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Revenue Sharing in Sports Leagues: The Effects on Talent Distribution and Competitive Balance

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Abstract

This paper uses a three-stage model of non-cooperative and cooperative bargaining in a free agent market to analyze the effect of revenue sharing on the decision of teams to sign a free agent. We argue that in all subgame perfect Nash equilibria, the team with the highest reservation price will get the player. We argue that revenue sharing will not alter the outcome of the game unless the proportion taken from high revenue teams is sufficiently high. We also argue that a revenue sharing system that rewards quality low-revenue teams can alter the outcome of the game while requiring a lower proportion to be taken from high revenue teams. We also argue that the revenue sharing systems can improve competitive balance by redistributing pivotal marginal players among teams.

JEL Classification Codes: C7, J3, J4, L83

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1. Introduction

Free agency has been cheered by players and agents but has been scorned by team owners. Player salaries under free agency have expanded to an extent that many fans believe small market teams now have difficulty in competing for playing talent and on the playing field, worsening the competitive balance between teams.

Under free agency, individual players own the rights to their talent while under a reserve clause, teams own the right to a player's talent. Consequently, replacing a reserve clause with free agency effectively reassigns property rights to talent from teams to players. In his seminal article on the baseball players' labor market, Rottenberg (1956) argued that the distribution of playing talent among teams would not be affected by such a reassignment. Under free agency, the team that acquires a player must compensate him in order to gain his services. Under the reserve clause, interested teams can acquire a player via a cash sale or a trade, but the trading team, instead of the player, obtains the compensation. In either case a given player would still play for the team that valued him the most, leaving competitive balance unchanged. The redistribution of property rights to playing talent from teams to players merely shifts compensation from trading teams to players.

El-Hodiri and Quirk (1971) argued that if teams in a sports league are profit-maximizers, all teams must have similar revenue functions[1] if it were to trend toward perfect competitive balance[2]. This implies that equalizing revenue functions could improve competitive balance.

If teams maximize profits and fans only care about the relative quality of the team, then revenue sharing in which revenue is shifted from high revenue teams to low revenue teams will not improve competitive balance (for example, see Rottenberg (1956), El-Hodiri and Quirk (1971), Rascher (1997), and Fort and Quirk (1995)). If teams maximize profits but fans care about the relative and absolute quality of a given team, then revenue sharing can improve competitive balance (Marburger, 1997). Furthermore, if teams maximize utility (for example, a team owner receives consumption value through the quality of the team), then revenue sharing can improve competitive balance (Rascher (1997) and Késenne (2000)). Consequently, the impact of revenue sharing on competitive balance depends on the objectives of teams and the factors that matter to fans.

This paper explores the workings of a revenue sharing system in a free agent market in a professional sports league. We develop a bargaining game that blends non-cooperative bargaining and cooperative bargaining to describe the market for free agents in a professional sports league with no revenue sharing. The results suggest that a given free agent will sign with the team that has the highest reservation price for him. Thus, in the spirit of El-Hodiri and Quirk (1971), a revenue-sharing system that can alter reservation prices sufficiently can alter competitive balance.

We then modify the free agent model to explore how revenue sharing affects the decision to sign a free agent. In Major League Baseball and the National Football League, every team pays the same proportion of its locally-generated revenues into a central pool. This is the type of revenue-sharing system examined in papers by El-Hodiri and Quirk (1971), Rascher (1997), Fort and Quirk (1995), (Marburger, 1997), and

Késenne (2000). Fort (2003) argues that if each team shares the same proportion of its revenues, the distribution of talent and the degree of competitive balance will remain unchanged. In the context of the game developed in the present paper, such a revenue sharing system will decrease the reservation prices of all teams, both high revenue teams and low revenue teams.

In contrast, similar to a progressive income tax, we allow for a higher proportion of revenue to be taken from "high revenue" teams than from "low revenue" teams. In general, this type of revenue sharing forces the reservation price of high revenue teams down farther than it will decrease the reservation prices of low revenue teams. If the proportion of revenue taken from high revenue teams is high enough, a free agent who would have signed with a high revenue team will sign with the low revenue team instead.

Another issue that has received some attention is the incentives for revenue-receiving teams to spend some of the shared revenue on player acquisition or development. The 2003-2006 Major League Baseball Collective Bargaining Agreement (MLB-CBA) includes wording specifically stating that shared revenue should be used to improve the quality of revenue-receiving teams. If shared revenue is not used for these purposes, the offending club must answer to the commissioner (page 106). Whether this threat is sufficiently credible to lead revenue-receiving clubs to spend these funds on their teams is debatable. We argue that a revenue-sharing system can be arranged to give revenue receiving teams an incentive to spend shared revenues on their clubs without the sort of threat stated in the MLB-CBA. To this end, we modify the revenue-sharing system to tie the amount of revenue received to the quality of the team. This modification increases the reservation price of the low revenue team and would thus cause that team's reservation price to be increasing in the proportion of revenue shared by the high-revenue team. Moreover, we argue that the proportion of revenue required to be taken from high revenue teams will be smaller if the proportion of revenue received by quality low revenue teams increases. Consequently, this type of revenue sharing system is more palatable to high-revenue teams.

The paper is organized as follows: section 2 presents the free agent theory and summarizes its key results; section 3 presents a discussion of how revenue sharing between high and low revenue teams affects the decision to sign a free agent; section 4 discusses and concludes. We now move to a formal description of the free agent bargaining model.

2. The Free Agent Theory

The following hypothetical example approximates the bargaining that occurs in a free agent market and sets the stage for the theory developed below. Consider a process in which teams make initial offers to a free agent. The free agent would prefer to sign with the team gave him the highest offer but he may not take it outright. Suppose that the free agent will generate total benefits of $5,000,000 to the team that gave him the highest offer and suppose the team offers him $3,000,000 in salary. This leaves a surplus of $2,000,000 if the player signs. Suppose that the team's next-best alternative is some player who will provide it with a surplus of $500,000. The substitute player may be another free agent, a player in the team's minor league system, or a player available from another team. In any case, by signing the free agent, the team will receive a net surplus of $1,500,000. If the free agent does not sign with the team for its initial offer, it will not attain this net surplus. Since the team is better off with the free agent, he has some hold-up power over the team and, consequently, has some bargaining power with which to acquire some of the net surplus.

The example involves a multi-stage game in which teams make non-cooperative bids to the free agent. He subsequently chooses to either sign for one of the initial bids or bargain with a particular team over a net surplus. We attempt to capture the spirit of this bargaining session in the formal model developed below.

The Model

Consider a two-team league with teams 1 and 2. Let the free agent have preferences represented by the utility function $U_p(.)$[3] and let the teams have preferences given by $U_1(.)$ and $U_2(.)$. For generality, we assume that all utility functions are strictly increasing. Let the free agent earn team i = 1,2 gross benefits of B_i. These benefits include ticket revenue, local media revenue, parking revenue, etc. If the team's owner gets utility directly from the talent on a team, then the talent level would be a component of these benefits. If team i signs the free agent at some wage, w_i, the team receives a surplus of $B_i - w_i$. If team i is unable to sign the free agent, then it will sign its next-best alternative, a substitute player who will generate a surplus of s_i. This substitute player may be another free agent. s_i and B_i are assumed to be exogenous. Note that s_i is the difference between the gross benefits generated by the substitute player for team i and his salary. For simplicity, assume that there is no shared revenue at this point. Hence, B_i represent all the benefits generated locally by the free agent and his substitute for team i. Note that the benefits generated by a player will be dependent upon the amount of talent possessed by the player. For simplicity in notation, we suppress this dependence in the current section. In section 3, we make the dependence explicit.

Team i's reservation price for the free agent, $\overline{w_i}$, is that wage where $U_i\left(B_i - \overline{w_i}\right) = U_i\left(s_i\right)$. Since U_i is strictly increasing, $\overline{w_i} = B_i - s_i$. Let the free agent have an exogenous reservation wage of w_r. This reservation wage is the highest wage that he could make outside the league if he, for example, sold insurance or operated a restaurant. Let the reservation prices of the teams and the free agent's reservation wage be ordered $\overline{w_1} > \overline{w_2} > w_r$. For simplicity, we assume that both teams and the free agent are perfectly-informed about one-another. While this assumption may not be exactly observed in practice, players and teams would find it in their interests to acquire accurate information about each other. The team will pay a free agent several hundreds of thousands of dollars. It will want to gather as much information on the player's proneness to injury, ability to get along with teammates, etc. Conversely, the player expects to be with a team for a year or more and will want to ensure that he will know as much about the team as possible. Will he get along with the manager? Is the team serious about winning a championship? Will he get along with his teammates? Consequently, it is reasonable to assume a given player and interested teams are perfectly-informed about each other regarding the value teams have for the player and what the player will generate for each team.

The game proceeds in three stages. In stage one, the teams make non-cooperative bids $w_i^{\circ} \leq \overline{w_i}$ to the free agent[4]. The initial bids can be conceived as indicators of which teams are interested in the free agent. Assume that the bid a particular team makes in the first stage commits it to sign the free agent for at least that amount. In stage two, the free agent chooses to either bargain cooperatively with one of the two teams or to sign outright with one of them for its initial bid.

If the free agent chooses to bargain cooperatively with one of the two teams but does not reach an agreement, then the game proceeds to a third stage. In the third stage, the free agent can choose to either bargain cooperatively with the other team or sign outright with that team for its initial offer. For convenience we assume that if the free agent is indifferent between two outcomes at equal stages of the game, he will choose that outcome associated with the team with the highest reservation price. If he is indifferent between bargaining (or signing outright) with a given team in stage two and bargaining (or signing outright) with that team in stage three, we assume he will choose to bargain in stage two. We will also assume that if the free agent is indifferent between signing with a team outright or bargaining with that team in stage two or three, he will sign outright. If the free agent is indifferent between signing for a team's initial bid in stage two and stage three, he will sign in stage two.

We also assume that no renegotiation is allowed: once a free agent chooses to bargain with a particular team, he cannot reopen negotiations with that team should negotiations with the other team fail, nor can he sign with that team for its initial bid. This is a simplifying assumption made to allow the use of backwards induction in the analysis, but we do see examples of this happening in actual negotiations. For example, the Cleveland Indians negotiated with free agent closer Ugueth Urbina during spring training of 2004. Urbina wanted to receive more than what the Indians wanted to pay him. The Indians thought that Urbina was not in physical shape to play, and they felt it would take several additional weeks for him to get into playing shape. Without any financial concession by Urbina, the Indians would not accept a deal (Hill, 2004). Negotiations broke down and Urbina eventually signed with the Detroit Tigers.

We now present a formal description of the game's stages and we summarize the analytical results. Since we use backwards induction to solve the model, we present the formal description in reverse.

Stage Three

Suppose that in stage two, the free agent has initially chosen to bargain with team j≠i but could not arrive at an agreement with it. In this stage, the free agent chooses to either bargain with team i or to sign outright for its initial offer. Regardless of what he chooses, team j will receive S_j. Let w_i denote the salary that the free agent receives in the second stage from cooperatively bargaining with team i. Following Nash, we assume that the player and the team will want to end up on the Pareto frontier bounded by their respective reservation prices. A reasonable bargain on that frontier is a point at which the relative loss suffered by one negotiator as a result of a move from that point exceeds the relative gain enjoyed by the other from such a move. This implies the optimal point to be that at which the product of the negotiators' utility functions is maximized. More generally, the utilities may be net of the utilities evaluated at the disagreement outcomes.

Hence the free agent and the team will choose that wage that maximizes the product of the differences in the utilities:

$$[U_p(w_i) - U_p(w_r)][U_i(B_i - w_i) - U_i(s_i)]. \quad (1)$$

Differentiating (1) with respect to w_i yields the first-order condition

$$\begin{aligned} &U_p'(w_i^*)[U_i(B_i - w_i^*) - U_i(s_i)] \\ &- U_i'(B_i - w_i^*)[U_p(w_i^*) - U_p(w_r)]. \\ &= 0 \end{aligned} \quad (2)$$

Where w_i^* is referred to as the Nash Solution. Note that in every non-disagreement solution $w_r \le w_i^* \le \overline{w}_i$. For convenience we will assume that if one of the equalities holds (they cannot both hold since $w_r < \overline{w}_i$), the Nash solution will be the outcome of the negotiations. Note that the second-order condition for a maximum is satisfied with the given assumptions. Therefore, (2) implicitly defines the function $w_i^* = w_i(w_r, s_i)$. Standard comparative statics analysis reveals that w_i^* is increasing in w_r and decreasing in s_i.

If the free agent chooses not to bargain cooperatively with team i in the third stage his only remaining alternative is to sign with that team for its initial bid. In this case the player receives w_i°, team i receives $B_i - w_i^o$, and team $j \neq i$ receives s_j.

Stage Two

In stage one, teams 1 and 2 made initial bids of w_1^o and w_2^o respectively to the player. In stage two, the player chooses to bargain with one of the two teams or chooses to sign with one of the teams for its initial bid. Suppose the player has chosen to cooperatively bargain with team $j \neq i$. Recall that once the player makes such a choice, he cannot sign with team j for its initial offer and he forgoes the option of cooperatively bargaining with team j in the third round should negotiations fail in the second round. He still has the option of cooperatively bargaining in the third stage with team i for a salary of $w_i^* = w_i(w_r, s_i)$ or signing with that team for its initial bid. Let w_j denote the salary that the free agent receives in the second stage from cooperatively bargaining with team j. Following Nash, we assume that the player and the team will want to end up on the Pareto frontier bounded by their respective reservation prices. Hence, the player and the team will choose that wage that maximizes the product of the differences in the utilities:

$$\left[U_p(w_j) - U_p\left(\max\left[w_i(w_r, s_i), w_i^o\right]\right)\right]\left[U_j(B_j - w_j) - U_j(s_j)\right]. \tag{3}$$

Maximizing over w_j yields the first-order condition

$$\begin{aligned} &U_p'(w_j^*)\left[U_j(B_j - w_j^*) - U_j(s_j)\right] \\ &- U_j'(B_j - w_j^*)\left[U_p(w_j^*) - U_p\left(\max\left[w_i(w_r, s_i), w_i^o\right]\right)\right]. \\ &= 0 \end{aligned} \tag{4}$$

Note that in every non-disagreement solution $max\left[w_i(w_r, s_i), w_i^\circ\right] \leq w_j^* \leq \overline{w}_j$ and the second-order condition for a maximum is satisfied with the given assumptions. Therefore, (4) implicitly defines the function $w_j^* = w_j\left(max\left[w_i(w_r, s_i), w_i^\circ\right], s_j\right)$. Standard comparative statics analysis reveals that w_j^* is increasing in $max\left[w_i(w_r, s_i), w_i^\circ\right]$ and decreasing in s_j.

If the free agent chooses not to bargain cooperatively with team j in the second stage he can choose to sign with team $i \neq j$ in the second stage for its initial bid. In this case the player receives w_i^o, team i receives $B_i - w_i^o$, and team $j \neq i$ receives s_j.

Stage One and the Summary of the Outcome

In the first stage, the teams make closed bids which they give to the free agent in the second stage. The player has no move in stage one. Since teams are choosing their bids and since they cannot legally collude with one-another in the free agent market, we assume the bids are chosen non-cooperatively. Since each team must pay its initial bid to the player if he accepts it, a strategy of team i in the first stage is a bid, w_i°, such that $w_i^o \leq \overline{w}_i$. Each team makes its bid to maximize its utility from proceeding to the second and third stages.

The details of the equilibrium analysis are given in Appendix 1. Summarizing, there are four subgame perfect Nash equilibrium solutions to the free agent game. In each equilibrium, the team with the highest reservation price, team 1, gets the free agent. Depending on the levels of the initial offers, the free agent will either sign for team 1's initial offer or he will choose to bargain cooperatively with team 1. The non-uniqueness of a solution results from the structure of the game - interested teams make initial bids to the free agent who then decides to either take that offer or bargain with the highest-bidding team.

Team 1 always gets the free agent because it can always outbid team 2 for his services. Team 1 can set its initial offer anywhere around the reservation price of team 2. If team 1's offer is above team 2's reservation price, it ensures itself that the free agent will not sign with team 2. If team 1 sets its initial bid below the reservation price of team 2 but sets it high enough so that the wage the free agent can get by cooperatively bargaining with it is no lower than any possible initial bid or any cooperative outcome with team 2, then team 1 will get the free agent. Consequently, team 1 can always outbid team 2 in the sense that the free agent can do no better than signing with team 1.

This result is essentially the same as that described by Rottenberg (1956). It is also essentially the same as that described by Quirk and Fort (1999), but with a minor adjustment. Quirk and Fort argued that the free agent will be paid somewhere between teams' reservation prices. We argue the he will earn a salary somewhere between $\overline{w_1}$ and w_2^o because w_2^o, the low-revenue team's initial bid, binds that team to pay him at least that amount.

Tables 1, 2, and 3 present a list of free agents who were active in the 2004-2005 Major League Baseball free agent market and for who reliable substitute information was available[5]. The list of free agents and their potential substitutes was gathered from various internet sources, including news stories on the official website of Major League Baseball and the Official Website of ESPN. Each free agent listed is a player who has signed a new contract on or before December 13th, 2004 – the end of the Winter Meetings. The table also contains potential substitute information. It also contains the contract length of each free agent as well as salary information of the potential substitutes. For substitutes who were also free agents and who signed contracts on or before December 15th, 2004, we include the contract length as well as the average salary during the new contract. For substitutes who were either not free agents or who were free agents but did not sign new contracts, the salary given is that player's 2004 salary obtained from USAToday.com. The average salaries do not include signing bonuses. The tables also provide productivity statistics for the 2004 season obtained from ESPN.com. For position players, we provide offensive and defensive statistics. For pitchers, we provide pitching statistics only. Each table provides the definition of the offensive and defensive productivity measures. Table 1 provides information of free agent position players, Table 2 provides information on relief pitchers, and Table 3 provides information on starting pitchers.

[Tables 1, 2, and 3 should be placed here]

Nomar Garciaparra came to the Cubs in 2004 in a midseason trade with the Boston Red Sox and was quickly accepted by his new teammates and the Cubs fans. After the season, Garciaparra, a shortstop, was on the market along with shortstops Edgar Renteria and Orlando Cabrera. Garciaparra signed a 1-year, $8 million contract with the Cubs. Renteria signed a 4-year $40 million contract with the Boston Red Sox, paying him, on average, $2,000,000 more than Garciaparra will earn with the Cubs. Even though Garciaparra was injured for a good portion of the 2004 season, he had a higher OPS (on-base plus slugging percentage, a commonly-used measure of offensive prowess) than either Cabrera or Renteria. This was somewhat offset by Garciaparra's lower range factor and zone ratio, measures of defensive productivity. The Cubs likely believe that the surplus that Garciaparra will provide is larger than what either Renteria or Cabrera could provide and their decision to sign him is thus consistent with the theory.

Troy Glaus signed with the Arizona Diamondbacks. Richie Sexson, the primary right-handed power hitter on the Arizona roster, signed a 4-year, $50 million contract with Seattle. When Glaus was signed, indications were that he would not be back with Arizona, and this created a need for some right-handed power. Consequently, Glaus was signed to replace Sexson in the lineup. Both Glaus and Sexson were injured for most of 2004, but when healthy, they both put up similar OPS's. Glaus's average salary is slightly lower than Sexson's suggesting that the Diamondbacks believe that Glaus will provide them with a higher surplus. The signing of Glaus is also consistent with the theory.

Jermaine Dye signed with the Chicago White Sox to replace Magglio Ordonez in the outfield. Dye's OPS was lower than Ordonez's, but Ordonez's 2005 salary will likely be much larger than Dye's average salary of $5.075 million. Consequently, Dye will likely not contribute as many wins to the White Sox as Ordonez would, but this will more than offset his lower salary. The White Sox likely believe that Dye will provide a greater surplus than Ordonez, implying that Dye's signing is consistent with the theory in the paper.

Troy Percival signed a 2-year contract with the Detroit Tigers that will pay him an average of $6,000,000 per year. His substitute, Esteban Yan, signed a 2 year, $2.25 million contract with the Anaheim Angels. While Percival's average salary is higher than Yan's, Percival was a more-effective pitcher in 2004. He had more saves and saved a greater proportion of his opportunities. He also had fewer walks and hits per nine innings. The Tigers likely believe that Percival will generate a higher surplus than Yan, and Percival's signing is consistent with the theory.

Kris Benson signed a 3-year $22.5 million contract with the New York Mets. Al Leiter, a potential substitute for Benson, signed a 1-year contract with the Florida Marlins. Benson is 9 years younger than Leiter, had more innings pitched, and slightly fewer walks and hits per nine innings. But Benson had a slightly higher ERA. The Mets probably believe that Benson will contribute more to their fortunes than Leiter, and Benson's signing by the Mets is consistent with the theory.

We now move to an examination of the effects of revenue sharing.

3. Revenue Sharing and the Distribution of Players

In this section, we examine a revenue sharing system that takes a higher proportion from high revenue teams than from low revenue teams. We remain within the confines of the free agent model described above, but with some minor modifications described below.

We continue to consider a two-team league with teams 1 and 2. For simplicity, let all benefits derived from having any player on a team be from revenue and let revenue be an explicit increasing function of player talent, measured in units of player talent "t". The rationale for this is that the representative fan's demand for baseball is an increasing and concave function of talent acquired by the team. Hence, $B_i = B_i(t)$. We assume that a player's talent level is exogenously determined. Hence, $B_i(t)$ is exogenously determined. Let team 1 be the high revenue team. Hence, for a given value of t, $B_1(t) > B_2(t)$. Also, assume that the free agent is more talented than his substitute for any team. Hence, if the free agent has talent level t_f and his substitute has talent level

t_s, then $B_1(t_f) > B_1(t_s)$. Initially assume there is no restriction on how teams spend the revenue they receive through revenue sharing.

The proportion of revenues that are paid out in revenue sharing in MLB and the NFL are the same whether the team is a high revenue team or a low revenue team. Fort (2003) argues that this sort of revenue sharing system will not alter the distribution of players nor will it alter competitive balance. Suppose a fixed proportion $\gamma \in (0,1)$ of each unit of gross revenue is transferred from team 1 to team 2. Hence, if the free agent signs with team 1, team 2 gets $\gamma B_1(t_f)$ and team 1 keeps $(1-\gamma)B_1(t_f)$. This decreases team 1's reservation price of the free agent to $\overline{w_1'} = (1-\gamma)B_1(t_f) - s_1'$. s_1' is the after-revenue-sharing surplus generated by the substitute player for team 1, given by $s_1' = (1-\gamma)B_1(t_s) - w_{s1}$. $B_1(t_s)$ is the gross benefit generated by the substitute player for team 1 and w_{s1} is the salary paid to him by team 1. Since γ, w_{s1} and t_s are exogenous to the model, $B_1(t_s)$ and s_1' are also exogenous. Lastly, under the above assumptions, $\frac{\partial \overline{w_1'}}{\partial \gamma} = -(B_1(t_f) - B_1(t_s)) < 0$. Hence, increasing the sharing proportion will decrease team 1's reservation price for the free agent.

Note that the reservation price of team 2, $\overline{w_2}$, is unchanged because the shared revenue is not generated by the free agent should he play for team 2 and there is nothing tying the shared revenue to team 2's reservation price for him. Therefore, altering the proportion of revenue shared will not change team 2's reservation price for the free agent.

The introduction of revenue sharing into the model does not alter the equilibrium analysis detailed in Appendix 1. In each equilibrium described in the appendix, the general result is that the team with the highest reservation price gets the player. Each equilibrium depends on the reservation prices of the teams which in turn depend on the utility functions being strictly increasing. The introduction of revenue sharing will alter the reservation price of team 1, but does not change the general result that the team with the highest reservation price still gets the player. The introduction may change which team has the highest reservation price.

Suppose that γ is set such that $\overline{w_1^{'}} > \overline{w_2}$. Although there is a continuum of solutions to the free agent bargaining game, the team with the highest reservation price always gets his services. Hence, if γ is set at this level, team 1 obtains the free agent and the distribution of talent (and thus competitive balance) is unaltered.

Substituting for $\overline{w_1^{'}}$ yields $(1-\gamma)B_1(t_f) - [(1-\gamma)B_1(t_s) - w_{s1}] > \overline{w_2}$. Rearranging this expression yields the condition

$$\gamma < 1 - \frac{(\overline{w_2} - w_{s1})}{(B_1(t_f) - B_1(t_s))}. \tag{5}$$

Hence, if γ is sufficiently small, the free agent will sign with team 1.

The outcome of the game depends on the particulars of the situation: the difference between team 2's reservation price and the salary that team 1 pays its substitute player relative to the additional revenue that team 1 receives from signing the free agent instead of signing the substitute player. Therefore, the model suggests that for revenue sharing to have the intended consequence of causing a high-revenue team to not sign a free agent it otherwise would, the proportion of revenue shared must be sufficiently high. If γ were sufficiently high, then $\overline{w_1^{'}} < \overline{w_2}$, and team 2 would get the free agent.

We can separate the expression $1-\frac{\left(\overline{w_2}-w_{s1}\right)}{\left(B_1\left(t_f\right)-B_1\left(t_s\right)\right)}$ into three possibilities. First, if we have a very cheap substitute who provides high gross benefits to team 1 such that $B_1\left(t_f\right)-B_1\left(t_s\right)<\overline{w_2}-w_{s1}$, then $1-\frac{\overline{w_2}-w_{s1}}{B_1\left(t_f\right)-B_1\left(t_s\right)}<0<\gamma$ and team 2 gets the free agent. Second, if the substitute is highly paid such that $\overline{w_2}-w_{s1}<0$, then $\gamma<1<1-\frac{\overline{w_2}-w_{s1}}{B_1\left(t_f\right)-B_1\left(t_s\right)}$ and team 1 gets the free agent. Third, if $1-\frac{\overline{w_2}-w_{s1}}{B_1\left(t_f\right)-B_1\left(t_s\right)}\in(0,1)$, then team 2 will get the player as long as $\gamma>1-\frac{\left(\overline{w_2}-w_{s1}\right)}{\left(B_1\left(t_f\right)-B_1\left(t_s\right)\right)}$. In this case, γ is sufficiently high. This will be the case if $\overline{w_2}-w_{s1}>0$ and $\overline{w_2}-w_{s1}<B_1\left(t_f\right)-B_1\left(t_s\right)$.

This suggests that the factors that could cause γ to be sufficiently high are a sufficiently low w_{s1}, a sufficiently high $B_1\left(t_s\right)$, a sufficiently low $B_1\left(t_f\right)$, or a combination of any of these three characteristics. Therefore, the better the substitute player (in terms the revenue he generates, the salary he is paid, or both), the more likely it will be that revenue sharing will have the intended consequences of redistributing talent. Moreover, the lower the revenue-generating capability of the free agent, the more likely that revenue sharing will alter with which team the free agent will sign.

Built-in Incentives for Increasing the Reservation Prices of Low Revenue Teams

Here we examine a revenue-sharing plan that effectively increases the reservation price of the next-best alternative team for the free agent. The 2003-2006 MLB-CBA specifically states that "… each Club shall use its revenue sharing receipts… in an effort to improve its performance on the field." If it appears that receiving teams are not doing this, then "…the Commissioner may impose penalties on any Club that violates this obligation" (page 106). Is this an effective way of forcing receiving teams to spend their shared revenues on players? It essentially relies on a threat of some action imposed by the commissioner, and whether that threat is credible is open to debate. However, the revenue-sharing system described above can be altered so that teams that receive revenue will make their decision whether to sign a free agent based on the amount of revenue that they will receive through sharing. They will do so without the sort of threat contained in the current MLB-CBA.

Suppose that $\overline{w_1} > \overline{w_2}$ with no revenue sharing. As argued above, an increase in the proportion of revenue taken away from the high-revenue teams may not cause $\overline{w_1}$ to fall far enough, in which case, a given free agent will still sign with team 1. Thus, to improve competitive balance, a relatively large proportion of revenue must be taken from high revenue teams.

Recall that, by definition, team 2's reservation price for the free agent is given by $\overline{w_2} = B_2 - s_2$. B_2 represents the benefits to team 2 generated by the free agent and s_2 is the surplus that it obtains by signing its next-best alternative player: $s_2 = B_{s2} - w_{s2}$

where B_{s2} represents the benefits generated by the substitute and w_{s2} is his salary. Hence if shared revenue can increase $\overline{w_2}$ "enough", this, along with revenue sharing decreasing $\overline{w_1}$ will have the intended consequence of changing team 1's and team 2's reservation prices so that $\overline{w_1} < \overline{w_2}$ and the player will end up with team 2.

Recall that a proportion of team 1's revenue, γ, is paid out in the revenue-sharing plan. Suppose that the high revenue team generates total revenue of R. Therefore, the total amount paid out by team 1 in this plan is γR. Now, suppose that a proportion of this revenue is paid out to the low revenue team while the rest is kept in some fund that does not reach the low-revenue team (e.g. a league slush fund). Suppose that this proportion is increasing in the quality of the team. Let there be two types of team 2, "good" and "bad". A "good" team is one that performs relatively well on the field. Let the proportion of total revenue received by "good" team 2 be $\Omega_g \in (0,1)$ while $\Omega_b \in (0,1)$ is received if the team performs poorly. Since teams are assumed to be rewarded in this system, $\Omega_g > \Omega_b$ Thus, if the team performs well, it will receive $\Omega_g\gamma R$ from the revenue sharing system and, if it performs poorly, $\Omega_b\gamma R$. $\Omega_g \gamma R - \Omega_b \gamma R$ is the total premium paid to the low revenue team if it is a quality team.

Team 2's reservation price for the free agent becomes $\overline{w_2}' = B_2 + \Omega_g \gamma R - s_2$ where $s_2 = B_{s2} + \Omega_b \gamma R - w_{s2}$. Substituting for s_2 and collecting terms yields

$$\overline{w_2}' = (P_2 - P_{s2}) + (\Omega_g - \Omega_b)\gamma R + w_{s2}\,. \tag{6}$$

Comparative statics analysis reveals $\frac{\partial \overline{w_2}'}{\partial(B_2 - B_{s2})} > 0$: the greater the difference between the gross revenue generated by the free agent and his substitute, the higher the reservation price of team 2. Also, $\frac{\partial \overline{w_2}'}{\partial \Omega_g} > 0$. Hence, the higher the proportion given to "good" low-revenue teams, the higher the reservation price of team 2 will be. $\overline{w_2}'$ is also an increasing function of γ because this sort of revenue sharing system forces the low-revenue team to include the benefits received from revenue sharing in the decision to sign the free agent. In short, the commissioner's office does not need to put resources into monitoring teams to ensure higher rates of compliance with the goals of the revenue sharing system. Teams monitor themselves.

The corollary is that since the proportion received by a good team 2 increases, a smaller proportion of total revenue paid out by team 1 is required to reverse the inequality between their reservation prices. This type of revenue-sharing system would be more palatable to the high revenue teams. Recall that El-Hodiri and Quirk (1971) argued that absolute competitive balance (i.e. the probability of any team winning any given game is 50%) will only occur if all teams have similar revenue functions. One way to approach this condition is to equalize reservations prices.

Lastly whether teams were subject to the sort of revenue sharing system described above, players such as Wayne Gretzky, Joe Greene, Michael Jordan, and Ernie Banks would likely still have played for the same teams. These star players had high revenue-generating capabilities and had few good substitutes, and the theory suggests that revenue sharing would have had little impact on where these sorts of star players would have played.

Revenue sharing will have the largest impact on competitive balance through the market for “marginal players” – those players who are not stars, but who are solid and productive players. Compared to the stars, these marginal players have lower revenue-generating capabilities and have more close substitutes available. Consequently, revenue sharing systems are more likely to redistribute these types of players between teams. These marginal players will include pivotal players – those players who can make the difference between being in contention and not being in contention. Because the revenue-sharing systems described above would redistribute more of these pivotal players, the revenue sharing system described above can improve competitive balance.

4. Discussion and Conclusion

This paper examines the decision of a professional sports team to sign a free agent, giving particular attention to the operation of a revenue sharing agreement in such systems. Unlike past analyses, we examine a system in which a larger proportion of revenue is taken from high-revenue teams. A free agent will sign with that team with the highest reservation price for him, and in the absence of revenue sharing, that will be the high revenue team. If there is revenue sharing, the decision of whether to sign this free agent will not be changed unless the proportion of revenue taken from the high revenue team is sufficiently high. We also argue that in the absence of specific incentives, revenue received through revenue sharing by a low revenue team will not change its reservation price for a given player because that revenue is not generated by the its players. We also examine a system that rewards quality low-revenue teams. Such a system would force low-revenue teams to account for revenue received through sharing in the objective function regarding a free agent. Such a system would cause a redistribution of some talent while requiring a smaller proportion of revenue to be taken from high-revenue teams. Both of these systems can improve competitive balance by redistributing pivotal marginal players.

One measure that has been suggested in past labor negotiations in Major League Baseball was a decrease in the number of years of major league service a player must have before he can become a free agent. Currently, that threshold is 6. During the 1994-1995 baseball players' strike, team owners proposed to do away with baseball's arbitration system in exchange for free agency after 4 years. Decreasing the threshold of free agency would likely bring more players into the free agent market in a given year. The existence of better substitutes for a given free agent would lower teams' reservation price of this player and, consequently, the proportion of revenues taken from high revenue teams would not need to be as large in order to improve competitive balance.

Therefore, coupled with revenue-sharing, lowering the free agency threshold would cause some free agents to sign with low-revenue teams who otherwise would have signed with large-revenue teams. For exceptional players with few good substitutes for their services (like Alex Rodriguez), the existence of revenue sharing would likely not cause them to sign with the Montreal Expos instead of, say, the Texas Rangers. Consequently, it is the marginal players who will be affected by this agreement. Of course, being "marginal" in this sense is endogenous to the sizes of the proportion of revenue taken from the high-revenue team. As this proportion becomes larger, we increase the number of relatively high-quality players whom we refer to as “marginal”.

Lastly, note that teams may find it beneficial to find creative ways of masking revenue sources so they do not appear as being generated by baseball sources (which we refer to as “masking revenue” below). History is replete with examples of teams masking their revenues. For example, Zimbalist (1994) describes how the St. Louis Cardinals baseball club hid its concession and parking revenue. In 1984, a division of the Anheuser-Busch corporation (called the Civic Center Redevelopment Corporation) kept all parking and concession revenues generated by the Cardinals. This revenue did not appear on the Cardinals balances sheet, making them look much poorer than the actually were. If masking revenue is costly for teams, they will not do so unless the expected net return from masking is positive. Revenue sharing systems increase the benefits from masking revenues. However, a revenue sharing system that explicitly rewards quality low-revenue teams would lessen this benefit because a smaller proportion of revenue needs to be taken from high-revenue teams to achieve more competitive balance. This sort of revenue-sharing system would make masking activity less-frequent than it otherwise would be. Although this paper does not examine this phenomenon, it would be an interesting study.

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Table 1: 2004 Selected Free Agent Position Player Signings as of 10:00 PM CST on 12/13/04

		Name	Position	Old Team	New Team	Years	Salary*	OPS	RF	ZR	CERA	Team ERA
1	**Player**	Todd Walker	2B	Chicago Cubs	Chicago Cubs	1	$2,500,000	0.820	4.50	0.824		
	Substitute	Mark Grudzielanek					$2,500,000**	0.779	5.10	0.854		
2	**Player**	Tony Womack	2B	St. Louis	NY Yankees	2	$2,000,000	0.735	4.98	0.825		
	Substitute	Miguel Cairo					$900,000**	0.763	4.94	0.795		
3	**Player**	Troy Glaus	3B	Anaheim	Arizona	4	$11,250,000	0.930	2.07	0.652		
	Substitute	Richie Sexson				4	$12,500,000	0.915	9.95	0.840		
4	**Player**	Vinny Castilla	3B	Colorado	Montreal	2	$3,100,000	0.867	3.08	0.781		
	Substitute	Tony Batista					$1,500,000**	0.728	2.65	0.767		
5	**Player**	Gary Bennett	C	Milwaukee	Montreal	1	$750,000	0.626	6.32	0.833	5.06	4.24
	Substitute	Einar Diaz				1	$600,000	0.595	7.65	0.833	5.89	4.33
6	**Player**	Henry Blanco	C	Minnesota	Chicago Cubs	2	$1,350,000	0.628	7.53	0.833	4.25	4.03
	Substitute	Paul Bako					$865,000**	0.571	8.63	1.000	3.53	3.81
7	**Player**	Damian Miller	C	Oakland	Milwaukee	3	$2,833,333	0.742	7.00	1.000	4.24	4.17
	Substitute	Gary Bennett				1	$750,000	0.626	6.32	0.833	5.06	4.24
8	**Player**	Mike Matheny	C	St. Louis	San Francisco	3	$3,500,000	0.690	7.36	1.000	3.89	3.75
	Substitute	A.J. Pierzynski					$3,500,000**	0.729	6.63	1.000	4.23	4.29
9	**Player**	Eric Young	LF	Texas	San Diego	1	$1,000,000	0.758	1.68	0.829		
	Substitute	Robert Fick					$800,000**	0.595	2.05	0.769		
10	**Player**	Jermaine Dye	RF	Oakland	Chicago Sox	2	$5,075,000	0.793	1.99	0.912		
	Substitute	Magglio Ordonez					$14,000,000**	0.836	2.37	0.897		
11	**Player**	Nomar Garciaparra	SS	Chicago Cubs	Chicago Cubs	1	$8,000,000	0.842	3.94	0.753		
	Substitute	Edgar Renteria				4	$10,000,000	0.728	4.41	0.855		
	Substitute	Orlando Cabrera					$6,000,000**	0.689	4.39	0.838		

*Salary is the average salary during the length of the contract and is not adjusted for signing bonuses.
**If a substitute was either not a free agent or had not signed elsewhere by 12/15/2004, salary is that player's 2004 salary

OPS: On-base percentage plus slugging percentage; RF: Range Factor - (Put-outs plus assists)*9/innings; ZR: The percentage of balls fielded by a player in his "zone" as determined by Stats, Inc.; CERA: Catcher's ERA - The ERA of the pitchers when the particular catcher was behind the plate; Team ERA: Overall Team ERA of the player's team. All productivity statistics are from the 2004 baseball season for the position listed in the third column

Sources: Free agent information and all productivity information: ESPN.com; potential substitute list: various internet resources including articles gathered from ESPN.com and mlb.com; 2004 salary information: USAToday.com

Table 2: 2004 Selected Free Agent Reliever Signings as of 10:00 PM CST on 12/13/04

		Name	Old Team	New Team	Years	Salary*	G	SV	Opp	WHIP
1	**Player**	Troy Percival	Anaheim	Detroit	2	$6,000,000	52	33	38	1.25
	Substitute	Esteban Yan			2	$1,125,000	69	7	17	1.43
2	**Player**	Matt Mantei	Arizona	Boston	1	$750,000	12	4	7	2.16
	Substitute	Scott Williamson				$3,175,000**	28	1	2	1.01
3	**Player**	Antonio Alfonseca	Atlanta	Florida	2	$2,375,000	79	0	0	1.34
	Substitute	Chad Fox				$1,200,000**	12	0	2	1.59
	Substitute	Josias Manzanillo				$500,000**	26	1	4	1.64
4	**Player**	Rudy Seanez	Florida	San Diego	1	$550,000	39	0	1	1.39
	Substitute	Antonio Osuna				$750,000**	31	0	2	1.17
5	**Player**	Armando Benitez	Florida	San Francisco	3	$7,166,667	64	47	51	0.82
	Substitute	Robb Nen				$9,150,000**	Did Not Play in 2003 or 2004			
	Substitute	Dustin Hermanson			2	$2,750,000	47	17	20	1.36

*Salary is the average salary during the length of the contract and is not adjusted for signing bonuses.
**If a substitute was either not a free agent or had not signed elsewhere by 12/15/2004, salary is that player's 2004 salary

G: Games Pitched; SV: Saves; Opp: Save Opportunities; WHIP: walks and hits per nine innings pitched. All productivity statistics are from the 2004 season

Sources: Free agent information and all productivity information: ESPN.com; potential substitute list: various internet resources including articles gathered from ESPN.com and mlb.com; 2004 salary information: USAToday.com

Table 3: 2004 Selected Free Agent Starter Signings as of 10:00 PM CST on 12/13/04

		Name	Old Team	New Team	Years	Salary*	IP	WHIP	ERA
1	**Player**	Russ Ortiz	Atlanta	Arizona	4	$8,250,000	204.2	1.51	4.13
	Substitute	Steve Sparks				$500,000**	120.2	1.52	6.04
	Substitute	Casey Fossum				$345,000**	142.0	1.65	6.65
	Substitute	Casey Daigle				$300,000**	49.0	1.84	7.16
2	**Player**	Al Leiter	NY Mets	Florida	1	$8,000,000	173.2	1.35	3.21
	Substitute	Carl Pavano				$3,800,000**	222.1	1.17	3.00
	Substitute	Ismael Valdez				$800,000**	170.0	1.48	5.19
3	**Player**	Kris Benson	NY Mets	NY Mets	3	$7,500,000	200.1	1.31	4.31
	Substitute	Al Leiter			1	$8,000,000	173.2	1.35	3.21
4	**Player**	Jon Lieber	NY Yankees	Philadelphia	3	$7,000,000	176.2	1.32	4.33
	Substitute	Kevin Millwood				$11,000,000**	141.0	1.46	4.85
	Substitute	Eric Milton				$9,000,000**	201.0	1.35	4.75
	Substitute	David Wells			2	$4,000,000	195.2	1.14	3.73

*Salary is the average salary during the length of the contract and is not adjusted for signing bonuses.
**If a substitute was either not a free agent or had not signed elsewhere by 12/15/2004, salary is that player's 2004 salary

G: Games Pitched; IP: Innings Pitched; WHIP: walks and hits per nine innings pitched; ERA: Pitchers' ERA. All productivity statistics are from the 2004 season

Sources: Free agent information and all productivity information: ESPN.com; potential substitute list: various internet resources including articles gathered from ESPN.com and mlb.com; 2004 salary information: USAToday.com

Appendix 1

Equilibrium Analysis of the Free Agent Game

There are four subgame perfect Nash equilibria to the free agent bargaining game. Below we describe each Nash equilibrium in a proposition.

Recall that when the Nash solution is chosen, it is required for the bargaining outcome to give no less utility than the disagreement point from the point of view of either party. Hence, we can discard the disagreement outcome in the third stage from consideration.

Proposition 1: A subgame perfect Nash equilibrium exists when team 2 bids w_2° *such that* $w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right) > \overline{w_2}$, *team 1 bids* $w_1^\circ \geq \overline{w_2}$, *and the player chooses to cooperatively bargain with team 1 in the second stage. The player earns a salary of* $w_1\left(\max\left[w_2^o, w_2(w_r, s_2)\right], s_1\right) > w_1^o \geq \overline{w_2}$

Proof: Consider the Nash solution obtained from bargaining with team 1 in the second round, $w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right)$. This is increasing in $max\left[w_2^\circ, w_2(w_r, s_2)\right]$. Since $max\left[w_2^\circ, w_2(w_r, s_2)\right] \leq \overline{w_2}$, we know that if $max\left[w_2^\circ, w_2(w_r, s_2)\right] = \overline{w_2}$, then $\overline{w_2} \leq w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right)$ and $max\left[w_2^\circ, w_2(w_r, s_2)\right]$ will never be chosen. Hence, if the player chooses to bargain with team 1 in the second round, he and the team will never disagree. However, the level of $max[w_2^\circ, w_2(w_r, s_2)]$ still plays a role in determining the subgame perfect Nash equilibria of this bargaining game.

Initially suppose that $max[w_2^\circ, w_2(w_r, s_2)]$ is such that $w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1) > \overline{w_2}$. If team 1 chooses $w_1^\circ > w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1)$ the player will choose to sign for w_1°. However, the team would prefer that the player choose to bargain with it and receive a salary of $w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1)$. Hence, team 1 has no incentive to set $w_1^\circ > w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1)$. Therefore, a Nash equilibrium bid by team 1 must be one that satisfies $w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1) \geq w_1^\circ$.

Note that the argument above is for a given w_2° such that $w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1) > \overline{w_2}$. As noted above, $max[w_2^\circ, w_2(w_r, s_2)] \leq w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1)$ and $max[w_2^\circ, w_2(w_r, s_2)]$ is never chosen. However, if $w_2^\circ > w_2(w_r, s_2)$ it would seem that team 2 could possibly lower its bid sufficiently enough so that it could decrease $w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1)$ below that which team 2 could negotiate with the player in stage two, $w_2(max[w_1^\circ, w_1(w_r, s_1)], s_2)$. However, if the offer of team 1 is such that $w_1^\circ \geq \overline{w_2}$, team 2 would still be unable to sign the player. Hence if team 2 lowers its offer, it would not be better off when $w_1^\circ \geq \overline{w_2}$. Therefore, a subgame perfect Nash equilibrium is one where team 2 bids w_2° such that

$w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right) > \overline{w_2}$, team 1 bids $w_1^\circ \geq \overline{w_2}$, and the player chooses to cooperatively bargain with team 1. The player earns a salary of $w_1\left(\max\left[w_2^o, w_2(w_r, s_2)\right], s_1\right) > w_1^o \geq \overline{w_2}$. Note that since there are many different values of $w_1\left(\max\left[w_2^o, w_2(w_r, s_2)\right], s_1\right)$ for different values of P_i, s_i, and $max\left[w_2^\circ, w_2(w_r, s_2)\right]$, there is a continuum of Nash solutions that satisfy this proposition.

Proposition 2: A subgame perfect Nash equilibrium exists when team 1 chooses w_1^o *and team 2 chooses* w_2^o *such that* $w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right) < \overline{w_2}$ *and* $w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right) \geq w_2\left(max\left[w_1^\circ, w_1(w_r, s_1)\right], s_2\right)$ *and the player chooses to bargain cooperatively with team 1 in the second stage. Note that in this proposition, the player earns less than the reservation price of his next-best alternative. But this salary is at least as what he could get by signing with the next-best team.*

Proof: Suppose w_2° is set so $max\left[w_2^\circ, w_2(w_r, s_2)\right]$ is such that $w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right) < \overline{w_2}$. Further suppose that $max\left[w_1^\circ, w_1(w_r, s_1)\right]$ is set such that $w_2\left(max\left[w_1^\circ, w_1(w_r, s_1)\right], s_2\right) > w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right)$. If this were the case, the player would choose to bargain with team 2 and they would agree. Team 1 thus has an incentive to change its offer.

First note that since $w_2(w_r, s_2) \geq w_r$ then $max[w_2^\circ, w_2(w_r, s_2)] \geq w_r$. Thus it must be that $w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1) \geq w_1(w_r, s_1)$. Hence, in equilibrium, the player would never choose actions that would eventually pay him $w_1(w_r, s_1)$.

However, $w_1(w_r, s_1)$ still has a potential role in how team 1 chooses to change its initial bid given that $w_2(max[w_1^\circ, w_1(w_r, s_1)], s_2) > w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1)$.

Suppose that $w_1^\circ > w_1(w_r, s_1)$. If this were the case, team 1 could choose to lower its initial bid to decrease $w_2(max[w_1^\circ, w_1(w_r, s_1)], s_2)$. If team 1 could lower its bid such that $w_1^\circ \geq w_1(w_r, s_1)$ and cause $w_2(max[w_1^\circ, w_1(w_r, s_1)], s_2)$ to decrease such that $w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1) \geq w_2(max[w_1^\circ, w_1(w_r, s_1)], s_2)$, then the player will choose to bargain with team 1 and they will agree. Team 2 can do no better by changing its offer because doing so would at most increase $w_1(max[w_2^\circ, w_2(w_r, s_2)], s_1)$. Note that, as before, since it does not matter what offer team 2 makes in this case, there is a continuum of subgame perfect Nash equilibria.

Proposition 3:   A subgame perfect Nash equilibria exists where team 1 sets its initial bid such that $w_1^\circ = \overline{w_2} + \varepsilon$, *team 2 sets* w_2^o *sufficiently low that if* w_1° *were set such* *that* $w_1^\circ < \overline{w_2}$*, then*

$w_2\left(max\left[w_1^\circ, w_1(w_r, s_1)\right], s_2\right) > w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right)$*, and the player signs outright for team 1's initial bid,.*

Proof: If team 1 sets its bid such that $w_1(w_r, s_1) \geq w_1^\circ$ but still finds that $w_2\left(max\left[w_1^\circ, w_1(w_r, s_1)\right], s_2\right) > w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right)$, then lowering its offer does no good. However, in this case, team 1 could raise its offer to $w_1^\circ = \overline{w_2} + \varepsilon$, where ε is small and positive, causing $w_2\left(max\left[w_1^\circ, w_1(w_r, s_1)\right], s_2\right)$ to increase such that $w_2\left(max\left[w_1^\circ, w_1(w_r, s_1)\right], s_2\right) > \overline{w_2}$. If the player chooses to bargain with team 2, they will not come to an agreement. The player could thus do no better by signing with team 1 in the second stage for this bid and team 2 can do no better by changing its initial bid. Hence, we have another continuum of subgame perfect Nash equilibria in which the player signs outright for team 1's initial bid, team 1 sets its bid such that $w_1^\circ = \overline{w_2} + \varepsilon$, and team 2 sets any bid sufficiently low so that if w_1° were set such that $w_1^\circ < \overline{w_2}$, then $w_2\left(max\left[w_1^\circ, w_1(w_r, s_1)\right], s_2\right) > w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right)$.

Proposition 4: Finally note that a subgame perfect Nash Equilibrium exists when team 1 sets its initial bids such that $w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right) = w_1^\circ > \overline{w_2}$ *and the player chooses to sign outright with team 1 given any initial bid made by team 2 such that* $w_1\left(max\left[w_2^\circ, w_2(w_r, s_2)\right], s_1\right) > \overline{w_2}$*. The player thus earns* w_1^o*.*

The proof of this proposition is trivial.

Hence, there is a continuum of subgame perfect Nash equilibria in this free agent bargaining process that we can describe by their observed payoffs to the player. Summarizing, the player will be paid:

* The free agent will sign with team 1 in the second stage for $w_1\left(max\left[w_2^\circ, w_2\left(w_r, s_2\right)\right], s_1\right)$ when this is greater than w_1° and no less than $w_2\left(max\left[w_1^\circ, w_1\left(w_r, s_1\right)\right], s_2\right)$ (Propositions 1 and 2)

* The free agent will sign with team 1 for its initial bid, $w_1^\circ = \overline{w_2} + \varepsilon$, when this is greater than all other possible wages of the player and $w_1\left(max\left[w_2^\circ, w_2\left(w_r, s_2\right)\right], s_1\right)$ is less than $w_2\left(max\left[w_1^\circ, w_1\left(w_r, s_1\right)\right], s_2\right)$ (Proposition 3)

* The free agent will sign with team 1 for its initial bid, w_1°, when it is equal to $w_1\left(max\left[w_2^\circ, w_2\left(w_r, s_2\right)\right], s_1\right) > \overline{w_2}$ (Proposition 4).

In each case, team 1 gets the player and receives B_1 minus the player's wage. Team 2 takes its best alternative which pays it s_2.

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[1] By "similar revenue functions," we mean that the revenue generated at a given amount of output by any two teams is similar.

[2] Perfect competitive balance occurs when any given team has a 50% of winning a game against any particular opponent.

[3] For simplicity, we assume that the player cares solely about the salary he will receive although many players find other sources of value. For instance, Karl Malone chose to play for the Los Angeles Lakers in hopes that he would win a championship with them.

[4] Since a given initial bid is a wage the player could potentially be paid if he plays in the league, an initial bid is *conceptually* not the same as the player's reservation wage.

[5] This empirical part was suggested by an anonymous referee.