

# Edgeworth's *Mathematical Psychics*: A Centennial Notice

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Francis Ysidro Edgeworth (1845-1926) published his *Mathematical Psychics* in 1881.<sup>1</sup> While not an architectonic classic, such as Alfred Marshall's *Principles*,<sup>2</sup> it was a major work in the history of economics. It was, in retrospect, a book remarkable for its time in its emphasis on mathematical formalism, its depth of technical sophistication and formulation, and its studied application of utilitarian analysis to matters of economic and political policy, as well as its particular substantive contributions to the discipline. A book now apparently largely unread, it is striking for what it did, for what it tried but did not accomplish, and for what it contained that was neglected by subsequent writers until redeveloped much later. More important, it prefigured both the form which economic analysis was subsequently to take and the problems of applying economic analysis to problems of policy. It is a book worthy of centennial notice, indeed, perhaps capable of providing further insight.

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<sup>1</sup>F. Y. Edgeworth, *Mathematical Psychics*. An Essay on the Application of Mathematics to the Moral Sciences. London: Kegan Paul, 1881. Subsequently reprinted for Economist's Book Club by Augustus M. Kelley. All page references are to this work.

<sup>2</sup>The *Mathematical Psychics* was published when Edgeworth was 36; the *Principles*, when Marshall was 48. Both men lived into their early eighties.

We may note, first, that Edgeworth equivocated in the terminology which he used to describe his use of mathematics to explicate the role of utility and maximization in exchange. The phrase "mathematical psychics" is used in the title and the text (p. 138) but Edgeworth also employed "physical mathematics" (p. 126), "economical mathematics" (p. 127), "mathematical economics" (pp. 127, 138), "mathematical theory of exchange" (p. 119), and "a general theory of contract" (p. 146). All these are found in the appendices (comprising almost one-half the book) in which he attempted to apply and to justify the application of his approach to problems of analysis and policy.

The book essentially was a brief for the use of what Edgeworth called "mathematical reasoning without numerical data" or "unnumerical mathematics." Although Edgeworth, always characteristically modest, was apparently anxious over the degree of his mastery of mathematics, he presented the case for the axiomatic method using formal mathematical symbolism. His mathematical formulation of economic problems was remarkable *vis-a-vis* the great bulk of the economic literature of his day and for some time thereafter. Edgeworth was aware, of course, of the relatively inconspicuous mathematical tradition which had been slowly developing in economics in the work of Augustin Cournot and Leon Walras, among others, and, in England, William Stanley Jevons and Alfred Marshall. Nonetheless, for 1881, Edgeworth published a high degree of sophisticated formalism and

abstract mathematical rigor. The growth of mathematicization was to be considerable in the years ahead but remained largely secondary until after Paul Samuelson's *Foundations* helped to rechannel economic theory two-thirds of a century later.

Samuelson's maximization economics rather precisely echoed Edgeworth's vision. Edgeworth argued for mathematical reasoning as constituting science. He included in his argument the idea of verification by mathematical reasoning (p. 3). But he justified the use of mathematics, particularly, of course, the calculus, on the grounds that the economics of exchange could be analyzed as a series of maximization problems in the hypothetico-deductive mold. Indeed, he envisioned the problems of social science as problems of individual and collective maximization, and the two sections of the substantive Part II of the *Psychics* treated precisely those problems. Economics as social mechanics was recognized and extolled, the conception of man as a pleasure machine both facilitating and justifying the employment of mechanical terminology and mathematical reasoning (pp. 12, 15).

It is not clear whether Edgeworth considered his economics to be a corpus of descriptive truth or a set of analytical tools or, for that matter, whether he seriously considered the question when writing the *Psychics*. Constrained maximization seems to have been, in his *Psychics*, both a technique and a substantive paradigm. In any event, from the perspective of the present, Edgeworth's mode of economic analysis, in contrast with the Walrasian or neo-Walrasian, is clear: an emphasis on interagent behavior rather than prereconciled choice; individual exchange rather than marketwide outcomes; recontracting rather than market adjustments as the exchange process; attention to disequilibrium difficulties; atomistic units in a completely unorganized market; and an

emphasis on contract(ing) rather than equilibrium.

The *Psychics* is recognized for its presentation of the contract curve, perhaps less for its author's understanding that it also is a conflict curve (p. 29). It also is recognized for Edgeworth's defense of a conceivable measure of cardinal utility (pp. 7ff, 59–60, 98–102 and passim). But also to be found within the pages of the *Psychics* are a willingness to settle for ordinal utility (pp. 8–9); the idea of interdependent utility functions (pp. 12, 53, 102–103); a theory of public choice or of utilitarian politics (pp. 128ff); a theory of combinations or of collective action (pp. 44–45, 134ff); and, actually a centerpiece of Edgeworth's substantive analysis, a theory of imperfect competition.

As interesting as the other topics may be, Edgeworth's handling of the question of imperfect competition (so called by him) is perhaps more instructive. For Edgeworth, the affirmative focus of his analysis of individual maximization was the "field of competition:" "A perfect field of competition professes in addition certain properties peculiarly favourable to mathematical calculation; namely, a certain indefinite *multiplicity* and *dividedness*, analogous to that *infinity* and *infinitesimality* which facilitate so large a portion of Mathematical Physics (consider the theory of Atoms, and all applications of the Differential Calculus)" (p. 18). But competition enabled not only the use of mathematical symbolism. It also engendered determinate solutions. "That contract in a state of perfect competition is determined by demand and supply is generally accepted, but is hardly to be fully understood without mathematics" (p. 30). Moreover, "The advantage of this particular method is that it is applicable to the particular cases of imperfect competition; where the conceptions of *demand and supply at a price* are no longer appropriate" (p. 31). Applicable it was, but imperfect competition yielded

indeterminate solutions for both economic actors and economic analysts.

In 1939, John Hicks<sup>3</sup> lamented that

It has to be recognized that a general abandonment of the assumption of perfect competition, a universal adoption of the assumption of monopoly, must have very destructive consequences for economic theory. . . .

It is, I believe, only possible to save anything from this wreck—and it must be remembered that the threatened wreckage is that of the greater part of economic theory—if we can assume that the markets confronting most of the firms with which we shall be dealing do not differ very greatly from perfectly competitive markets. . . . At least, this getaway seems worth trying.

Much the same response was made by Edgeworth to his recognition of the problem posed by noncompetitive conditions:

. . . it does not seem very rash to infer . . . a considerable extent of indeterminateness.

Of this inference what would be the consequence. To impair, it may be conjectured, the reverence paid to *competition*; in whose results—as if worked out by a play of physical forces, impersonal, impartial—economists have complacently acquiesced. . . . But if it should appear that the field of competition is deficient. . . ; if competition is found wanting, not only the regularity of law, but even the impartiality of chance. . . —economics would be indeed a 'dismal science,' and the reverence for competition would be no more. (p. 50)

In the real-world economy, the "characteristic evil of indeterminate contract" brought about by noncompetitive conditions was "deadlock, undecidable opposition of interests" between buyers and sellers (p. 29). Such situations permitted strategic bargaining and gains from combination. There also would result "a general demand for a *principle of arbitration*" (p. 51), which would lead to the resolution of economic and political conflicts in accordance with some principle of justice or equity, especially distributive equity. This

was of great theoretical and practical concern to Edgeworth and he argued that "*Justice* requires to be informed by some more definite principle . . . a superior luminary—utilitarianism" (p. 52): "*competition requires to be supplemented by arbitration, and the basis of arbitration between self-interested contractors is the greatest possible sum-total utility*" (p. 56). Edgeworth thus was led, reluctantly, by consideration of the importance of economic power to the necessity for deliberative conflict resolution and a strategy for economic policy.

The greatest happiness principle was, to Edgeworth, the first principle of economics and ethics (p. v) and more of the *Psychics* was devoted to the practical application of it than to the economics of individual exchange in competitive markets. Both Keynes<sup>4</sup> and Schumpeter<sup>5</sup> distinguished Edgeworth's philosophical utilitarianism from his theoretical, positive economics but Edgeworth firmly believed that the two were inseparable and he directed considerable attention to the subtleties of utilitarian reasoning and the contemporary psychological literature.

It is possible and perhaps often necessary to debate the degree of Edgeworth's appreciation of the complexities and conditions of individual and collective utility maximization, especially the degree to which the greatest happiness principle is capable of providing unambiguous and unequivocal answers to problems of policy in the absence of severely limiting and prefiguring, channeling assumptions (see below). In Edgeworth's favor is his recognition that realization of the greatest possible quantity of happiness is "under conditions" (p. 12) and that the conclusions, or rules, provided by his mathematical

<sup>4</sup>John Maynard Keynes, *Essays and Sketches in Biography* (New York: Meridian Books, 1956), p. 107.

<sup>5</sup>Joseph A. Schumpeter, *History of Economic Analysis* (New York: Oxford University Press, 1954), pp. 408–409, 780, 831, 1056.

<sup>3</sup>John R. Hicks, *Value and Capital* (Oxford: Oxford University Press, 1939), pp. 83–85.

psychics were only general instructions, not detailed regulations; a very general, abstract and rough pattern requiring checking by experience and modification in practice (pp. 71, 126). It would be unfair to attribute to Edgeworth the sins of extreme rationalism, determinateness, and exclusiveness of policy implications perpetrated by later generations of economic analysts.

Two important characteristics of Edgeworth's work in collective maximization warrant attention. First, Edgeworth's utilitarianism constituted an *administrative*, not a methodological individualist approach to collective utility maximization.<sup>6</sup> Using Vilfredo Pareto's terminology, Edgeworth dealt with utility *for* a society, not *of* a society. Thus, his definition of the problem of the collective utilitarian calculus is to "find the distribution of means and of labor and the quality and number of population, so that there may be the greatest possible happiness" (p. 56, paraphrased). We shall see in a moment that Edgeworth's utilitarianism was distinctly conservative (or hierarchic) but it was a problem-oriented, activist utilitarianism, one taking the central decision maker's or planners' point of view and not the optimality of markets *per se*. His was a utilitarianism for the statesman, a style of policy analysis which typified the late nineteenth century departure from the eighteenth century natural order approach to social phenomena.<sup>7</sup>

The second characteristic of Edgeworth's analysis of collective maximization was his insistence on individuals' differential capacity for happiness (p. 57; and also for work, pp. 59, 66), as well as the diminishing marginal util-

ity of means (p. 61). Accordingly, we find in the *Psychics* a conservative theory of economic policy. He would allocate the means of education to the highest in order of evolution, as the group most capable of improvement (pp. 68, 79). He would limit population differentially for each section of population in order to maximize the happiness of the next generation (pp. 69, 71). He would sacrifice the interests of the least favored class in the utilitarian community to the happiness of the higher classes (compare John Rawls!) (but the limit is somewhere above the starvation point, determined by reference to disincentive effects, the relation of diminishing marginal utility to privation, population control, and political instability (p. 75 and *passim*)). Finally, *inter alia*, the happiness of the present generation may have to be sacrificed to that of the future (pp. 74-75). Overall, he felt "that in order to the greatest possible sum total of happiness, the more capable of pleasure shall take more means, more happiness" (p. 10), thus "negating the assumption that *Equality* is necessarily implied in Utilitarianism. For, if sentiments differ in *Capacity for happiness* . . . there is no presumption that equality of circumstances is the most felicitous arrangement; especially when account is taken of the interests of posterity" (p. vii).

Edgeworth insists that the importance of each class is not necessarily in proportion to its numbers (p. 89n) and that true utilitarianism is not that each person counts for one but that each increment of marginal utility counts for one (p. 122). Unequal capacity means unequal distribution of means (pp. 124-125).

Edgeworth does insist that the final result may be aristocratic or egalitarian but that equality, although part of, is not the whole of distributive justice, weight having to be given to differences in the capacity for pleasure, considerations which enter his discussions of income, work, and the treatment of women (pp. 77ff). A critical question, of course, is the

relation of existing institutions to the "natural" distribution of such capacity. Edgeworth's answer with respect to women is clear: "Altogether, account being taken of existing, whether true or false, opinions about the nature of woman, there appears a nice consilience between the deductions from the utilitarian principle and the disabilities and privileges which hedge round modern womanhood" (p. 79; Edgeworth never married).

There are other defenses of inequality, of course, but Edgeworth's admits several ironies. First, insofar as differential capacity for happiness is institutionally determined (unequal distribution of means produces unequal capacity), Edgeworth's utilitarian calculus operates to reproduce social structure and its artificial differential capacity for happiness. Second, Arthur Cecil Pigou's later assumption that rich and poor likely had similar temperaments (that is, had roughly equal capacity to derive utility), involved a quite different variant of utilitarianism. After all, Bentham's greatest happiness principle had long posed the further problem whether the object to be maximized was the number or percentage of happy people or the happiness of those most intensively happy;<sup>8</sup> and the Benthamite left and right took quite different positions on this, the fundamental distributive issue. Third, the indifference curve used by Edgeworth to further his utilitarian analysis was later used to exorcise utilitarianism from economic analysis.<sup>9</sup>

Edgeworth's combination of constrained maximization, unorganized markets, and mathematics went through several reformula-

tions until it is now lauded as a promising alternative to neo-Walrasian model building<sup>10</sup>—if it is not already, through Samuelson, the principal and most distinguishing characteristic of economics although this writer, for one, believes that there is more than this to economics.

But there are several problems which have been, it is tempting to say, perennial in economics. Edgeworth's *Mathematical Psychics* is significant, in part, because these problems arose in modern form for the first time with its publication. Let me state the problems from the point of view of the "critic." First, neither Edgeworth nor many modern economists seem to have adequately recognized the limits of formalism with regard to description, explanation, and policy. Formal rationalism, while productive of deep, pointed theoretical insights, is unable uniquely to solve questions of policy. Second, Edgeworth's marriage of utilitarianism with a conservative, hierarchic view of social order seems to illustrate the ease with which utilitarianism can be combined with antecedent normative premises with regard to whose interests ought to or can count. Conversely, Edgeworth's felt necessity to resort to some version of the greatest happiness principle in order to reach determinate solutions to conflict problems suggests that determinate solutions are achieved only by introducing, explicitly or implicitly, channeling and prefiguring assumptions as to whose interests are to or can count. Third, Edgeworth's resort to the greatest happiness principle as an approach to collective utility maximization is (as I already have noted) an inadequate treatment of the methodologically collectivist (not normatively collectivist) forces at work with respect to social organization and structure.

It may be said that no economist really has

<sup>6</sup>See S. Todd Lowry, "Recent Literature on Ancient Greek Economic Thought," *Journal of Economic Literature*, vol. 17 (March 1979), pp. 65-86; and Keith Tribe, *Land, Labour and Economic Discourse* (London: Routledge & Kegan Paul, 1978), especially chapters 3 and 5.

<sup>7</sup>Ellen Frankel Paul, *Moral Revolution and Economic Science* (Westport: Greenwood Press, 1979).

<sup>8</sup>Overton H. Taylor, *A History of Economic Thought* (New York: McGraw-Hill, 1960), p. 134: "... leaving unanswered the question *which* is really to be maximized, the number or percentage of (more or less) happy people, or the intensity of the happiness of those most largely benefited."

<sup>9</sup>Henry W. Spiegel, *The Growth of Economic Thought* (Durham: Duke University Press, 1971), p. 526.

<sup>10</sup>E. Roy Weintraub, *Microfoundations* (New York: Cambridge University Press, 1979).

believed differently. Certainly, Edgeworth (and Pareto, for that matter) would have agreed, but he carried his utilitarianism so far, abetted by his conservative assumption of differential capacity for happiness, that the limits were easily obscured if not forgotten. Paradoxically, Edgeworth's utilitarianism implies (1) that atomistic individualism is inevitably informed by considerations (whatever they are) of social structure and (2) that economic analysis necessarily contains and should make explicit its normative premises (whatever they are).

To Edgeworth's credit, he made his normative premise explicit. That he considered differential capacity for happiness a *fact*, not a valuational premise, does not weaken that point very much. (In a different connection, Randall Bartlett<sup>11</sup> has shown that implicitly treating differentially or unequally situated persons equally may give unobserved effect to the actual inequality, whereas making the inequality of circumstance explicit enables quite different conclusions to be reached.) Rather, it suggests the difficulties which the discipline subsequently has had in identifying the normative premises, and their grounds, in its analysis.<sup>12</sup> Edgeworth subtitled his *Mathe-*

*matical Psychics* "An Essay on the Application of Mathematics to the Moral Sciences." He surely went too far in declaring that the limits of mathematics can "only . . . be decided by the authority and in the presence of Mathematics herself" (p. 3) and that "Mecanique Sociale' may one day take her place along with 'Mecanique Celeste,' throned each upon the double-sided height of one maximum principle, the supreme pinnacle of moral as of physical science" (p. 12). He did recognize, however, that the realization of the maximum sum-total of happiness was subject to constraints, including structural considerations (pp. 9, 11-12). To his credit, notwithstanding his conservatism, he recognized the applicability of technique "to the political struggle for power, as well as to the commercial struggle for wealth" (p. 16.). As to whether mathematical psychics has anything to teach with regard to the political (read: power) aspects of problems, Edgeworth said, "Nothing as to practical politics; but as to the first principles of political theory perhaps something" (p. 127; see also 133), and that "about a subject so illusory, where the vanity and the very virtues of our nature, oligarchical pride, democratical passion, perturb the measurements of utility; not slight the advantage of approaching the inquiry in the calm spirit of mathematical truth" (p. 134). Edgeworth surely overestimated that advantage, as he overestimated the non-normative character of the uses of mathematics in social science, but he was not totally captivated or blinded by his principle of the differential capacity for happiness.

<sup>11</sup>Randall Bartlett, *Economic Foundations of Political Power* (New York: Free Press, 1973). See Warren J. Samuels, Book Review, *Journal of Economic Issues*, vol. 10 (March 1976), pp. 181-185.

<sup>12</sup>See Warren J. Samuels, "Ideology in Economics," in Sidney Weintraub, ed., *Modern Economic Thought* (Philadelphia: University of Pennsylvania Press, 1977), pp. 467-484; and "Normative Premises in Regulatory Theory," *Journal of Post Keynesian Economics*, vol. 1 (Fall 1978), pp. 100-114.