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Competitive balance in sports leagues and the paradox of power

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Abstract

It is generally thought that competitive equilibrium in sports leagues involves too little competitive balance (the strong dominate the weak too much- a more even contest would be more attractive). However, it is possible to sow in a standard logit contest model that the reverse is true – the strong do not win "enough"- i.e. more wins by the strong team would increase attendance or revenues. This is consistent with Hirshleifer's paradox of power. However, this is only true so long as the strong do not become too dominant- otherwise the regime switches to one of pre-emption: the strong never lose. This paper identifies the conditions under which the paradox of power and pre-emption will manifest themselves.

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1. Assumptions

This paper develops a simple model to illustrate Hirschleifer's "paradox of power". Applied in the context of a sports league, this means that teams which have greater potential to draw fans will in end up with less success than would be optimal from the league's point of view, because the weak teams "try too hard". The model is developed on the following assumptions:

1. There are only two teams in a league
2. Team owners maximise profits
3. The revenue function for each team depends upon win percentage (w), where $w_1 + w_2 = 1$
4. Team 1's revenue function is $(\sigma - w_1) w_1$, team 2's revenue function is $(1 - w_2) w_2$ where $\sigma > 1$, meaning that team 1 can generate a higher revenue than team 2 for any given level of win percentage.
5. Win percentage depends on each team's share of total talent employed (T^D), according to the logit "contest success function" $w_1 = \frac{t_1}{t_1 + t_2}$ where $t_1 + t_2 = T^D$.
6. Talent undifferentiated and perfectly divisible.

2. Competitive Equilibrium

From these assumptions it follows that we can write the profit function for each team as:

(1) $$\pi_1 = (\sigma - w_1) w_1 - ct_1 \ , \ \ \pi_2 = (1 - w_2) w_2 - ct_2 \ , \sigma > 1$$

Here "c" is the marginal cost of talent. If we assume the supply of talent is fixed (as is conventional in models of the US major leagues), then marginal cost must adjust to ensure that supply and demand are equal in equilibrium. If the supply of talent were perfectly elastic (an assumption that seems closer to the reality of European soccer leagues), then marginal cost must equal the reservation wage. We illustrate this point after deriving the equilibrium demands.

First note that the derivative of the contest success function with respect to talent is

(2) $$\frac{\partial w_1}{\partial t_1} = \frac{t_2}{(t_1 + t_2)^2} = \frac{w_2}{T^D}$$

Hence the first order conditions for profits to be a maximum (from (1)) are

(3) $$\frac{\partial \pi_1}{\partial t_1} = (\sigma - 2w_1)\, w_2 - cT^D = 0, \quad \frac{\partial \pi_2}{\partial t_2} = (1 - 2w_2)\, w_1 - cT^D = 0$$

If we subtract the first order condition for team 2 from the first order condition for team 1 and rearrange terms we can easily derive the following expression for the win percentages at the Nash equilibrium

(4) $$w_1^* = \sigma / (1 + \sigma), \qquad w_2^* = 1 / (1 + \sigma)$$

We can estimate the demand for talent for each team simply by rewriting (3) in terms of talent rather than wins:

(5) $$\frac{\partial \pi_1}{\partial t_1} = \left(\sigma - \frac{2t_1}{t_1 + t_2}\right)\frac{t_2}{(t_1 + t_2)^2} - c = 0, \quad \frac{\partial \pi_2}{\partial t_2} = \left(1 - \frac{2t_2}{t_1 + t_2}\right)\frac{t_1}{(t_1 + t_2)^2} - c = 0$$

note also, using (4) that in equilibrium

(6) $$\frac{w_1}{w_2} = \frac{t_1}{t_2} = \sigma$$

and therefore $t_1 = \sigma\, t_2$. Substituting for t_1 in (5) gives us

(7) $$\frac{\partial \pi_1}{\partial t_1} = \left(\sigma - \frac{2\sigma t_2}{\sigma t_2 + t_2}\right)\frac{t_2}{(\sigma t_2 + t_2)^2} - c = 0$$

From which we can solve for team 2's demand for talent:

(8) $$t_2^D = \frac{\sigma(\sigma-1)}{(1+\sigma)^3 c}$$

and from which it follows that

(9) $$t_1^D = \sigma t_2^D = \frac{\sigma^2(\sigma-1)}{(1+\sigma)^3 c}$$

and therefore total demand T^D is

(10) $$T^D = t_1^D + t_2^D = \frac{\sigma(\sigma-1)}{(1+\sigma)^2 c}$$

We can consider now the two polar cases of a fixed labour supply (US model) and an elastic labour supply (European model).

Case 1: fixed labour supply.

If the labour supply is fixed then demand must adjust to meet that fixed supply. In the model the only way that can happen is through the adjustment of the marginal cost of labour. This is illustrated in figure 1.

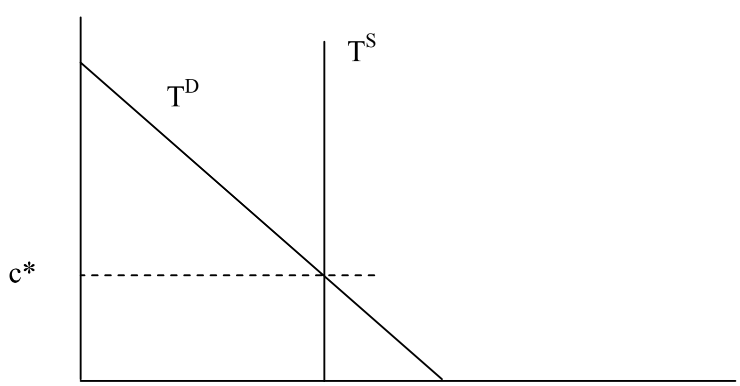

Figure 1: Inelastic labour supply

In this case the market clearing wage rate is given by

(11) $$c^* = \frac{(\sigma-1)\sigma}{(\sigma+1)^2 T^S}$$

Case 2: perfectly elastic labour supply

In this case, talent will be available at a constant marginal cost equal to its reservation wage r, and

(12) $$T^S = \frac{(\sigma-1)\sigma}{(\sigma+1)^2 r}$$

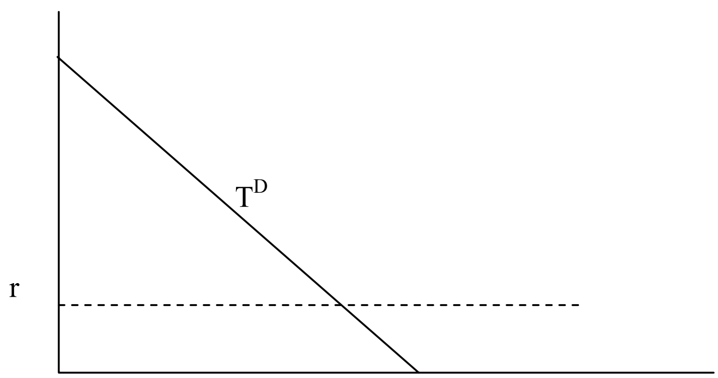

Figure 2: Elastic labour supply

Note that in this model the elasticity of supply has no effect on total profits, even though it affects the marginal cost of talent. To see why this is so, note that in the talent demand equations (8) and (9) the marginal cost of talent appears in the denominator. When we substitute this expression into the profit equations (1), the marginal cost cancels out. Hence increasing the wage rate reduces demand for talent but does not reduce profits. Profits depend only on the asymmetry parameter σ, and using (1), (4), (8) and (9) we can show that profit for each team equals

(13) $$\pi_1 = \frac{(\sigma^2+1)\sigma^2}{(\sigma+1)^3}, \quad \pi_2 = \frac{2\sigma}{(\sigma+1)^3}$$

3. Two implications of the model.

Implication 1: inefficiency of the Nash equilibrium

The Nash equilibrium is inefficient for the teams and involves too much success for the weaker team. To see this write joint profits as

(14) $$\pi_1 + \pi_2 = (\sigma - w_1)\, w_1 - ct_1 + (1 - w_2)\, w_2 - ct_2 = (1 + \sigma)w_1 - 2w_1^2 - cT^D,$$

Efficiency (from the point of view of the teams) requires that the share of wins allocated to each team maximizes joint profits. From (14) we can simply find the profit maximizing share for team 1 from the first order condition

(15) $$\frac{\partial(\pi_1 + \pi_2)}{\partial t_1} = 1 + \sigma - 4w_1 = 0,$$

and hence

(16) $$w_1^M = (1 + \sigma)/4 > w_1^*$$

Note that for $\sigma \geq 3$ the joint profit maximizing solution is for team 1 to win 100% of its games. Another to express these results is to note that at the Nash equilibrium the marginal revenue of a win for team 1 exceeds the marginal revenue of team 2:

(17) $$\frac{\partial R_1}{\partial w_1} = \sigma - 2w_1{}^* = \frac{\sigma(\sigma-1)}{(\sigma+1)} > \frac{\sigma-1}{(\sigma+1)} = 1 - 2w_2{}^* = \frac{\partial R_2}{\partial w_2}$$

showing that wins could profitably be redistributed from team 2 to team 1.

Hence the quantity bidding mechanism entails "too much" competitive balance at the Nash equilibrium. Intuitively, this result is a consequence of asymmetry. Competition always involves an externality- each team's actions under competition fails to account for the negative effect that these actions have on rivals' profits. The externality imposed by the team with the lower win percentage in equilibrium is bigger precisely because the big team loses more than the small team when its rival wins more. This is a version of Hirshleifer's paradox of power. The "weaker" team devotes relatively greater resources to competition and hence ends wins more than is optimal.

Implication 2: The attractiveness of pre-emption

Dakhlia and Pecorino (2004) consider a rent-seeking model where teams not only bid for a quantity of talent but also submit a bid for the wage rate per unit of talent. If each team offers the same wage rate then the Nash equilibrium distribution of talent will be the same as above. However, if one team bids higher than the other it can attract all the talent, generating a corner solution. In their model, where teams only have a demand for winning and there is no value in competitive balance, they show that the dominant team will be willing to pre-empt all of the talent by offering a bid at with its rival's demand for talent is zero, as long as the quantity of talent is not too great. However, if the supply of talent is large enough, pre-emption is not profitable, given that the team would have to hire all of the talent in order to pre-empt the market.[2] Here we only consider the case where the supply of talent is limited enough to produce an interior equilibrium where talent is paid a market clearing wage c* identified in (11).

The incentive to pre-empt can be identified by comparing the profit level at an interior equilibrium for the interior market clearing marginal cost of talent, c*, with the profit made by one team raising price by ε above marginal cost, hiring all the talent and winning all the time. If this deviation can be shown to be profitable then a form of Bertrand competition will ensue.

From equation (13) we know that at the interior equilibrium the profits of the dominant team is simply $\pi_1{*} = \dfrac{\sigma^4 + \sigma^2}{(1+\sigma)^3}$. If team 1 pre-empts by offering a wage rate c* + ε, it acquires all the talent ($T^S = T^D = t_1{*} + t_2{*}$, from (10)), has a win percentage $w_1 = 1$, and, for ε small enough, profits equal to

[2] Efficiency in their model of pure rent seeking (à la Tullock) is slightly peculiar, in that the most efficient result is for team 1 to win all the time since it values the payoff more. Moreover, even if team 1 pre-empts all the talent, it only needs to employ ε of it to win with certainty, since the efficient employment level for team 2 is zero. The point here is that the simple rent seeking game requires more structure in order for an interior solution to be efficient. If, for example, there is a demand for competitive balance, then an interior solution can be efficient.

$$(18) \qquad \sigma - 1 - c^{*}T^{S} = \frac{\sigma^{3}-1}{(1+\sigma)^{2}}$$

Comparing (18) with (13), pre-emption can therefore be profitable if

$$(19) \qquad \sigma^{3} > \sigma^{2} + \sigma + 1 \Rightarrow \sigma > \sigma^{*} \approx 1.84$$ [3]

Under these conditions firm 1 will start to bid up the price of talent, but at the same time firm 2 may also bid to retain a share of the total talent so long as it can continue to earn positive profits. Hence for $\sigma > \sigma^{*}$ we can define a pre-emption constraint for the marginal cost of talent and participation constraint for firm 2 to establish whether firm 2 concedes the contest (and firm 1 pre-empts):

(20) Pre-emption constraint: $c = c^{**}$ where $\pi_1(w_1 = 1, c^{**}) = \pi_1(w_1 = w_1^{*}(\sigma), c^{**})$

(21) Participation constraint: $\pi_2(w_2 = w_2^{*}(\sigma), c^{**}) \geq 0$

In other words, the pre-emption constraint requires the marginal cost of talent to be bid up to the point where pre-emption is no more profitable than not pre-empting (in both cases while paying c^{**}), while the participation constraint requires that firm 2 is willing to pay c^{**} and hire t_2 when this is more profitable than exiting the contest. For $\sigma > \sigma^{*}$ one or other of these constraints must bind.

To solve for c^{**} when (20) binds, the potential profit from pre-emption is derived from (18) while the profit without pre-emption depends on the value of profits at the profit maximising win percentage (4).

$$(22) \qquad \sigma - 1 - c^{**}T^{S} = \frac{\sigma^{3}}{(1+\sigma)^{2}} - \frac{\sigma}{1+\sigma}c^{**}T^{S}$$

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[3] The exact value of σ^{*} is $\frac{1}{3}\left(1 + \sqrt[3]{19 \pm 9\sqrt{\frac{11}{3}}} + \frac{4}{\sqrt[3]{19 \pm 9\sqrt{\frac{11}{3}}}}\right)$

From this we derive

(23) $$c^{**} = \frac{1+\sigma}{T^S}\left(\sigma - 1 - \frac{\sigma^3}{(1+\sigma)^2}\right)$$

The participation constraint requires

(24) $$\pi_2(w_2 = w_2^*(\sigma), c^{**}) = \frac{\sigma}{(1+\sigma)^2} - \frac{c^{**}T^S}{1+\sigma} \geq 0$$

From (23) and (24) we can see that both constraints will bind when

(25) $$\frac{\sigma}{(1+\sigma)^2} - \sigma + 1 + \frac{\sigma^3}{(1+\sigma)^2} = 0 \quad => \quad \sigma = 1 + \sqrt{2}$$

Beyond this point the participation constraint binds, and the wage level must be such that firm 2 would not find it profitable to re-enter the contest. From (24) this requires

(26) $$\frac{1}{(1+\sigma)^2}\left(\sigma - (1+\sigma)c^{***}T^S\right) = 0 \quad \Rightarrow \quad c^{***} = \frac{\sigma}{(1+\sigma)T^S}$$

Note that, because firm 2 must still be deterred from re-entering the contest, the wage is still increasing in σ, even after pre-emption has occurred.

Finally, note also that pre-emption reduces profits as long as

(27) $\pi_1(w_1 = 1, c^{**}) < \pi_1(w_1 = w_1^*(\sigma), c^*)$

This is certainly true when $\pi_2 > 0$, since from (20)

(28) $\pi_1(w_1 = 1, c^{**}) = \pi_1(w_1 = w_1^*(\sigma), c^{**}) < \pi_1(w_1 = w_1^*(\sigma), c^*)$

Thus profits are lower when the threat of pre-emption is credible at c* even though pre-emption does not occur because wages are bid up to c**. This implies that firm 1 would like to commit itself not to pre-empt for these values of σ and so increase profits. It can also be the case that firm 1 would prefer not to pre-empt even though it ends up doing so. Hence when $\pi_1(w_1 = 1, c^{**}) = \pi_1(w_1 = w_1^*(\sigma), c^{**})$ and $\pi_2 = 0$ we can obtain the value of σ for which $\pi_1(w_1 = w_1^*(\sigma), c^{**}) < \pi_1(w_1 = w_1^*(\sigma), c^*)$, i.e.

$$(29) \qquad \sigma - 1 - c^{**}T^S = \sigma - 1 - \frac{\sigma}{1+\sigma} < \frac{\sigma^4 + \sigma^2}{(1+\sigma)^3}$$

which in turn implies

$$(30) \qquad \sigma^3 > 3\sigma^2 + 3\sigma + 1 \Rightarrow \sigma > \sigma^{***} \approx 3.85$$

Thus for $\sigma < \sigma^{***}$ firm 1 would prefer not to pre-empt.

We can summarise these results as follows:

Critical values	**Type of equilibrium**	**Value of wages**
$1 < \sigma < 1.84$	Interior equilibrium	$c^* = \frac{(\sigma-1)\sigma}{(\sigma+1)^2 T^S}$
$1.84 < \sigma < 1 + \sqrt{2}$	Pre-emption constraint binds (wins distributed according to same formula as at the interior equilibrium), firm 2 makes positive profits	$c^{**} = \frac{1+\sigma}{T^S}\left(\sigma - 1 - \frac{\sigma^3}{(1+\sigma)^2}\right)$
$1 + \sqrt{2} < \sigma < 3.85$	Firm 2 makes zero profits (participation constraint binds), pre-emption occurs, but profits are lower than if firm 1 could commit not to pre-empt	$c^{***} = \frac{\sigma}{(1+\sigma)T^S}$
$\sigma > 3.85$	Pre-emption more profitable for firm 1 than not pre-empting	$c^{***} = \frac{\sigma}{(1+\sigma)T^S}$

4. Conclusions

Hirshleifer's paradox of power is the proposition that the strong may not get stronger in a contest, but may actually get weaker. The weak have an incentive to exert more effort relative to their resources/potential. In this paper this idea is applied to competition in a league. Conventionally it is argued that strong teams will become too strong – hence the need for intervention to maintain competitive balance in the interests of the league as a whole. The paper shows that as long as the dominant team is not too strong, this is unlikely to be a problem. However, once a dominant team has potential that is far enough in excess of rivals, there is an incentive "pre-empt" and eliminate competition altogether. It is perhaps this that people have in mind when they express concern over the dominance of very rich individuals over particular clubs.

References

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