'000</td>
</tr>
<tr>
<td></td>
<td>Denver Nuggets</td>
<td>2'000'000</td>
<td>13'600'000</td>
</tr>
<tr>
<td></td>
<td>Miami Heat</td>
<td>-</td>
<td>8'300'000</td>
</tr>
<tr>
<td></td>
<td>Boston Celtics</td>
<td>-</td>
<td>8'200'000</td>
</tr>
<tr>
<td></td>
<td>Minnesota Timberwolves</td>
<td>1'000'000</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>LA Lakers</td>
<td>-</td>
<td>5'100'000</td>
</tr>
<tr>
<td></td>
<td>Phoenix Suns</td>
<td>-</td>
<td>3'900'000</td>
</tr>
<tr>
<td></td>
<td>San Antonio Spurs</td>
<td>200'000</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>League</th>
<th>Club</th>
<th>Season 2006 ($100'000)</th>
<th>Season 2007 ($100'000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLB</td>
<td>New York Yankees</td>
<td>26'000'000</td>
<td>23'880'000</td>
</tr>
<tr>
<td></td>
<td>Boston Red Soxs</td>
<td>498'000</td>
<td>6'060'000</td>
</tr>
</tbody>
</table>

This paper provides a game-theoretic model with respect to a professional team sports league in order to analyze the effect of luxury taxes on competitive balance, club profits and social welfare. We show that a luxury tax increases aggregate salary payments in the league as well as produces a more balanced league. Moreover, a higher tax rate increases the profits of large-market clubs, whereas the profits of small-market clubs only increase if the tax rate is not set inadequately high. Finally, we show that social welfare increases with a luxury tax.

The effect of luxury taxes on social welfare has not yet been studied in the sports economic literature. There are, however, some studies that analyze the effect of luxury taxes on competitive balance and player salaries. Based on a dual
supply and demand model for star and regular players, Gustafson and Hadley (1996) find that a luxury tax will depress the demand curve for star players on high-payroll teams and will not alter the demand for star players by low-payroll teams. The result is a lower equilibrium salary for star players. The new equilibrium is characterized by a higher level of competitive balance, because the high-payroll teams will hire fewer star players and the low-payroll teams will hire more star players as compared to the period prior the introduction of the luxury tax.

Marburger (1997) develops a model with two profit-maximizing clubs, including one large-market club and one small-market club, and a fixed talent supply. He shows that luxury taxes that are uniformly imposed as a linear function of a club’s payroll and that are not redistributed to other clubs do not affect club profitability because the decline in salaries equals the increase in taxes. Luxury taxes that are redistributed according to a linear subsidy function result in lower salaries and higher profits, but they do not affect competitive balance. In order to reward small-market clubs and improve competitive balance, the proceeds of luxury taxes must be distributed uniformly among all clubs.

Ajilore and Hendrickson (2005) analyze the effect of luxury taxes on competitive balance in MLB by empirically estimating the impact of luxury taxes on team competitiveness. Their results show that the introduction of a luxury tax in MLB has reduced the competitive inequality of teams. However, most of their results are driven by a single team, the New York Yankees.

The remainder of the paper is organized as follows. The next section presents the model’s framework, including its underlying assumptions, and discusses the results. Section 3 concludes.
2 Model

The following model describes the impact of luxury taxes on social welfare in a professional team sports league consisting of an even number of profit-maximizing clubs. The league generates total revenues according to a league demand function. League revenues are then split among the clubs that differ with respect to their bargaining power. We assume that there are two types of clubs, namely, large-market clubs with strong bargaining power and small-market clubs with weak bargaining power. In order to maximize profits, each club independently invests in playing talent. Note that we regard the salary payment of each club as an investment in talent.\(^2\)

League demand depends on the quality of the league \(q\) and is derived as follows.\(^3\) We assume a continuum of fans who differ in their willingness to pay for a league with quality \(q\). Every fan \(k\) has a certain preference for quality that is measured by \(\theta_k\). The fans \(\theta_k\) are assumed to be uniformly distributed in \([0, 1]\), i.e., the measure of potential fans is one. The net utility of fan \(\theta_k\) is specified as \(\max\{\theta_kq - p, 0\}\). At price \(p\), the fan that is indifferent between consuming the league product or not is given by \(\theta^* = \frac{p}{q}\). Hence, the measure of fans that purchase at price \(p\) is \(1 - \theta^* = \frac{q - p}{q}\). The league demand function is therefore given by \(d(p, q) := 1 - \frac{p}{q}\). Note that league demand increases in quality, albeit with a decreasing rate; that is, \(\frac{\partial d}{\partial q} > 0\) and \(\frac{\partial^2 d}{\partial q^2} < 0\). By normalizing all other costs (e.g., stadium and broadcasting costs) to zero, league revenue is simply \(LR = p \cdot d(p, q)\). Then, the league will choose the profit-maximizing price

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\(^2\)In the subsequent analysis, we use the terms "salary payments" and "talent investments" interchangeably.

\(^3\)Our approach is similar to Falconieri et al. (2004), but we use a different quality function. The quality function \(q\) in Falconieri et al. (2004) always increases with a club’s own talent investments, i.e., \(\frac{\partial q}{\partial x_i} > 0\), regardless of how unbalanced the league becomes. In contrast, in our model, quality decreases if the league becomes too unbalanced. Also see Dietl and Lang (2008), Dietl, Lang and Rathke (2009) and Dietl, Lang and Werner (2009)
\[ p^* = \frac{q}{2}. \] Given this profit-maximizing price, league revenue depends solely on the quality of the league as follows:

\[ LR = q \frac{q}{4}. \] (1)

Following the sports economic literature (e.g. Szymanski, 2003), we assume that league quality depends on the level of the competition as well as the potential suspense associated with a close competition (i.e., competitive balance).

The level of competition is measured by the aggregate talent within the n-club league. We assume that the marginal effect of the salary payment, denoted by \( x_i \), on the level of the competition \( T \) is positive but decreasing:

\[ T(x_1, \ldots, x_n) = \alpha \sum_{j=1}^{n} x_j - \left( \sum_{j=1}^{n} x_j \right)^2. \] (2)

This is guaranteed in our model if \( \frac{\partial T}{\partial x_i} > 0 \iff \sum_{j=1}^{n} x_j < \frac{\alpha}{2} \) and \( \frac{\partial^2 T}{\partial x_i^2} < 0 \), which will always be satisfied in equilibrium.

Competitive balance, \( CB \), is measured by minus the variance of salary payments, that is:

\[ CB(x_1, \ldots, x_n) = -\frac{1}{n} \sum_{j=1}^{n} (x_j - \bar{x}_n)^2 \text{ with } \bar{x}_n = \frac{1}{n} \sum_{j=1}^{n} x_j. \] (3)

Note that a lower variance of salary payments among the \( n \) clubs implies closer competition and, therefore, a higher degree of competitive balance. If all clubs invest the same amount in talent, then the measure for competitive balance attains its maximum and equals zero.

\[ ^4 \text{Note that the optimal price increase with quality, i.e., } \frac{\partial p^*}{\partial q} > 0. \]

\[ ^5 \text{For an analysis of competitive balance in sports leagues, see e.g., Humphreys (2002), Buraimo et al. (2007) and Buraimo and Simmons (2008). Moreover, see Frick et al. (2003) who investigate the consequences of wage disparities on team performance.} \]
League quality is now defined as:

\[ q(x_1, \ldots, x_n) = \mu T(x_1, \ldots, x_n) + (1 - \mu) CB(x_1, \ldots, x_n). \]  \tag{4}

The parameter \( \mu \in (0, 1) \) represents the relative weight that fans place on aggregate talent and competitive balance. Given aggregate salaries \( \sum_{j=1, j \neq i}^{n} x_j \) of the other \( (n - 1) \) clubs, league quality increases with club \( i \)'s salary payment \( x_i \) until a threshold value \( x'_i(\mu) \), i.e., \( \frac{\partial q}{\partial x_i} > 0 \iff x_i < x'_i(\mu) \). Since fans have at least some preference for competitive balance, excessive dominance by one club causes quality to decrease.

League revenues are split between the two types of clubs according to the bargaining power of the clubs. For the sake of simplicity, we assume that half of the clubs have strong bargaining power, \( m_l \), and half of them have weak bargaining power, \( m_s \). Each of the large clubs receives a fraction \( \frac{m_l}{n/2} \) of league revenues, and each of the small clubs receives a fraction \( \frac{m_s}{n/2} \) of league revenues, with

\[ m_l > m_s \text{ and } m_l + m_s = 1. \]

We denote \( J_l \) and \( J_s \) as the set of large-market and small-market clubs, respectively, i.e., \( J = \{1, \ldots, n\} = J_l \cup J_s \).

Furthermore, our league features a luxury tax system with an endogenously-determined luxury tax and subsidy.\footnote{See also Marburger (1997).} A club must pay a luxury tax if its salary payment lies above the league's average salary level. The club with a salary payment below the league’s average salary level then receives this tax as a subsidy. We model the endogenously determined tax or subsidy, \( \theta_i(x_1, \ldots, x_n) \), as follows

\[ \theta_i(x_1, \ldots, x_n) = -r \cdot \left( x_i - \frac{1}{n} \sum_{j=1}^{n} x_j \right), \]  \tag{5}
where the parameter $r \in [0, 1]$ represents the tax rate. Note that if club $i$ spends more than the league’s average salary level, then this club has to pay a luxury tax, whereas it receives a subsidy if it spends less than the average level in the league, i.e., $\theta_i > 0 \iff x_i < \frac{1}{n} \sum_{j=1}^{n} x_j$. Moreover, note that $\sum_{i=1}^{n} \theta_i = 0$: that is, the luxury tax or subsidy involves a pure redistribution among clubs.

The profit function $\Pi_i(x_1, \ldots, x_n)$ of club $i \in J$ is given by

$$\Pi_i(x_1, \ldots, x_n) = \frac{m_{\delta}}{2n} q(x_1, \ldots, x_n) - x_i + \theta_i(x_1, \ldots, x_n),$$

$$= \frac{m_{\delta}}{2n} \left( \mu \alpha \sum_{j=1}^{n} x_j - \mu \left( \sum_{j=1}^{n} x_j \right)^2 - \frac{1 - \mu}{n} \sum_{j=1}^{n} (x_j - \bar{x}_n)^2 \right) - x_i$$

$$- r \cdot \left( x_i - \frac{1}{n} \sum_{j=1}^{n} x_j \right),$$

(6)

with $\delta = l$ for $i \in J_l$ and $\delta = s$ for $i \in J_s$.

Social welfare is given by the sum of the aggregate consumer (or fan) surplus, the aggregate club profit and the aggregate player salaries. Aggregate consumer surplus, $CS$, corresponds to the integral of the demand function, $d(p, q)$, from the equilibrium price $p^* = \frac{q}{2}$ to the maximum price $p = q$, which is the maximum price fans are willing to pay for quality $q$, i.e.,

$$CS = \int_{p^*}^{p} d(p, q)dp = \int_{\frac{q}{2}}^{q} \frac{q - p}{q} dp = \frac{q}{8}.$$

Summing up aggregate consumer surplus, aggregate club profits and aggregate salary payments, social welfare is defined as

$$W(x_1, \ldots, x_n) = \frac{3}{8} q(x_1, \ldots, x_n).$$

(7)

Note that neither salary payments, taxes nor subsidies directly influence social
welfare, because salaries merely represent a transfer from clubs to players, and
the tax or subsidy involves a pure redistribution among clubs.

As mentioned above, clubs are assumed to be profit-maximizing and thus, each club \( i \in J \) aims to solve the following maximization problem:\(^7\)

\[
\max_{x_i \geq 0} \Pi_i(x_1, \ldots, x_n)
\]

The solution to the maximization problem is given in the next lemma:

**Lemma 1**

The equilibrium salary payments for club \( i \in J \) are given by

\[
x^*_i = \frac{\alpha}{2n} - \frac{v_1 w_2 - v_2 w_1}{2m_1 m_s n (1 - \mu) \mu} =: x^*_i \forall i \in J_l,
\]

\[
x^*_j = \frac{\alpha}{2n} - \frac{v_1 w_1 - v_2 w_2}{2m_1 m_s n (1 - \mu) \mu} =: x^*_j \forall j \in J_s,
\]

with \( v_1 := m_s (r(n - 1) + n), v_2 := m_l (r(n - 1) - n) \) and \( w_1 = 1 - \mu (n^2 + 1), w_2 = 1 + \mu (n^2 - 1) \).

**Proof.** See the Appendix. ■

The lemma shows that all large (small) clubs choose the same salary level, \( x_l \) (\( x_s \)). Moreover, the large clubs spend more on player salaries than the small clubs because the marginal revenue of talent investments is higher for the former type of clubs. As a consequence, each large club has to pay a luxury tax, and each small club receives a subsidy, which is financed by the large clubs.

We note that a higher tax rate \( r \) induces small clubs to increase their talent investments, i.e., \( \frac{\partial x^*_s}{\partial r} > 0 \). This result is intuitively clear: a higher tax rate increases the subsidies to small clubs, which are financed by large clubs, such that the investment costs of small clubs decrease. The effect of a higher tax rate

\(^7\)For a discussion of the club objective function, see e.g., Sloane (1971), Hoehn and Szymanski (1999), Fort and Quirk (2004), Késséne (2000b, 2007), Dietl, Lang and Werner (2009).
on the talent investments of large clubs, however, is ambiguous and depends on
the preference parameter $\mu$. Note that

$$
\frac{\partial x^*_i}{\partial r} = \frac{(n - 1)(m_s - m_l + \mu(m_l - m_s + n^2))}{2m_l m_s (1 - \mu) \mu} \begin{cases} 
> 0 & \text{if } \mu \in \left(0, \frac{m_l - m_s}{m_l - m_s + n^2}\right), \\
= 0 & \text{if } \mu = \frac{m_l - m_s}{m_l - m_s + n^2}, \\
< 0 & \text{if } \mu \in \left(\frac{m_l - m_s}{m_l - m_s + n^2}, 1\right).
\end{cases}
$$

A higher tax rate induces large clubs to increase their investment level if fans have
a high preference for competitive balance, i.e., $\mu < \frac{m_l - m_s}{m_l - m_s + n^2}$, and to decrease
their investment level if fans have a high preference for aggregate talent, i.e.,
$\mu > \frac{m_l - m_s}{m_l - m_s + n^2}$. The rationale for this result is as follows. If $\mu$ is relatively small,
the equilibrium (8) is characterized by a high level of competitive balance and a
low level of aggregate talent. At these equilibrium levels, the marginal benefit of
a higher level of aggregate talent, which translates into higher revenues, is larger
than the higher investment costs due to a higher tax. As a consequence, large
clubs will increase their investment level.

In contrast, if $\mu$ is relatively high, the equilibrium is already characterized by
a high level of aggregate talent. In this case, the marginal benefit of a higher
level of aggregate talent is small, and the higher investments costs compensate
for the higher revenues.

On aggregate, however, the investment level always increases with a higher
tax rate. That is, even if large clubs decrease their investments (i.e., if $\mu > \frac{m_l - m_s}{m_l - m_s + n^2}$), they never compensate for the increase of talent among small clubs.

The luxury tax paid by each large club, $i \in J_l$, in equilibrium is given by

$$
\theta_i = -\frac{rn((m_l - m_s)n - (n - 1)r)}{2m_s m_l (1 - \mu)} < 0.
$$
Meanwhile the subsidy received by each small club, \( j \in J_s \), is given by

\[
\theta_s = \frac{rn((m_l - m_s)n - (n - 1)r)}{2m_sm_l(1 - \mu)} > 0.
\]

Note that a higher tax rate, \( r \), increases the subsidy \( \theta_s \) received by small clubs and decreases the luxury tax, \( \theta_l \), paid by large clubs until the maximum and minimum, respectively, is reached for

\[
r = \hat{r} := \frac{n(m_l - m_s)}{2(n-1)}.
\]

In equilibrium, the aggregate level of salary payments, \( S^*(r) \), and competitive balance, \( CB^*(r) \), are given by

\[
S^*(r) = \sum_{j=1}^{n} x_j = \frac{\alpha}{2} - \frac{m_l(n(1 - r) + r) + m_s(n + r(n - 1))}{2m_tm_sm_l},
\]

and

\[
CB^*(r) = -\left( \frac{n(m_l - m_s) - r(n - 1))}{2m_tm_sm_l(1 - \mu)} \right)^2.
\]

We thus derive the following proposition.

**Proposition 1**

A higher tax rate increases the level of competition and produces a more balanced league.

**Proof.** See the Appendix. ■

Remember that on aggregate, the investment level increases with a higher tax rate: that is, the net effect of a higher tax rate is positive, and aggregate player salaries in the league will increase, i.e., \( \frac{\partial S^*(r)}{\partial r} > 0 \). It follows that a higher level of aggregate salary payments in the league translates through the talent function (2) into a higher level of the competition, \( T \).

The proposition further shows that a higher tax rate produces a more balanced league and, thus, increases competitive balance, i.e., \( \frac{\partial CB^*(r)}{\partial r} > 0 \). The rationale for this result is that a higher tax diminishes differences among clubs.
That is, even if large clubs increase their investment levels, small clubs will always respond with even higher investment levels such that \( \frac{\partial x^*}{\partial r} > \frac{\partial x^1}{\partial r} \).

Since both the level of the competition, \( T \), and competitive balance, \( CB \), increase through a higher tax rate, it is clear that league quality, as given by \( q = \mu T + (1 - \mu)CB \), will also increase. A higher league quality will then result in higher league revenues, \( LR \).

As a consequence, we are able to establish the following proposition:

**Proposition 2**

A higher tax rate increases social welfare in a team sports league comprised of profit-maximizing clubs.

**Proof.** Straightforward. ■

The proposition posits that the introduction of a luxury tax system that redistributes revenues from large-market clubs to small-market clubs increases social welfare in a team sports league comprised of profit-maximizing clubs. Since a higher tax rate increases league quality, it will also increase social welfare because welfare is directly proportional to league quality. Note that the result of the proposition is independent of the fans’ preferences for aggregate talent and competitive balance.

In the following proposition, we analyze the effect of a higher tax rate on club profits.

**Proposition 3**

A higher tax rate always increases the profits of large clubs, whereas the profits of small clubs only increase until the profit maximum is reached for a tax rate given by \( r = r^* \in (0, 1] \).

**Proof.** See the Appendix. ■
This proposition posits that even though large clubs must subsidize small clubs, the large clubs always benefit from a higher tax, whereas the small clubs only benefit up to a certain tax level, $r^*$. The rationale for this result is as follows. On the revenue side, both clubs benefit from higher league revenues as a result of a higher luxury tax. Large clubs, however, benefit from the higher league revenues at an above-average rate because they receive a larger share of league revenues. On the cost side, small clubs face higher investment costs due to higher salary payments, while the investment costs for large clubs decrease (increase) if fans have a high preference for aggregate talent (competitive balance). For small clubs, the higher subsidies and higher revenues compensate for the higher salary payments only until the tax rate reaches $r^*$. For large clubs, however, the higher revenues always compensate for the higher costs, and thus, profits increase with a higher tax rate.

3 Conclusion

Luxury taxes are an important way to increase competitive balance in professional sports leagues. In this paper, we analyze the effects of a luxury tax on competitive balance, club profits, and social welfare under the assumption that clubs maximize profits. We develop a game-theoretic model of an n-club league consisting of small-market and large-market clubs and derive fan demand from a general utility function by assuming that a fan’s willingness to pay depends on the quality of the league. Our league features the combination of an endogenously-determined luxury tax and subsidy. Clubs with payroll exceeding the average salary level must pay a luxury tax on the excess amount. These proceeds are then redistributed proportionally to those clubs with a payroll below the league average.

Our analysis shows that a higher luxury tax induces small clubs to increase
their salary payments. If fans have a high preference for aggregate talent, however, large clubs will respond by decreasing their salary payments. Aggregate payrolls will increase with a higher tax rate, as the increase in salary payments by small clubs is always larger than the decrease in salary payments by large clubs. As a consequence, both competitive balance and total salary payments will increase. The effect of luxury taxes on social welfare is positive, because league quality will always increase as a result of the combination of luxury taxes and its resulting subsidies. Finally, our model shows that a luxury tax will increase the profits of large-market clubs, whereas the profits of small-market clubs only increase if the tax rate is not set inadequately high. This result holds despite the fact that large-market clubs must finance the subsidies for small-market clubs.

Further research is necessary, for example, to model the bargaining game among clubs and league authorities in the distribution of league revenues. Moreover, luxury taxes have not yet been analyzed in the context of so-called mixed leagues, that is, in leagues in which some clubs maximize profits, while others aim to maximize wins.
A Appendix

A.1 Proof of Lemma 1

The first-order conditions of equation (6) are given by\(^8\)

\[
\frac{m_\delta}{2n} \left( \mu \left( \alpha - 2 \sum_{j=1}^n x_j^* \right) - \frac{2(1 - \mu)}{n} \left( x_i^* - \frac{1}{n} \sum_{j=1}^n x_j^* \right) \right) - 1 - r \cdot \left( 1 - \frac{1}{n} \right),
\]

if \(x_i^* > \frac{1}{n} \sum_{j=1}^n x_j^*\) and

\[
\frac{m_\delta}{2n} \left( \mu \left( \alpha - 2 \sum_{j=1}^n x_j^* \right) - \frac{2(1 - \mu)}{n} \left( x_i^* - \frac{1}{n} \sum_{j=1}^n x_j^* \right) \right) - 1 + r \cdot \left( 1 - \frac{1}{n} \right),
\]

if \(x_i^* < \frac{1}{n} \sum_{j=1}^n x_j^*\). Note that \(\delta = l\) for \(i \in J_l\) and \(\delta = s\) for \(i \in J_s\). Solving the system of first-order conditions yields the following equilibrium investment levels:

\[
x_i^* = \frac{\alpha}{2n} - \frac{v_1 w_2 - v_2 w_1}{2 m_l m_s n (1 - \mu) \mu} =: x_i^* \forall i \in J_l,
\]

\[
x_j^* = \frac{\alpha}{2n} - \frac{v_1 w_1 - v_2 w_2}{2 m_l m_s n (1 - \mu) \mu} =: x_j^* \forall j \in J_s,
\]

with \(v_1 := m_s (r(n - 1) + n), v_2 := m_l (r(n - 1) - n)\) and \(w_1 = 1 - \mu(n^2 + 1), w_2 = 1 + \mu(n^2 - 1)\).

In order to guarantee positive equilibrium investments, we assume that \(\alpha\) is sufficiently large. Moreover, in order to guarantee that large clubs always invest more than small clubs, we assume that \(m_l > \hat{m} := m_s \frac{n + (n-1)r}{n(1-r) \Gamma(r+1)}\).

\(^8\)It is easy to show that the corresponding second-order conditions for a maximum are satisfied.
A.2 Proof of Proposition 1

First, we prove that a higher tax rate increases the level of competition. Substituting the equilibrium talent investments (8) in the talent function, \( T \), given by (2) and computing the derivative with respect to \( r \) yields

\[
\frac{\partial T^*(r)}{\partial r} = \frac{(m_l - m_s)(n - 1)(n - r(m_l - m_s)(n - 1))}{2(m_lm_s)^2}
\]

We derive that \( \frac{\partial T^*(r)}{\partial r} > 0 \) for all \( n > 2, 1 > m_l > m_s > 0, 1 > r > 0 \) and \( 1 > \mu > 0 \).

Second, we show that competitive balance increases with a higher tax rate. We find that

\[
\frac{\partial CB^*(r)}{\partial r} = \frac{n^2(n - 1)(m_l n + r - n(m_s + r))}{2(m_lm_s(1 - \mu))^2}
\]

Since \( m_l > \hat{m} \), it holds that \( \frac{\partial CB^*(r)}{\partial r} > 0 \), which completes the proof.

A.3 Proof of Proposition 3

In order to analyze the effect of a luxury tax on club profits, we consider a two-club league with \( n = 2 \) and set the fan preference parameter to \( \mu = 1/2 \). Substituting the equilibrium talent investments (8) into the profit function (6) and maximizing it with respect to the tax rate, \( r \), yields the following profit-maximizing tax rate for large clubs

\[
r^*_l = \frac{10m_l - 10m_s(m_l + 1) + 26m^2_s}{4 + m_l^2 - 2m_s(m_l + 8) + m^2_s},
\]

and for small clubs

\[
r^*_s = r^* := \frac{2m_l(5 + 3m_l) - 2m_s(11m_l + 5)}{4 + m_l^2 - 2m_l(m_s - 8) + m^2_s}.
\]

\[9\text{This parameterization allows us to derive closed-form solutions. The results remain qualitatively the same for other parameter configurations.}\]
We can show that \( r_l^* \geq 2 \), i.e. that the maximum profit for large clubs is not within the interval of feasible tax rates. As a consequence, profits for large clubs increase for all \( r \in [0, 1] \). In contrast, for small clubs, the profit-maximizing tax rate, \( r_s^* = r^* \), is in the interval \( (0, 1] \). It follows that the profits of small clubs increase when \( r < r^* \) and decrease when \( r > r^* \). This completes the proof.

**References**


