Back to Basics: A New Look at Gate-revenue Sharing and Competitive Balance

Robert Sandy†, Peter Sloane††, and John Treble†††

June 2006

Abstract
Most models with profit maximizing teams conclude that competitive balance is unchanged or reduced in response to gate sharing. We critique these models and then develop three alternatives: adding unshared post-season revenue; modelling the largest market team as a dominant firm with a rising marginal cost of talent; and a new general model that incorporates both a consumer demand for athletic talent and close competition. All three approaches can cause gate sharing to increase competitive balance.

JEL Classification Codes: J0, L0, L83

Keywords: sports, monopsony, monopoly power

†Indiana University Purdue University Indianapolis (IUPUI)
††University of Swansea, P.J.Sloane@Swansea.ac.uk (corresponding author)
†††University of Swansea
1. Introduction:

Although there is no direct way of observing owners’ motivations, it is almost an article of faith among economists that the owners of professional sports teams in North America are profit maximizers. In such a profit-maximizing environment the invariance of competitive balance to gate sharing is a disputed result. In contrast, that competitive balance increases in response to gate sharing in a league with utility maximizing owners is widely accepted. (Kesenne, 2005) The claim that gate sharing would have no effect on competitive balance, provided the share given to the visiting was less than 50%, was first put forward by Quirk and El-Hodiri (1974).¹ Their key assumption was that the home team’s revenue over a season depended on its win ratio, which in turn was determined by the home team’s talent level compared to the talent levels of visiting teams. This revenue function assumption has been retained, albeit with some refinements, in models by Fort and Quirk (1995), Vrooman (1995), Késenne (1996), Marburger (1997), Rascher (1997) and Szymanski and Késenne (2004). More recently, Szymanski has shown that the invariance result for gate-revenue sharing is inconsistent with a Nash equilibrium and concludes that perhaps the underlying model, an orthodoxy that has been accepted for thirty years, “just makes no sense.” (Szymanski, 2004, p. 124) We think the underlying model is based on false premises for reasons we outline below.

Fort and Quirk (1995) do find that under profit maximization the presence of unshared local broadcast revenues can overturn the invariance result. Still, the effect of

---
¹ This chapter was an extension of El-Hodiri and Quirk (1971). The 1971 paper included gate-sharing in the model, but did not examine the impact of changing gate-sharing arrangements. Their invariance result is seen as a particular application of the Coase theorem, similar to the argument that the distribution of talent is unaffected by whether the market for players is free or restricted. See, for example Szymanski 2003, p. 1159). However, it is not clear that the Coase theorem applies to changes in gate-sharing arrangements, which concern the redistribution of income between teams within a league rather than changes related to the assignment of ownership rights to the players.
gate sharing in a world with no broadcasting revenue is interesting because the original
gate sharing regimes were set up before broadcasts existed.\(^2\) Also, it makes sense to try to understand the simple case of gate sharing before addressing the many new and complex revenue sharing schemes in professional sports.

The key issues in modelling the effect of gate sharing under profit maximization are the arguments of the revenue function and the shape of the supply of talent function to the individual team. A team’s revenue function can be modelled as depending solely on its relative talent, solely on the sums of the absolute talent of the team and its opponent, or some combination of the relative and absolute talent levels. These are all simplifications of the complex determinants of gate revenue. For example, gate revenues could be sensitive to the presence of “stars” on the visiting team even if the sum of the talents on the teams was low and the teams were mismatched.\(^3\) Some “purist” fans care about how well the game is played and do not identify strongly with their local team. Other fans just care about wins by their local team no matter how sloppily their team or the visiting team plays. Obviously, most fans are somewhere in between the purist and the diehard supporter. There are also possible inter-temporal variations in demand. The long-suffering fans of a low ranked team might be very responsive to an improvement in its fortunes. While the determinants of demand are not well understood, it is clear that the standard linear downward sloping marginal revenue function in win percent is counter-factual. In all leagues the top drawing visiting teams are the ones with the best records. If fans simply valued wins by their home team or valued close competition low-ranked

\(^2\) According to Quirk and Fort (1995) baseball’s gate sharing began in 1876 and NFL’s began in the 1920s. Similarly, in UK soccer gate sharing was adopted in the 1920s (Dobson and Goddard, 1998).

\(^3\) For example, at the end of his career Michael Jordan drew full houses around the NBA even though his team was not competitive. More recently, Lebron James was a big draw around the league during his rookie season even though his team, the NBA Cleveland Cavaliers, were doing poorly.
visiting teams would draw better than high-ranked visiting teams. The supply of talent can be modelled as fixed to the league but perfectly elastic to the individual team or having a rising price to some individual team, particularly the large market team. For soccer leagues in Europe it might be reasonable to model the supply of talent as being perfectly elastic to the entire league since the teams in the individual leagues buy soccer talent in a worldwide market. Not surprisingly, changing either the marginal revenue or the marginal cost assumptions can alter the conclusions from gate sharing increasing competitive balance, leaving it unchanged or reducing it.

This paper offers three points of departure from almost all previous models. One is simply to propose dropping the standard assumptions of the continuity and monotonicity of the marginal revenue functions. We argue that wins are discrete and that some threshold number of wins may have particular value to a team. Second is to model the league as an industry with a dominant firm. Third is to tie the revenue function to both the closeness of the two teams’ talent levels and the absolute level of talent. We develop a model that allows different weights on these aspects of fan preferences and allows for different preferences of fans for success by their home teams. In this very general framework we show that gate sharing increases competitive balance.

We do not believe that it is appropriate to show the impact of revenue-sharing as a downward shift in the marginal returns to winning schedule. This schedule is presumably derived from the fans’ demand curve. But why should this alter as a result of revenue-sharing? The fans may not even know what the revenue-sharing arrangements are for the games they watch. In the standard diagrammatic representation no sharing actually takes place, both teams are simply taxed on their home gates. Revenue-sharing should,
however, affect the willingness of owners to supply winning teams, as indicated by their marginal cost of winning schedule. The tax on the (larger) gate of the bigger club is a quasi subsidy to the smaller club and exceeds the value of the tax on the (smaller) home gate of the smaller club. The net effect is an ad valorem tax on the larger club which is transferred as an ad valorem subsidy to the smaller club. Further, it seems plausible that the supply curves of providing a winning team are upward sloping rather than horizontal, unless (smaller) clubs are price takers in the labour market.

The organization of the paper follows (1) a review of the literature, (2) a detailed critique of the 1995 Fort and Quirk’s (hereafter FQ) static model with no broadcast income, primarily to discuss their assumptions, (3) modifying their model to allow for some post-season money that is not shared. This modification can create an exception to the invariance result in the direction of gate sharing increasing competitive balance. In section (4) we present a dominant firm model. In section (5) we develop a new model that includes the fans’ demand for the absolute quality of athletic talent. Again, in this more general framework the invariance result can be overturned in the direction of more competitive balance. Section (6) concludes and offers suggestions for further research.

2. Literature Review

Fort and Quirk (1995) and Vrooman (1995) model the effects of more equal gate-sharing by using a simple example of a two-team league with team $i$ being a strong drawing team and team $j$ a weak drawing team. The assumption of two teams is convenient for exposition. Their models yield the same results with an $n$ team league where $n$ is an arbitrary finite number of teams. They assume that each club is a profit
maximizer that initially retains all of its home gate receipts. Their models have uncertainty of outcome as the sole draw for fans in each city. This implies that as the winning percentage of either team increases its total revenue will rise but at a decreasing rate, so that the marginal returns to winning schedule will be downward sloping when the winning percentage is measured on the horizontal axis from left to right. The zero-sum nature of the league requires that $\sum_{i=1}^{n} w_i = n/2 = 1.0$ where $w_i$ equals the winning percentage of the $i$th team.

The marginal cost per unit of athletic talent is assumed to be equal for both teams and invariant to winning percentage. The implicit assumption behind the horizontal cost function is that enough additional athletic talent can be purchased to increase the win percentage by one point at a constant marginal cost over the entire range of win percentages. Under these assumptions about marginal cost and marginal revenue each team equates the common $MC$ with marginal returns to winning ($MR$) and the large market team $i$ wins more games than the weak drawing team $j$.

Fort and Quirk and Vrooman introduce gate-sharing arrangements in which the home team receives $\alpha$ percentage of gate revenue and the away team $(1 - \alpha)$ percent, where $\alpha$ is a number between 0.5 and 1.0. They argue that gate sharing above 50% of local gate receipts would make the wins of away teams in their home stadiums more valuable to a team than its home wins and thus destroy the incentive to field winning teams. No gate sharing schemes have gone beyond sharing 50% of the local gate revenue. The after-gate sharing marginal revenues for teams $i$ and $j$ are 

$$MR_i' = \alpha MR_i + (1-\alpha)MR_j$$ and $$MR_j' = \alpha MR_j + (1-\alpha)MR_i'$$

They argue that the intersection point
will occur directly below the original point as if both MR curves shift down by the same amount. The introduction of gate sharing has no effect on competitive balance. Further, the fact that a given winning percentage will have less effect on revenues than previously implies that gate sharing will tend to reduce player salaries, so that costs will be forced down. The teams in the league collectively bid up the marginal cost of athletic talent to the point where it is equal to marginal revenue of enough additional talent to generate a percentage point of wins. If the wins are worth less revenue the teams will bid less. That is why the players’ unions, at least in the US, have opposed revenue sharing.

Fort and Quirk acknowledge that there may be an indirect effect on competitive balance if the reduced player costs enable weak drawing franchises, which would otherwise be loss making, to become profitable. The gate sharing may allow some small market teams to exist. Vrooman also points out that competitive balance might increase if the revenue that is transferred from large to small drawing teams is win inelastic, in the sense that revenue rises less than proportionately as the percentage of games won rises. This restriction is necessary because of the negative interdependence between the teams’ MR and MC functions implied by the zero sum nature of the wins. That is, when a team shares revenue, the away team’s share depends directly on the demand of the opponents’ fans for winning, which varies inversely with its own ability to win away. If the elasticity of revenue to changes in the win proportion is greater than 1 and teams are subject to the same sharing formula there will be no effect on competitive balance to changes in \( \alpha \) in the range of 0.5 to 1.0.

Marburger (1997) differs from the above in modelling attendance as an increasing function of the quality of both the home and visiting teams and shows that increased
sharing of gate receipts can enhance competitive balance. This is because revenue sharing in his model affects large market $M_{RP}$s (which replace $MR$ to winning) more negatively than small market $M_{RP}$s. This ensures that revenue-sharing will enhance league balance by shifting $M_{RP}$ of the small city club upwards. He also allows the slopes of the $M_{RP}$ curves to differ and suggests that equal sharing of the subsidies among clubs will reward small city teams without any accompanying quality disincentives. This paper is the first to allow for redistribution effects, though it retains the linear downward sloping marginal revenue of wins functions.

Késenne (1996) substituted a win-maximising model with a break-even constraint for profit maximisation and found that revenue-sharing under this assumption resulted in a more equal distribution of playing talent. In a later paper (Késenne 2000) he introduced absolute quality (as does Rascher, 1997) as well as relative quality into the analysis and found that revenue-sharing improved competitive balance under both profit and utility maximising hypotheses. A preference for absolute quality implies that spectators prefer to watch two high quality teams rather than two poor teams, given their relative quality. It is the inclusion of the winning percentage of the visiting team in the model that makes the difference, but this only applies where there are more than two clubs in the league. Revenue-sharing will not change the distribution of playing talent in the profit-maximising case if only the winning percentage of the home team matters, but it will do so regardless of this in the utility-maximising case.

Szymanski and Késenne (2004(a)) depart from all other authors in claiming that increased gate-revenue sharing will reduce competitive balance. They use a logit contest

---

4 He also assumes that the impact of a winning percentage on home gates is greater than on away game attendance, which is supported by empirical evidence.
success function in which they treat the total supply of talent to the league as fixed as in the Quirk and El Hodiri (1974), Fort and Quirk (1995) and Vrooman (1995) models, or variable, the latter case being more appropriate to European soccer. Not only does competitive balance decline in this model, but absolute quality does as well. This result depends on the logit formulation which ensures that the marginal impact on the large market team’s winning chances from an increase in talent by the weakest team exceeds that of a similar increase in investment in the strongest team. In Quirk and Fort’s model a unit of talent buys one more win for any team in the league if the other teams’ talent levels are held constant.

The picture that emerges from the previous literature on the impact of more equal gate-revenue sharing on competitive balance is one of considerable disagreement over the fundamental set-up of the model and consequently the results of gate sharing. Changing assumptions in relation to the objectives of teams and of the importance of absolute versus relative quality lead to quite different outcomes.

3. Detailed Review of the Assumptions in Quirk and Fort Model:

There are \( n \) teams indexed from \( i = 1, \ldots, n \) in a league. Teams play a home and an away game with every other team. Each team has a local market of a different size, \( A^i \). Teams have one choice variable, the amount of talent they hire for a season, \( t^i \). The vector \( t \) represents the talent levels of every team. The relative levels of talent of the two teams in a game determine the probability that the home team wins that game. We have no quarrels with any of these assumptions.
In the FQ model there is uncertainty about which team will win an individual game but no uncertainty about the number of games a team will win over the season. Consequently, one implicit assumption is that the realized talent level is exactly what the team intended to purchase. The absence of chance variations in talent rules out having more than some expected number of injuries, or legal problems or personal conflicts—random events that beset real teams. One possible avenue for modelling the effect of gate sharing is how season-level uncertainty affects stocking additional talent and under what circumstances such stocking would affect competitive balance. What is subtle about this issue is that many units of talent can be embodied in one star player and the risks of injury are at the level of the player rather than the units of talent. In leagues with fixed rosters the large market team has a greater incentive to insure against injuries but the incentives could change with gate sharing, depending on assumptions on the team level marginal cost of talent function. We do not pursue this line of research in this paper but we think the stocking of talent to cover injuries is common and that gate sharing would have different effects on this stocking for large and small market teams.

A second implicit assumption in the FQ model is that the season is so long that the realized number of wins for each team matches its expected number of wins even though each individual game is an independent random draw with the known probability of a win based on the teams’ relative talent levels. North American teams currently have regular seasons ranging from 16 games in the NFL to 162 in MLB. The assumption that the season is so long that the realized number of wins equals the expected number of wins is a reasonable approximation for MLB and perhaps for the NBA and NHL, which are both at 82 games a season, but clearly not for the NFL. The Premier League in the
England has a 38 game season. A team that had a 0.5 probability of winning each game would have just a 0.13 probability of winning exactly 50% of 38 games. Thus even if the talent levels for the season itself had no randomness, random variation in the number of wins over a relatively short season could affect choices of talent levels and differentially impact on the large and small market teams’ reactions to gate sharing revenue.

In the FQ model revenue at a home game of team \( i \) with team \( j \) depends on the market size of team \( i \) and the closeness of the teams’ talent levels. Implicitly, there are no away-team visitors attending games in the FQ model. This assumption is reasonable in North America but European teams’ games within their national leagues are physically closer, and are connected by convenient rail links. Also, European teams typically set aside stands for visitors. If the large market teams send more fans to away games that would introduce an asymmetry in which gate sharing would affect competitive balance.

\[ Z_{ij}^{ij} \text{ measures of “closeness” of the talent levels of teams } i \text{ and } j. \]

\[ Z_{ij}^{ij}(t) = w^i(t) - w^j(t) \]

where \( w \) is the proportion of wins in a season by a team. We think this equation is simply wrong. For example, suppose \( w_i \) is 0.60 and \( w_j \) is 0.40, \( w_i - w_j \) is then 0.20. If \( w_j \) increases to 0.45 and \( w_i \) falls to 0.55, then \( w_i \) and \( w_j \) are closer, but the measure of closeness \( w_i - w_j \) has fallen to 0.10. It is not a measure of closeness, unless \( w_j \) is restricted always to be greater than \( w_i \). However, the relative sizes of \( w_i \) and \( w_j \) are to be determined by the model rather than \textit{a priori}. The FQ closeness measure goes to zero as the teams’ talent levels get closer and increases as they get further apart. The term \( A^i \) is the market size of team \( i \). It is not a choice variable in the model, implying that no teams relocate. The home
team revenue is:

\[ R^i \left[ Z^i(t), A^i \right] \]  \hspace{1cm} (2)

Revenue increases as \( Z^i \) approaches zero, at which point the two teams are perfectly matched, and also as the home market increases.

The FQ revenue function has some implicit assumptions that are crucial. The FQ model ignores fans’ interest in the absolute level of athletic talent in the league and the effort levels of individual players. Both league-wide talent and individual effort are fixed. Marburger (1997) has a good discussion of how the FQ assumption that demand depends solely on relative talent is contradicted by empirical studies of attendance.

The FQ model is silent on whether fans buy tickets on a season basis or for individual games and whether there is differential pricing for the more attractive (i.e. closely matched) games. Whatever the mechanism for selling tickets, there is a deterministic relationship between a team’s gate revenue for the season, its number of wins over the season, and in turn the talent of the team’s players relative to all other teams in the league. The team hires a particular level of talent at the beginning of the season, the fans know how many wins that level of talent will produce and how “close” each game will be (defined as the difference in the season-long win ratios of each pair of teams) and then they somehow buy enough tickets to yield the corresponding gate revenue.

Another problem is the connection between the closeness variable that drives the fans’ interest and the linear marginal revenue functions in the diagrams of the FQ model. If fans cared only about closeness each team’s total revenue function would be maximized when the teams had fifty percent win ratios. QF examples are linear marginal
revenue functions equalling zero for a large market team at a win ratio of 1.00 and a win ratio of 0.8 for the small market team. Their diagram simply doesn’t line up with the story they tell about their model. It would be more reasonable to describe fans as valuing close contests and wins by their local team. The fans’ utility from wins by their team lets each team have a maximum point on their total revenue function at higher than a 0.5 win ratio. The contrast between FQ’s equation (2) and the revenue function in the El-Hodiri and Quirk (1971) paper is striking. Their total revenue function $R_{ij}$ is simply assumed to be concave in the win ratio with a maximum at a win ratio above 0.5. League rules favouring a home field advantage help to bridge the contradictory desires for home team wins and close competition.

In the highly deterministic FQ world the marginal revenue of wins declines monotonically. There is no threshold number of wins that is more valuable to fans because it leads to post-season or Europe-level play or gains promotion or avoids relegation. Every year is the same so the long-suffering fans of a small market team would never pay more collectively than the fans of the large market team for given number of wins. Conversely, the fans of the team with the largest market are never bored with their perennial first place finish.

If the $i$th team could (contra the fixed supply assumption in the FQ model) increase its talent level while all of the other teams keep their talent levels the same, the season-long gain in the proportion of wins for team $i$ would come at the expense of reductions in the proportions of wins for every other team. Thus, there is no specialization within the league such as being able to hire talent better suited to indoor football stadiums or short left-field fences.
\[
\frac{\partial w^i}{\partial t^i} = -\sum_{j \neq i}^n \frac{\partial w^j}{\partial t^i}
\]  
(3)

A further implication of this symmetry assumption is that there is no home-field advantage. A given level of talent in a team generates as many away wins as home wins. Talent is scaled such that a unit of talent added to team \( i \) while holding the talent on all other teams constant would add one win to \( i \)’s season total. An explicit assumption by Quirk and Fort is that there are no scale effects—one unit of talent always adds one win. Diminishing returns to scale is a more plausible assumption because of the difficulties in coordinating a team full of stars. That the additional losses due to \( i \)’s increase in talent will be spread evenly over the other teams results in

\[
\frac{1}{n-1} \sum_{j \neq i} \frac{\partial w^i}{\partial t^i} = -\frac{\partial w^j}{\partial t^i}.
\]

This assumption implies that \( w \) is a continuous variable, which is mathematically convenient but not realistic. If team \( i \) wins one more game in a season only one other team can lose an additional game. The definition of a unit of talent as enough to generate one more win in a season sets \( \frac{\partial w^i}{\partial t^i} \) to equal 1. Thus,

\[
\frac{\partial w^i}{\partial t^i} = \left( \frac{1}{n-1} \right) \text{ for } i, j = 1, \cdots, n
\]  
(4)

The league can set the ratio, \( \alpha \), that the team retains from a home game’s gate revenue. The term \((1 - \alpha)\) is the share given to the visiting team. With gate sharing the total revenue function becomes:

\[
R^i[t;\alpha,n,A] = \alpha \sum_{j \neq i}^n R^j[Z^j(t)\wedge A] + (1 - \alpha) \sum_{j \neq i}^n R^j[Z^j(t)\wedge A]
\]  
(5)

Profits for team \( i \) are total revenue minus total cost. The fixed costs are ignored.

\[
\pi^i = R^i[t;\alpha,n,A] - c t^i
\]  
(6)
The per-unit cost of talent is $c$. It is the same for every team. Implicitly, the league size, $n$, is so large enough that one team can draw away a unit of talent from the other teams without bidding up the price of a unit of talent. Since there are just a few star free agents available each year and just a few teams using the free agent market to raise substantially their standing within their league, a rising price of talent to at least the large market team seems more realistic. The first order condition for profit maximization is:

$$\frac{\partial \pi}{\partial t_i} = \left( \frac{n}{n-1} \right) \sum_{j \neq i} \left( \alpha \frac{\partial R_{ij}}{\partial Z_{ij}} - (1-\alpha) \frac{\partial R_{ji}}{\partial Z_{ji}} \right) - c = 0, \; i = 1, \ldots, n$$  

(7)

In other words, the first order condition is that a team adds talent until the marginal revenue of the dollar gain from additional wins minus the dollar losses from imposing additional away losses on competitors is equal to the marginal cost of a unit of talent. Since the total talent available to the league is perfectly inelastic in supply adding talent to team $i$ means that every other team has more losses. Define the marginal home revenue of talent for team $i$ when playing team $j$ as:

$$a_{ij} = \left( \frac{n}{n-1} \right) \frac{\partial R_{ij}}{\partial Z_{ij}}$$  

(8)

Here team $i$ is adding talent and changing its closeness to team $j$. The marginal home revenue of team $j$ to changes in team $i$’s talent is defined as:

$$a_{ji} = \left( \frac{n}{n-1} \right) \frac{\partial R_{ji}}{\partial Z_{ji}}$$  

(9)

The first order condition can be restated as:

$$\alpha \sum_{j \neq i} a_{ij} - (1-\alpha) \sum_{j \neq i} a_{ji} - c = 0, \; i = 1, \ldots, n$$  

(10)
By substitution into (10) the equilibrium level of marginal home game revenue of team \( i \) in games with opponent \( j \) can be expressed in terms of the parameters \( \alpha, c, \) and \( n \).

\[
a_{ij} = \frac{c}{(2\alpha - 1)(n - 1)}
\]  
(11)

A team’s home-game marginal revenue function is:

\[
MRGi = \sum_{j \neq i} \alpha R_{ij} = \sum_{j \neq i} a_{ij}
\]  
(12)

In equilibrium the home game marginal revenue equals:

\[
MRGi = \frac{c}{(2\alpha - 1)(n - 1)}
\]  
(13)

The equilibrating mechanism is changes in common marginal cost of talent facing each team. Because of the fixed supply the cost of a unit of talent rises as all teams bid for units of talent that are priced below their marginal revenue product.

Quirk and Fort use a diagram of a two-team league, which has since appeared in at least four textbooks on the economics of sports, to convey the intuition of their model. Since our main interest is exploring exceptions to the invariance result with profit maximizing teams, the two-team version of the model suffices as a starting point. Here we use a specific example for the \( MRG \) functions. The large market team’s marginal revenue is:

\[
MRGi = 100 - 100w_i
\]  
(14)

the small market team’s marginal revenue is:

\[
MRGj = 80 - 80w_j
\]  
(15)

Without gate sharing the equilibrium is \( w_i = 5/9 \) and \( w_j = 4/9 \). The marginal revenue of both teams is 44.44. After adding a 60-40 gate share the equilibrium win ratios are still
\( w^i = 5/9 \) and \( w^j = 4/9 \) but the marginal revenue of a home win falls to 28.89. The gate sharing marginal revenue curves can be represented by equal parallel downward shifts of the two original marginal revenue curves.

Quirk and Fort’s main conclusions is that gate sharing does not affect competitive balance except in the limited sense that it might keep the small market team in business.

4. Adding Unshared Financial Awards:

All North American leagues have some sort of post-season play-offs. The World Series in MLB began in 1903 although there were other post-season championships in baseball going back to 1884. The gate revenues and payments to players for all World Series games are listed in: http://www.baseball-almanac.com/wswshares.shtml. Starting in 1918 players in the top-three finishing teams in each league also received a share of the World Series gate. Before television broadcasts, such as from 1903 through 1950, the players’ share of the gate revenue was approximately 40%. “Loser's shares in 1903, 1905 and 1907 and winners’ shares in 1906 and 1907 included the club owners' slices, which were added to their teams’ players’ pool.” By the time the World Series gate revenues reached a million dollars in 1923, voluntary owner contributions were a distant memory. Clearly, unshared post-season gate receipts were substantial in baseball’s pre-broadcast era.

The NFL the championship games began in 1930. However, there is much less

---

6 http://www.findarticles.com/p/articles/mi_m0FCI/is_10_63/ai_n6189425
7 eleven million dollars adjusted for the change in the CPI from 1923 to 2004
8 http://www.4nflpicks.com/NFL%20Championship%20Games.html
information on the early NFL revenues compared to baseball. Since the NBA and the
NHL do not have gate sharing their post-season histories are not relevant examples of the
possible connections between gate sharing, competitive balance and unshared post-season
money. Since 1954 in European soccer there has been unshared post-season money for
the UEFA championship for which eligibility is tied to within-league standings. There
used to be within-season gate sharing. “European soccer leagues have practised various
forms of gate sharing in their history. In England, visiting teams in League matches
received up to 20% of the gate until the early 1980s.” (Szymanski and Késenne, 2004, p.
166). Another source has gate sharing in English soccer ending in 1985.9

Another possible origin of the unshared money associated with a particular
number of wins is extra concession revenue for games that might determine relegation or
promotion. The assumptions required for this source of revenue are that such concession
revenue is not shared and that these crucial games generate more purchases of food or
memorabilia.

If the unshared post-season money or concession-derived money from “in-
contention” games was equal for the two teams there would be no effect on competitive
balance. However, it is plausible for the extra money to be greater for the small market
team. Attendance is not purely driven by market size. Success by a team that has had a
long drought can spark more fan interest. Although the two-team league model cannot
capture this situation, the largest market team almost always makes the play-offs or the
European championship. A small market team has not won a championship in living

__________

9 Scott Plagenhoef “Will the sun set? The shaky world economy has been unkind to
English soccer—and it's not only the small clubs that are hurting”, Soccer Digest
(October-November, 2003)
memory. In the relegation/promotion context, the large market team is hardly ever in danger of relegation. Generally only a small market team has the crucial game for avoiding relegation.

Different unshared prizes for a particular win, can be attached to that win. Even if the unshared prize is larger for the small market team, depending on the values of the unshared prizes, the competitive balance could be kept the same. However, it is straightforward to give examples in which the same two unshared prizes do change the win ratios toward more competitive balance after gate sharing is introduced. The basic idea is that the unshared prizes are a bigger share of the revenue after adding the gate sharing and can tip the competitive balance in this discrete win set-up.

Whether the differential unshared prizes are in practice important enough tip the competitive balance is uncertain. We do not know if the inter-temporal responses of fans to the occasional successes by a small market team are high enough to make a small market team’s prize for post-season play greater than a large market team’s prize. Therefore, we do not know if the differential unshared prizes could be enough to change the competitive balance under gate sharing. All we know is that there are substantial unshared prizes.

5. Gate Sharing in a Model with a Dominant Team

A second alternative to the FQ model is to treat the large market team as a dominant firm that is the first mover. This model assumes that more equal gate-sharing does not shift demand but affects the willingness of teams to supply talent to win games. Rather than simply treating gate-sharing as a tax on home gates which disappears into a
black hole it explicitly treats revenue from away games as a subsidy to the smaller team. The key idea is that the large market team has a rising marginal cost for hiring talent. Real leagues typically have 20 to 33 teams. If every team had equal home markets it would be reasonable to treat every team as a price taker in the talent market. However, some large market teams buy so much talent relative to the entire league that they alone can bid up the price of talent. The New York Yankees are the best example of a team whose payroll far exceeds the payroll of any other teams, but is hiring in a sport that has almost entirely a US market. In the model below the large city team chooses its winning percentage on the basis of maximizing its profits and the small market team follows by hiring enough talent to achieve the remaining wins.

The outcome before and after gate-revenue sharing is illustrated in Figure 1 below. We assume that the marginal cost of wins is an increasing function of its win ratio for the large market team. Initially the large city chooses the quality of talent to maximise its profits, where its marginal revenue equals its marginal costs, or point A. The large city team wins 70% of the games. The small market team is a price taker in the talent market. Its marginal cost is constant at the price set by the last unit of talent hired by the large market team. If the small city team could chose its talent level at this marginal cost it would maximize its profits at $100 - 55 = 45\%$ win ratio, but it cannot because the large market team’s initial move has committed it to contracts for the season ensuring enough talent for it to win 70% of its games. It is not literally the case that in a duopoly the second mover is a price taker. Here the small market team metaphorically represents many small market teams. The inability of the small market team to reach its profit-maximizing point may be one reason why small clubs struggle financially.
Now suppose gate sharing is introduced, say 60/40. The ad valorem tax on home gate revenues will raise the marginal cost of winning games for the large market team more than its gain from the 40% share on its away games. The after-gate sharing marginal cost for the large market team is $MC_i'$. At the origin there is no change in the large team’s marginal cost but as it wins more games it has to pay more to the small market team in net subsidies. Winning more games is increasingly more costly. Thus, the $MC_i'$ rotates anti-clockwise around the origin relative to the original $MC$ of the large market team. The large city club now selects point $B$ at the intersection of its original marginal revenue curve and the new marginal cost curve. At point $B$ the large market team has a lower winning percentage (50%). The small city team has lower costs as given by the $MC_j'$ after the subsidy. If there were no subsidy going to the small market team but just a tax on the talent hired by the large market team, the marginal cost of the small market team would be the horizontal marginal cost line determined by the before-tax cost of the last unit of talent hired by the large market team. The small market team is still a price taker in the talent market. The after-subsidy marginal cost is lower than the before subsidy marginal cost line by exactly the difference between the before-and-after-tax marginal costs of the large market team.

If it was unconstrained the small market team would maximise its profits winning 65% of its games, but as the second mover it cannot buy the contracts of enough talent to reach 65% and it does not want to get into a bidding war with the large market team. Though its costs are reduced it foregoes profits equal to the shaded triangle $XYZ$. Yet, it is better off financially than it was before as the greater percentage of games won together with the subsidy increases its net revenue overall. Thus, the argument that more equal-
gate sharing may improve the financial viability of small clubs is upheld, but it must also improve competitive balance.

One possibility is that the $MR$ curve is shaped as an inverted U. If a team wins hardly any games it cannot be in contention for the league championship. If revenue-sharing enables it to win more games so that it comes into contention this would raise revenue – a benign outcome for a small club which will face both higher revenue and lower costs. However, to model this properly we require more than two teams and a dynamic framework.
Figure 1
Gate-Revenue Sharing Under The Dominant Firm Model

LARGE CITY
CLUB
REVENUE
AND COSTS

SMALL CITY
CLUB
REVENUE
AND COSTS

% GAMES WON

MC_i before subsidy

MC_i’ after subsidy

MC_j after subsidy

MC_j’ before subsidy

MR_i

MR_j

MC_i’

MC_j

MC_i

A

B

Y

Z

0

35

50

55

70

100

MC_j’ after subsidy
6. Modelling with Demand for Absolute Quality of Athletic Talent:

In the FQ model talent is inherently a relative concept. A unit of talent adds one win when the talent levels of all other teams are held constant. However, fans are drawn to star athletes known for extraordinary ability. New leagues have to establish their credibility by hiring some famous players. The NFL used the star power of Red Grange, the most famous college football player of his era, to draw huge crowds and convince the public that its players were better than the college players.\(^{10}\) When the American Football League was formed as a rival to the NFL it hired the most famous college football player at that time and hired away some of the best-known NFL players.\(^{11}\)

We begin by assuming that fans value both the closeness of competition and the absolute quality of the teams using the simplest formulation possible. The stripped-down model is nonetheless rich in insights. An earlier version of this model is in Treble (2005). The only other model we have found that utilizes both closeness and absolute quality is by Palomino and Rigotti (2000). Their model is about shared broadcast revenue. Another basic difference is that they treat the league as an independent agent with its own motivation. In the subsection labelled League Politics below we discuss modelling league-level decisions on gate-sharing rate based on a voting mechanism. Most North American leagues decisions are based on majority or super-majority votes.

Again, the league consists of \( n \) teams, each of which is run by a profit-maximizing entrepreneur. Let \( t^i \) denote quality or talent level of team \( i \). This variable is best thought

---

\(^{10}\) A history of the early NFL is at www.nfl.com

\(^{11}\) A history of the AFL is at http://aflfootball.tripod.com/
of as the mean of a probability distribution of performance\(^{12}\), and relative performance
determines the outcome of each game. If two teams of equal quality meet, the probability
of either of them winning is 0.5. Suppose that \( t^i > t^j \), then \( i \) has a higher probability of
winning than \( j \), and the larger the difference, the higher that probability will become. This
formulation has the agreeable property that no ‘adding-up constraint’ is necessary.
Adding-up is a natural feature of the model.

Suppose that each team has a fan base, \( A^i \), and that it is indexed in such a way that
\( i < j \Rightarrow A^i \geq A^j \). The fan-base for any particular team may be simply thought of as the size
of the city in which it is based, but may be more elaborate than that, taking into account
such factors as local cultural differences, the presence or absence of competing events in
a city, and so on. If \( A^i > A^j \), we will refer to team \( i \) as the large market team and team \( j \) as
the small market team. Below we will add a third team, \( h \), that has a larger market than
team \( i \). The manager of team \( i \) can choose \( t^i \) at a cost. Once a team of quality \( t^i \) has been
chosen, each player on a team receives a fixed fee for the season based on the number of
units of talent the player embodies. As we argued above, we view the cost per unit of
talent as rising for individual teams. We make the simplest possible assumption as to the
structure of this cost, that is that the cost per season of a team with quality \( t^i \) is
\[
C(t^i) = \frac{1}{4} (t^i)^2; \quad i = 1, \ldots, n. \]
That is team \( i \)’s purchases of talent have no affect on the price
paid by \( j \), but \( i \) affects its own price. This implies a linear marginal cost function equal to

\(^{12}\) In a more elaborate formulation one might think of managers as choosing their squads in order
manipulate the shape of the probability distribution of performance in more subtle ways than just shifting
the mean. We leave consideration of such devices for later development.
In this set-up the total talent available to the league is not fixed but the costs go up fairly rapidly if all teams try to hire more talent.

A simple formulation in which demand for a game between teams $i$ and $j$ by any individual consumer depends equally on the closeness of the competition $|t^i - t^j|$ and on sum of the athletic talent on the two teams is $d^{ij} = \frac{1}{2}\left[(t^i + t^j) - |t^i - t^j|\right] = \min(t^i, t^j)$. The term $d^{ij}$ is the demand measured in dollars for total athletic talent and close contests. Our formulation differs from Marburger’s in that the fans in a city in our model have no identification with their home team (home and visiting team talent is treated symmetrically). They simply like close contests and high levels of athletic skill. However, weights can be added into this formulation to capture home team advantage and other characteristics of demand without changing the fundamental character of the equilibrium. The key point about this demand function is that it is not differentiable where $t^i = t^j$ and that the left partial derivatives as either $t^i$ approaches $t^j$ or $t^j$ approaches $t^i$ are positive, while the right partial derivatives as either $t^i$ approaches $t^j$ or $t^j$ approaches $t^i$ are negative. As long as these conditions are maintained, the weights on total talent and the closeness terms could be varied from the equal weighting assumed here without changing the model’s qualitative predictions. Indeed, there is no reason why, for instance $t^i$ and $t^j$ should not be weighted differently to take account of the fan interest in the home team on the part of its fan base. The non-differentiability induces a discontinuity in the marginal revenue function, so that the conventional twice-differentiable revenue function introduced by El-Hodiri and Quirk is inconsistent with the approach taken here. Also the equal weighting of total talent and closeness could be
changed without altering the conclusions. As shown in the plot below, this ‘demand’\textsuperscript{13} function increases in the quality of both teams, and is weakly maximal when team qualities are equal.\textsuperscript{14} Viewed as a three-dimensional object the function looks like the corner of the roof of a house (See Figure 2).

\textsuperscript{13} It could equally be described as a production function of competitive quality.
\textsuperscript{14} Other formulations are available, that some readers may prefer. We choose this on grounds of simplicity.
Revenue Functions when Fans Value Both Athletic Talent and Closeness of Competition

Optimal team quality decisions without gate-sharing

Using these specifications, the profit functions for each team can be formulated. For clarity, we do this initially for a 2-team league that plays an equal number of games on each home ground with no gate-sharing. The revenue functions depend on the size of the fan base and the individual demand function that captures the idea that both closeness of competition and total talent are valued.

\[
\Pi_i = A_i \min(t', t^i) - \frac{1}{2}(t^i)^2 = \begin{cases} 
A^i t^i - \frac{1}{2}(t^i)^2 & \text{if } t^i \leq t^j \\
A^i t^j - \frac{1}{2}(t^j)^2 & \text{if } t^i > t^j 
\end{cases}
\]

\[
\Pi_j = A_j \min(t', t^j) - \frac{1}{2}(t^j)^2 = \begin{cases} 
A^j t^i - \frac{1}{2}(t^i)^2 & \text{if } t^i \leq t^j \\
A^j t^j - \frac{1}{2}(t^j)^2 & \text{if } t^i > t^j 
\end{cases}
\]
Analyzing profit-maximising behaviour in this structure is most easily done by examining the profit functions themselves, since the demand structure implies that interior optima may not be feasible for both teams. For the 2-team case, diagrams are useful.

In Figure 3, the left hand panel shows the profit function for team \( j \), including the constraint on its optimisation implied by the minimum function. The light curve is the part of the profit function relevant when \( t_i \leq t_j \), the heavy is relevant otherwise. The profit function is thus given by the light curve up to the intersection at \( t_i = t_j \), and by the heavy curve to the right of the intersection. In the right-hand diagram, the profit function for team \( i \) is shown. The heavy curve is the part of the profit function relevant when \( t_j \leq t_i \), the light is relevant otherwise. The profit function is thus given by the heavy curve up to the intersection at \( t_i = t_j \), and by the light curve to the right of the intersection. The configuration shown is the Pareto dominant Nash equilibrium\(^\text{15}\), characterised by \( t_j = t_i = A_j \). Talent and fan bases are commensurate because both have invisible coefficients that convert units of talent and units of population to dollars. The invisible coefficient on the talent is $1 divided by one unit of talent and the invisible coefficient on the fan base is a $1 per “fan consuming unit”. In this formulation the optimal quality of both teams is determined by the fan base of the team with the smaller fan base (team \( j \)), and the league is competitively balanced.

\(^\text{15}\) It is easy to show that any choice of \( t_j = t_i \), where \( t_j \leq A_i \) is a Nash equilibrium. However, the equilibrium in which \( t_j = A_j \) yields higher profits for both teams than any other. See Hirshleifer (1983)
To see why this is the equilibrium, suppose that team $i$ were to choose a lower quality of team, this would shift the heavy curve on the left hand panel to the left and lower the profits of both teams. If team $i$ were to choose a higher level of quality, the heavy curve would shift to the right on the left hand panel, but team $j$ would not change its chosen quality, since it is already at the global maximum of its unconstrained profit function. Therefore team $i$’s profits would fall (along the light curve in the right hand panel). Now consider team $j$. If it were to choose a lower level of quality, the light curve on the right hand panel would shift to the left and the profits of both teams would fall. By choosing a larger level, it can shift the light curve on the left hand panel to the right, and firm $i$ will then choose a higher quality level, but this would not increase firm $j$’s profits since the right shift of the heavy line on the left diagram does not enable firm $j$ to reach anything other than its global optimum.
The economics of this equilibrium work as follows: The smaller team chooses the quality level that equates marginal revenue and marginal cost. But there is nothing to be gained by the larger team from increasing its quality above that provided by its opponent, because the consumers would not be prepared to pay as much for the reduced competitive balance, and the extra quality adds to costs. Note that in this situation, it may be in team $i$’s interests to give a side payment to team $j$, enabling it to increase quality, in turn enabling team $i$ to increase quality as well.

Now suppose that a third team, $h$, larger than the other two joins the league. The small market team, $j$, can still achieve its global optimum at $t^j = A^j$. Team $j$’s revenues from its games with team $i$ remain the same, but team $i$ now possibly has an incentive to raise its quality so that it can take the best advantage of its games with team $h$. (Its revenue from games with team $j$ are unaffected.) The global optimum for team $i$ is $t^i = \frac{1}{2} A^i$. If this is less than team $j$’s talent then team $i$ will continue to match team $j$’s talent, but if it is greater, it will be worthwhile for it to incur the additional costs. Team $h$ will be constrained by the quality of team $i$, for the same reasons as constrained team $j$ in the 2-team league. The equilibrium in this 3-team model is:

\[
\begin{align*}
t^*_j &= A^j \\
\max_i & = \frac{t^*_j}{1/2 A^i} \\
\end{align*}
\]
More generally equilibrium quality choices are given by Equation (18), where \( n \) teams are indexed in increasing order of size by \( i_1, i_2, \ldots, i_n \):

\[
\begin{align*}
{i_i^*} & = A^i \\
{t_i^*} & = \max_{i_{k-1}} \left( t_{i_{k-1}}^*, \frac{n-k}{n-l} A^k \right) \text{ for } h \geq k > 1 
\end{align*}
\]

Roughly speaking, the equilibrium of this model without gate-sharing, is that each team chooses quality in accordance with the size of its fan base. “Big cities have winning teams and small cities have losing teams”\(^{16}\). This is a rough description, because even though the ranking of fan bases may be strict, the ranking of optimal quality may be weak. This arises because the higher the position of a team in the hierarchy of fan base size, the lower its marginal revenue. Investments in extra effort do not increase revenues from games played against teams with lower effort choices. It is possible, therefore, that marginal revenues are smaller than marginal costs at the talent choice of the next smallest team. In this case, talent will be restricted to that of the team with the next lowest talent choice. This effect is strongest for the team with the largest fan base, which will always set talent equal to that supplied by the next smallest team.

\(^{16}\) Quirk and el-Hodiri (1974)
Table I illustrates some equilibrium configurations in a 5 team league:

<table>
<thead>
<tr>
<th>Team Number</th>
<th>Case 1</th>
<th></th>
<th>Case 2</th>
<th></th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fan base</td>
<td>Supply of talent</td>
<td>Fan base</td>
<td>Supply of talent</td>
<td>Fan base</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>22.5</td>
<td>25</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>25</td>
<td>40</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>110</td>
<td>27.5</td>
<td>75</td>
<td>20</td>
<td>110</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
<td>27.5</td>
<td>120</td>
<td>20</td>
<td>120</td>
</tr>
</tbody>
</table>

In each case, the range of fan base sizes is the same, but the resulting optimal effort decisions are quite different. This illustrates the importance of subtle distributional issues in the model. In the first example, the fan base sizes are spread more or less evenly through the range. The result is a wide spread of supply of talent. In the second example, bunching at the bottom of the distribution of fan base sizes generates perfect competitive balance, while in the third the league splits into two groups, one with a high, but balanced supply of talent, the other with a low, but balanced supply of talent.

The model’s predictions about competitive balance are thus rather complex, as might be expected from a structure in which co-operation and competition are vying with each other in the way they are here. However, one thing is clear: the variation of optimal
quality is less than the variation in fan base size: the league itself is an equalizing institution.

*Gate sharing*

The basic model analysed above assumes that no gate-sharing takes place, and that decisions are decentralised. It should be clear, however, that there may be conditions under which gate-sharing may be a Pareto-improving institution. To the extent that the revenue of the larger teams is restricted by the effort of smaller ones, it may be worthwhile for a larger team to ‘lend’ the smaller team a fraction of its fan base, so that the larger team increases its revenue from its games with the smaller team. Indeed, in the equilibrium demonstrated in the previous section, if there is a largest team, part of its fan base is always ‘wasted’, in the sense that it adds nothing to revenues. Thus transfers of fan base from a larger team to a smaller team may increase the profits of both teams.

Treble (2005) shows that gate-sharing up to a rate of 50% unambiguously improves equilibrium quality. It is intuitively clear why this should happen to the smallest team, less so for the larger. Gate-sharing implies that the smaller team makes more revenue in all its games, since all its opponents are larger, so that the share of the revenue that it gives up to visiting teams is smaller than the share that it receives from them. Why do larger teams increase quality with gate sharing? On the face of it they are potentially exchanging a share of a large home gate, for the same share of a smaller home gate, but this observation ignores the fact that equilibrium revenues in the model are constrained for larger teams by the quality choices of smaller ones. Gate-sharing relaxes these constraints, *and always does so sufficiently to outweigh the simple exchange relation.*
The idea of competitive balance is perhaps not quite as simple as it appears. In a 2-team model it is simple enough: competitive balance is achieved when the quality of the two teams is the same. Our model predicts that competitive balance will always hold in the two team case, but when there are more than 2 teams, there are definitional issues that need to be confronted. To understand this issue consider a three team league with \( t_j < t_i < t_h \). Suppose that \( t_i \) rises without exceeding \( t_h \). Does this increase competitive balance or reduce it? Games between 2 and 3 clearly are more balanced, but games between 1 and 2 are equally clearly not.

Fortunately, for reasonable rates of gate-sharing (up to 50%) Treble’s model predicts that an increase in gate-sharing will improve the balance between each pair of teams, so that the aggregation issue raised above does not arise.

League Politics

While competitive balance is valued by the consumers in the model, this does not imply that teams seek it. Teams differ in their fan bases, and a large team may lose more revenue from sharing with a smaller team, than it gains from the revenue generated by relaxing the constraint that the lower quality of the smaller team imposes on its revenues. To see what light the model can shed on these political issues, we first ask: What gate-share maximises the profits of an individual team?

Since the smallest team will have the quality of all its games increased, and will get a greater share of the revenue from away games which are all against teams with larger fan bases, the smallest team must prefer gate-sharing to no gate-sharing. The largest team’s preference is less easy to understand. The quality of all its games increases,
but each visiting team has a smaller fan base, so that the revenue that it receives from away games is less (given quality) than the revenue it shares from home games. This intuition turns out to be correct. In this model, the attitude of largest team to gate-sharing can be computed for any given configuration of fan bases, but whether team the largest market team in the league prefers some gate-sharing to no gate-sharing cannot be determined in general.

What can be said about teams other than the largest and the smallest? The model predicts that the rates of gate-sharing that maximize equilibrium profit levels for the \( n \) teams in the league have the same ordering as fan base size. This means that if the largest team benefits from gate sharing at some rate, then all the teams will benefit from it. If the largest team does not benefit from gate sharing, then there will be a critical team and all teams larger than the critical team will not benefit from profit-sharing, while those smaller will. But whatever the degree of unanimity about the desirability of gate-sharing as an institution, there are always likely to be disputes about the desirable rate of gate-sharing. This is because, in general, any team’s optimal rate of gate-sharing is different from any other team’s. Indeed, they are ordered in accordance with the ranking by size.

Recap

In this section of the paper, we have given a summary of recent work using conventional methods of industrial economics to examine the way in which profit-maximizing team owners would handle their teams in the context of a league whose consumers value both the absolute quality of the teams and the balance of the competition
in each game. The only factor that distinguishes teams in the model is the size of their fan base. Apart from this they are identical. The model predicts that

- The ranking of optimal quality choices is the same as the ranking of fan bases, although some adjacent teams with different fan bases may choose the same quality.
- Gate-sharing leaves the ranking of these optimal quality choices undisturbed, subject to the same proviso about possible weakening of the ordering.
- Gate-sharing up to a rate of 50% unambiguously increases quality for all teams.
- Gate-sharing up to a rate of 50% unambiguously improves competitive balance for any given pair of teams.
- The smallest team’s profits are increased by gate-sharing up to a rate of 50%, and the largest team’s profits may be increased or decreased.
- All teams in the league may agree that gate-sharing is desirable. But if they do they will disagree about the optimal rate. In fact, the optimal rate of gate-sharing decreases with size.

We have concentrated on gate-sharing and competitive balance in the present paper because of our dissatisfaction with the extant literature on this issue. This relies on a model that has some rather strange features: in particular, gate-sharing seems to treated as a tax, and labour supply is treated as perfectly elastic. Neither of these features seem desirable to us, so we have sought to rethink the issues involved using as simple and as conventional a model as we could manage.
Seeking to keep the model simple while retaining the key features of demand that are stressed by most writers in the field, inevitably means that some issues have remained unexplored. For instance, it seems unlikely that the cost structures facing teams are identical. The draft system commonly used in the US is one way in which real world leagues seek to give a cost advantage to smaller teams. Some readers may object to the rather stark character of the demand function that we have adopted. The function can, of course, be changed. One possible amendment would be to adopt a structure that yielded positive marginal revenues even if a team were the stronger in any given tie. We have not investigated such variations because they would likely lead to a weakening of our results, without adding any further insights.

Even within the context of a simple model, we have not exhausted the field of questions that could be pursued. We have not, for instance, said anything about sources of revenue other than those that vary directly with the size of the local fan base, neither have we said anything about the implications of special prizes in the league (such as entrance to some other league, or the ‘warm glow’ of coming top)\textsuperscript{17}, drafting arrangements, revenue sharing other than the gate-share, or salary caps. Finally, we have not broached any tactical issues involving intra-season decisions, although we believe that the structure proposed here provides a suitable vehicle for investigations of that sort. We leave these for later work, or, with any luck, for someone else to do, in the expectation that the present paper will provide a firmer footing for these investigations than the ‘conventional’ model provides.

\textsuperscript{17} Initial investigations suggest that the only effect of these factors will be to spread out the quality distribution, and possibly breaking the equality relations in the quality ranking, and definitely breaking it between the two largest clubs. This is work in progress.
7. Conclusion

We have offered three approaches to modelling the effect of gate sharing in a league with profit maximizing teams. The first is the smallest modification to the FQ framework because it retains the constant marginal cost assumption and treats gate sharing as a reduction in home marginal revenues. Even with this small modification the invariance result can be overturned if there are unequal and unshared post-season prizes. The importance of post-season prizes is not in doubt but whether the small market’s team prize could be greater than the large market team’s prize is a conjecture that requires empirical support. The second approach abandons the constant marginal cost of talent assumption and treats gate sharing as altering the cost of increasing home wins. It raises the costs of adding wins for the large market team and lowers the cost for the small market team. The empirical relevance of this model depends on whether the large market team has monopsony power in the market for talent and such deep pockets that the small market team is forced to accept the large market team’s preferred win ratios. The third approach explicitly models preferences for both total athletic talent and close competition. In this model closeness is defined in a consistent manner. A second departure is that both teams, or more generally the \( n \) teams, have rising marginal costs for talent.

Our overall conclusion is that several reasonable models of league behavior can result in competitive balance increasing in response to gate sharing. We do not think leagues adopted gate sharing regimes early in their history simply to lower wages or to
keep some teams from bankruptcy. We have not tested our model empirically because changes in gate-sharing occur infrequently. A weak form of test is to compare competitive balance in sports with relatively equal gate-sharing with those where it is less. The NFL has the most equal gate-sharing arrangements and five of the past seven Super Bowls have featured first-time winners, but this may be influenced by factors other than gate-sharing. At the minimum however, economists should be wary before attempting to dissuade team sports organisers from adopting more equal gate-sharing arrangements.
References

Dobson S.M. and Goddard J.A.,
Performance, Revenue and Cross-subsidisation in the Football League, 1927-1994,

El-Hodiri and Quirk J.,
An Economic Model of a Professional Sports League,

Fort R. and Quirk J.
Cross-subsidisation, Incentives and Outcomes in Professional Team Sports,

Hirshleifer, J.
From Weakest-Link to Best-Shot: the Voluntary Provision of Public Goods

Kesenne S.,
League Management in Professorial Team Sports with Win Maximising Clubs,

Kesenne S.,
Revenue Sharing and Competitive Balance in Professional Team Sports,

Kesenne S.,
Competitive Balance and Revenue Sharing: When Rich Clubs Have Poor Teams,

Marburger D.R.,
Gate Revenue Sharing and Luxury Taxes in Professional Sports,

Palomino F. and Rigotti L.,
The Sport League’s Dilemma: Competitive Balance Versus Incentives to Win,
unpublished manuscript, Tilburg University, November, 2000

Quirk J. and El-Hodiri M.,
The Economic Theory of a Professional League, in Noll R., editor

Rascher D.A.,
A Model of a Professional Sports League, in Hendricks W., editor,
Szymanski S.,
The Economic Design of Sporting Contests,

Szymanski S.,
Professional Team Sports Are Only a Game: The Walrasian Fixed-Supply Conjecture Model, Contest-Nash Equilibrium and the Invariance Principle

Szymanski S. and Kesenne S.,
Competitive Balance and Gate Revenue Sharing in Team Sports,

Vrooman J.,
A General Theory of Professional Sports Leagues,

Treble J.,