

# **SYMPOSIUM:**

## **AGGREGATE PRODUCTION FUNCTIONS**

What is Wrong with the Aggregate Production Function?	Steven Pressman
"A Theory of Production" The Estimation of the Cobb-Douglas Function: A Retrospective View	Jesus Felipe F. Gerard Adams
Nonlinear Dynamics and Pseudo-Production Functions	Anwar Shaikh
How Sound are the Foundations of the Aggregate Production Function?	Jesus Felipe J. S. L. McCombie
Aggregate Production Functions—A Pervasive, but Unpersuasive, Fairytale	Franklin Fisher

---

**Steven Pressman:** Professor of Economics and Finance, Monmouth University, West Long Branch, New Jersey 07764. E-mail: [pressman@monmouth.edu](mailto:pressman@monmouth.edu).

*Eastern Economic Journal*, Vol. 31, No. 3, Summer 2005

## What is Wrong with the Aggregate Production Function?

Steven Pressman

*Monmouth University*

The Capital Controversy between Cambridge, U.K., and Cambridge, U.S., took place in the third quarter of the 20<sup>th</sup> century (for a detailed history of these debates, see Blaug [1974; 1980, Ch. 10], Bliss [1975], Burmeister [1980], Cohen and Harcourt [2003], Harcourt [1972; 1982, Part V; 1995]). It started when Joan Robinson [1953-54] raised some fundamental questions about the aggregate production function. Stemming from the work of John Bates Clark [1889], economists at that time accepted three propositions: (1) that aggregate output ( $Q$ ) is a function of capital ( $K$ ) and labor ( $L$ ); (2) that the rate of profit was determined by the marginal productivity of capital while the wage rate was determined by the marginal productivity of labor; and (3) that the rate of profit (or the price of capital) measures the relative scarcity of capital. Thus, the higher the capital-labor ratio, the lower the rate of profit should be.

Ignoring many possible problems with respect to measuring labor (involving, for example, how to account for the different types and qualities of labor used in the production process), Robinson questioned how it was possible to measure aggregate capital. Since capital was extremely heterogeneous, she argued that we could not simply add together different pieces of capital as we might add up different individual workers. Capital equipment includes various types of sophisticated machinery, each of which has depreciated by different degrees, as well as simple machinery and tools. The robots used to make a car, therefore, cannot just be added together with palm pilots and with hammers and screwdrivers to arrive at a figure for aggregate capital that could then be placed in a production function. At issue here was whether there might be something that these different forms of capital had in common that would allow one to count up units of capital the same way that one counted up workers when measuring the quantity of labor. For Robinson, aggregate capital was an inherently fuzzy notion, and there was nothing that would allow for an adding up of individual pieces of capital.

One thing that all capital has in common, of course, is that it contributes to firm profits. In practice, the value of capital is measured by determining its value or its contribution to firm profits. To know the value of capital, however, requires knowing the rates of profit on different sorts of capital and then applying these rates to determine the value of each sort of capital. Using this procedure, all the different types of capital employed by the firm will have a value component to them and so capital can be added up or aggregated. The problem here, however, is that the profit rate is something that the marginal productivity theory was supposed to explain. As Robinson pointed out, if economic theory is supposed to *explain* the profit rate, it cannot assume it knows the profitability of capital in order to measure the quantity of capital. This procedure is circular. Because of its circularity, and also because she thought it was impossible to find a unit of capital, Robinson argued for abandoning the marginal productivity theory of distribution and growth.

Robinson's questions about aggregate capital had further consequences and extensions. An important one has come to be known as "reswitching," a rather perverse phenomenon from the perspective of neoclassical economic theory. The possibility of reswitching was first recognized by Robinson [1953-54; 1956, 109f.] and David Champernowne [1953-54], but it was brought to the attention of economists by Piero Sraffa [1960] as a significant paradox in the theory of capital.

The logic of reswitching is as follows. If capital was a real factor of production, then we could categorize each production technique by the amount of capital per worker that is being employed. Firms would have many choices about which technique of production to use, and they would make their decision on techniques based on the relative costs of capital and labor. More costly capital would cause firms to substitute labor for capital, and more costly labor would cause firms to use more capital and less labor when producing output. There would be a nice inverse relationship between the relative quantity of capital used (or the capital-labor ratio) and the rate of profit. Reswitching is the possibility that a firm would use one technique at low rates of profit, abandon that technique at intermediate profit rates, and then adopt it again at high rates of profit.

A simple example may help illustrate what is going on in the case of reswitching. Consider Adam Smith's famous pin factory. An employer hires workers to manufacture pins and must decide on how much capital and how much labor to use in the production process. When profit rates are very low, it will not make sense to automate the factory. Instead, firms will just hire more and more workers to produce pins using little capital since there is little return to automating through capital investment. As profit rates increase, at some point it will make sense to automate the production process partially so that one person does one task and a conveyor belt then moves the partly-finished product to the next person on the assembly line. Now consider much higher rates of profits (due to much higher prices for pins). Since it will take time to acquire a factory with an assembly line, and since low profits will be made during the year that this factory is being built, at very high rates of profit the more labor-intensive means of production may again be the one favored by the firm.

These logical critiques were regarded by many (including Robinson and Sraffa) as being devastating for neoclassical economic theory. At the very least, neoclassical theory required some defense (as all theories under attack do). This defense was mounted on the U.S. side of the pond, in Cambridge, Massachusetts, mainly by Paul Samuelson and Robert Solow. Basically, their defense was to accept the logical arguments of Robinson and others, but to defend the use of the aggregate production function on practical grounds.

Samuelson [1966, 578, 582] called the reswitching debate "perverse" and "empirically insignificant" and he attempted to defend the use of aggregate production in empirical work. Solow also tried to get away from philosophical debates about measuring capital. Instead, he sought to focus attention on the empirical questions that flow out of the aggregate production function. For example, is there an empirical relationship between high capital-output ratios and low rates of profit in real world data? Is there an empirical relationship between high capital-labor ratios and low rates of profit? Finally, in competitive environments, do factors of production receive their marginal productivities?

Solow [1987, 16-7; 1974, 121; 2000] also defended the aggregate production function on practical grounds—it seemed to help us understand how inputs are used to produce more output and it seems to give us the distributional results that we find in the real world. In fact, Solow [1974, 121] even went so far as to maintain that if empirical results obtained by using the Cobb-Douglas function showed that the share of wages was 25 percent and the share of capital 75 percent, he would tend to mistrust the aggregate production function methodology. His justification for continuing to use aggregate production functions was that there is a good fit when estimating aggregate production functions in the real world and because the marginal productivities that are found in the real world approximate the shares of capital and labor found in the real world.

Solow [1956, 1970] also attempted to show the practical value of the aggregate production function when developing his growth theory. Starting with a Cobb-Douglas production function, and a one-commodity world, Solow analyzed growth as stemming from either growth in capital or growth in labor. Going even further, Solow [1957] showed how to include technological progress in the model and how to divide the contributions of capital into capital deepening and technological progress.

In sum, the first issue in the Cambridge Controversy was how to measure capital at the aggregate level. Is there something called “capital” that we can put into a neoclassical production function, or is this a meaningless concept, like phlogiston, that economics is best off abandoning? But soon the U.S. protagonists shifted the focus of the debate to an empirical or practical level. Is using a neoclassical production function a good pragmatic decision, even when acknowledging that there may be some theoretical problems with the notion of aggregate capital? Or are the good empirical results that have been used to justify aggregate production functions merely measuring something meaningless? It is these latter questions that the papers in this symposium attempt to answer.

In the first symposium paper, Jesus Felipe and F. Gerard Adams go back to the original Cobb-Douglas data set and reproduce the results that convinced economists of the efficacy of the aggregate production function. Then they introduce an exponential time trend as a third independent variable in order to capture technical progress. This yields empirical results that are quite unsatisfactory. In particular, the coefficient to capital is both negative and insignificant, thereby raising questions about the decision to accept aggregate production function results on pragmatic or empirical grounds.

The second paper, by Anwar Shaikh, performs a quasi-experiment in order to ascertain why we get good results with aggregate production functions. He uses a Cobb-Douglas production function on both actual U.S. data and on a non-neoclassical data set derived from a fixed coefficient model with no marginal products. Shaikh finds that Cobb-Douglas provides good empirical results for both data sets. Furthermore, Shaikh finds that as long as any data set exhibits a relatively constant wage share, aggregate production functions will always provide a good fit of the data.

In the third symposium paper, Jesus Felipe and John McCombie argue that the good empirical estimates obtained when using aggregate production functions are merely a result of the fact that they are picking up accounting identities. They then

perform numerous simulations to test this hypothesis. These simulations show that it is nearly impossible *not* to find a close relationship between output elasticities and factor shares using a Cobb-Douglas aggregate production function.

Finally, Franklin Fisher discusses some of the problems involved with aggregating production functions at the firm level to production functions at the level of the entire economy. Fisher argues that since such aggregations are possible only under the most stringent of conditions, production functions at the aggregate level are therefore pretty much fictions and the empirical estimates that come from employing this technique give us the equivalent of imaginary numbers as results. He also draws out the implications of this for the Cambridge Controversy, arguing that given the fictitious nature of the aggregate production function, reswitching may not be as paradoxical as Sraffa and the other critics of marginal productivity theory make it out to be.

## REFERENCES

- Blaug, M.** *The Cambridge Revolution: Success or Failure?* London: Institute of Economic Affairs, 1974.
- \_\_\_\_\_. *The Methodology of Economics*. Cambridge: Cambridge University Press, 1980.
- Bliss, C.** *Capital Theory and the Distribution of Income*. Amsterdam: Elsevier North-Holland, 1975.
- Burmeister, E.** *Capital Theory and Dynamics*. New York: Cambridge University Press, 1980.
- Champernowne, D.** The Production Function and the Theory of Capital. *Review of Economic Studies*, Winter 1953-54, 112-35.
- Clark, J. B.** The Possibility of a Scientific Law of Wages, *Publication of the American Economic Association*, March 1889, 39-63.
- Cohen, A. and Harcourt, G.** Whatever Happened to the Cambridge Capital Controversies? *Journal of Economic Perspectives*, Winter 2003, 199-214.
- Harcourt, G.** *Some Cambridge Controversies in the Theory of Capital*. Cambridge: Cambridge University Press, 1972.
- \_\_\_\_\_. *The Social Science Imperialists*. London: Routledge & Kegan Paul, 1982.
- \_\_\_\_\_. The Capital Theory Controversies, in *Socialism, Capitalism and Post-Keynesianism*, edited by G. C. Harcourt. Aldershot: Edward Elgar, 1995, 41-46.
- Robinson, J.** The Production Function and the Theory of Capital. *Review of Economic Studies*, Winter 1953-54, 81-106.
- \_\_\_\_\_. *The Accumulation of Capital*. London: Macmillan, 1956.
- Samuelson, P.** A Summing Up. *Quarterly Review of Economics*, November 1966, 568-83.
- Solow, R.** A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*, February 1956, 65-94.
- \_\_\_\_\_. Technical Change and the Aggregate Production Function. *Review of Economics and Statistics*, August 1957, 312-20.
- \_\_\_\_\_. *Growth Theory: An Exposition*. Oxford: Clarendon Press, 1970.
- \_\_\_\_\_. Laws of Production and Laws of Algebra: The Humbug Production Function: A Comment. *Review of Economics and Statistics*, February 1974, 121.
- \_\_\_\_\_. Second Thoughts on Growth Theory, in *Employment and Growth: Issues for the 1980s*, edited by A. Steinherr and D. Weiserbs. Dordrecht: Martinus Nijhoff, 1987, 13-27.
- \_\_\_\_\_. The Neoclassical Theory of Growth and Distribution. *Banca Nazionale de Lavoro Quarterly Review*, December 2000, 349-81.
- Sraffa, P.** *Production of Commodities by Means of Commodities*. Cambridge: Cambridge University Press, 1960.

